Search Strategy of Detector Position For Neutron Source Multiplication Method by Using Detected-Neutron Multiplication Factor

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1. Introduction

In this paper, an alternative definition of a neutron multiplication factor, *detected-neutron multiplication factor* k_{det}, is produced for the neutron source multiplication method (NSM). By using k_{det}, a search strategy of appropriate detector position for NSM is also proposed.

The NSM is one of the practical subcritical measurement techniques, *i.e.*, the NSM does not require any special equipment other than a stationary external neutron source and an ordinary neutron detector. Additionally, the NSM method is based on steady-state analysis, so that this technique is very suitable for quasi real-time measurement. It is noted that the correction factors play important roles in order to accurately estimate subcriticality from the measured neutron count rates [1-3]. The present paper aims to clarify how to correct the subcriticality measured by the NSM method, the physical meaning of the correction factors, and how to reduce the impact of correction factors by setting a neutron detector at an appropriate detector position.

2. Methods and Results

2.1 Detected-Neutron Multiplication Factor

The steady state of subcritical system with external neutron source is assumed. In this subcritical system, neutron count rate $\langle \Sigma_{\mathrm{d}}\psi \rangle$ is measured by detector, where ψ is neutron flux, and Σ_{d} is macroscopic detection cross-section. Let us classify the neutron flux into two terms:

$$\psi(\vec{r},E,\vec{\Omega}) = \psi_{\mathrm{s}}(\vec{r},E,\vec{\Omega}) + \psi_{\mathrm{f}}(\vec{r},E,\vec{\Omega}), \qquad (1)$$

where ψ_{s} is source-flux due to the external source S:

$$\mathbf{A}\psi_{\mathrm{s}}(\vec{r},E,\vec{\Omega}) = S(\vec{r},E,\vec{\Omega}); \qquad (2)$$

ψ_{f} is fission-flux due to the fission source:

$$\mathbf{A}\psi_{\mathrm{f}}(\vec{r},E,\vec{\Omega}) = \mathbf{F}\psi(\vec{r},E,\vec{\Omega}). \qquad (3)$$

In Eqs. (2) and (3), **A** and **F** are neutron annihilation and production operators, respectively. Now, the detected neutron multiplication factor k_{det} is defined as follows:

$$k_{\mathrm{det}} \equiv \frac{\langle \Sigma_{\mathrm{d}}\psi_{\mathrm{f}} \rangle}{\langle \Sigma_{\mathrm{d}}\psi \rangle}, \qquad (4)$$

The physical meaning of k_{det} is a ratio of total number of detected fission-neutrons to total number of detected all neutrons.

By the aid of the definition of k_{det}, $\langle \Sigma_{\mathrm{d}}\psi \rangle$ is expressed as follows