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Dynamic Characteristics of Cylindrical Shells Considering FSI

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1. Introduction

There are many finite element models used for assessing the structural integrity of cylinders or shells with fluid-filled annulus. In the past the fluid region was not modeled explicitly and their mass was added to the structural mass as a form of an added mass for simplicity. In this case the fluid-structure interaction effect, so called annulus effect, is not considered in the analysis, generating the unrealistic or unconservative results in some cases. Therefore it is necessary to make a 3-dimensional model including fluid region and to couple two nodes which are assigned to the fluid and the structure. Fortunately commercial programs such as ANSYS [1] can model couplings between the fluid and the structure easily and can consider the annulus effect for various types of analyses very efficiently.

Therefore in this study, theoretical background and several finite element models are developed for coaxial cylindrical shells with fluid-filled annulus considering fluid-structure interaction. The effect of the inclusion of the fluid-filled annulus on the natural frequencies is investigated by comparing frequencies between various finite element models. Using the modal characteristics, typical dynamic analyses such as responses spectrum, power spectral density (PSD) and unit load excitation are performed and their response characteristics are addressed with respect to the various representations of the fluid-structure interaction.

2. Analysis

The equations of motion can be represented for the displacements x on the structure as:

$$\begin{bmatrix} m_{aa} & m_{ab} \\ m_{ba} & m_{bb} \end{bmatrix}\begin{bmatrix} \ddot{x}_a \\ \ddot{x}_b \end{bmatrix}+\begin{bmatrix} c_{aa} & c_{ab} \\ c_{ba} & c_{bb} \end{bmatrix}\begin{bmatrix} \dot{x}_a \\ \dot{x}_b \end{bmatrix} + \begin{bmatrix} k_{aa} & k_{ab} \\ k_{ba} & k_{bb} \end{bmatrix}\begin{bmatrix} x_a \\ x_b \end{bmatrix}=\begin{bmatrix} 0 \\ p_b \end{bmatrix} \tag{1}$$

where the displacements x_b are forced to vary in a defined manner with prescribed functions of time and p_b represents the column matrix of unknown forces causing the displacements x_b.

If the hydrodynamic couplings are considered in the analysis, the mass matrix is

$$M = M_s + M_h = \begin{bmatrix} m_{aas} & 0 \\ 0 & m_{bbs} \end{bmatrix}+\begin{bmatrix} m_{aah} & m_{ab} \\ m_{ba} & m_{bbh} \end{bmatrix} \tag{2}$$

where M_s and M_h are the structural and hydrodynamic mass matrices, respectively. Considering two mass points with hydrodynamic couplings, Eq. (1) becomes

$$\begin{bmatrix} m_{aas}+m_{aah} & m_{ab} \\ m_{ba} & m_{bbs}+m_{bbh} \end{bmatrix}\begin{bmatrix} \ddot{x}_a \\ \ddot{x}_b \end{bmatrix}=\begin{bmatrix} F_a \\ F_b \end{bmatrix} \tag{3}$$

where F_a and F_b are the spring and damping forces at nodes a and b defined as:

$$\begin{bmatrix} F_a \\ F_b \end{bmatrix}=\begin{bmatrix} -(c_{aa}\dot{x}_a + c_{ab}\dot{x}_b + k_{aa}x_a + k_{ab}x_b) \\ -(c_{ba}\dot{x}_a + c_{bb}\dot{x}_b + k_{ba}x_a + k_{bb}x_b) + p_b \end{bmatrix} \tag{4}$$

In a solution which is based on a direct integration of the equations of motion, the spring and damping forces are evaluated at each instant of time and then the accelerations are solved. Integration of the accelerations gives the velocities and displacements needed to reevaluate the accelerations for the next time step [2].

The hydrodynamic mass matrix can be calculated from the fluid velocity potential for the two long concentric cylinders separated by a gap filled with ideal and compressible fluid. The governing continuity equation is written for any instant [3] as:

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}=0 \tag{5}$$

where ϕ is the velocity potential, r and θ are the radial and angular coordinates. Applying the boundary conditions for the radial component of fluid velocity at $r = a$ and $r = b$ yields the following solution for ϕ:

$$\phi=\frac{b^2a^2}{b^2-a^2}\left\{\left(\frac{\dot{x}_j}{a^2}-\frac{\dot{x}_i}{b^2}\right)r+\frac{\dot{x}_j-\dot{x}_i}{r}\right\}\cos\theta \tag{6}$$

Therefore fluid forces on the cylinders are obtained by integrating fluid pressure in the annulus along the circumference, resulting in matrix form as:

$$M_h=\begin{bmatrix} m_1 f_c & -m_1(f_c+1) \\ -m_1(f_c+1) & m_2 f_c \end{bmatrix} \tag{7}$$

where $m_1(=\rho\pi a^2)$ is the mass of fluid displaced by the inner cylinder, m_2 $(=\rho\pi b^2)$ is the mass of fluid contained by the outer cylinder and f_c is the magnification factor for hydrodynamic mass depending on the size of the annulus defined as:

$$f_c = \frac{b^2 + a^2}{b^2 - a^2} \quad (8)$$

If the gap is infinite, the magnification factor f_c becomes zero and only off-diagonal term exists with the added mass of the inner cylinder as shown in Figure 1. If the gap is so small, the factor becomes infinite and the fluid force tends to cushion one cylinder from the other [4].

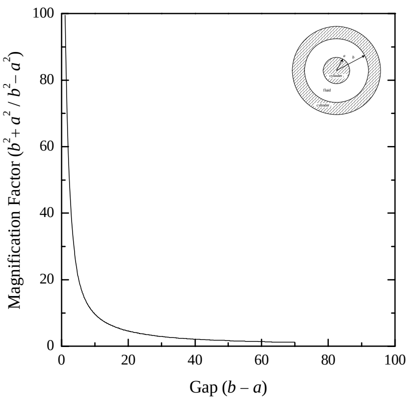

Fig. 1. Magnification factor of hydrodynamic mass

3. Results and Discussion

The maximum deflections and equivalent stresses are summarized from the response spectrum analysis. As indicated in the table, the responses decrease significantly when the fluid is not included in the model. If the fluid is considered as added mass only, or the fluid coupling effect is not considered in the model, the responses are generally lowered. Therefore it is concluded that not considering fluid coupling may give unconservative results, which should not be missed in the structural integrity assessment for the nuclear components.

The velocity PSDs at nodes for inner and outer shells are investigated, which shows that there are much differences of response PSDs between models. The amplitude and frequency depend on the modal characteristics which also depend on the existence of fluid and its modeling technique. The maximum deflections and equivalent stresses are summarized from the PSD analysis. As indicated in the table, the deflection and stress increase significantly when the fluid is included as an added mass in the model. If the fluid is not considered in the model, the responses are generally lowered.

The displacements at node where the pulse is applied and at node of the inner shell are investigated, which shows that there is a small difference of responses between models. The maximum deflections are summarized from the transient analysis. As indicated in the table, the deflection is almost the same irrespective of the fluid model. If the fluid is not considered in the model, the response after the pulse is applied decreases very rapidly. But if the fluid is considered, the response due to the pulse appears for a long time. As shown, there is no displacement at the node of the inner shell if the fluid is not included in the model because there is no load path from the outer shell where the pulse is applied to the inner shell via couplings between shell and fluid.

The same kind of transient analyses are performed for the 2-dimensional axisymmetric model. The displacements at corresponding nodes are investigated. By comparing displacements between 3-dimensional and 2-dimensional models, it is not clear that the 2-dimensional axisymmetric model can simulate the transient analysis.

The displacements and velocities from the harmonic analysis at node where the unit load is applied are investigated, which shows that there is a difference of responses between models due to the different modal characteristics. If the fluid is not considered in the model, the responses are generally lower than those for the model with fluid. But if the fluid is considered, the responses are almost the same irrespective of the fluid model representations such as added mass or fluid mass.

4. Conclusions

- The effect of fluid on the frequencies is more significant for out-of-phase mode and inner shell than in-phase mode and outer shell, respectively.
- Representing fluid by added mass gives higher frequencies for in-phase modes and lower frequencies for out-of-phase modes.
- Axisymmetric-harmonic element is found to be a very efficient one to investigate the modal characteristics, suggesting the use of this element instead of 3-dimensional element for modal analysis.
- Axisymmetric model is not recommended for the dynamic analysis except modal analysis.
- Not considering fluid coupling effect besides added mass for the response spectrum, PSD, transient and harmonic analyses may give unconservative results, which should not be neglected in the structural integrity assessment for the nuclear components.

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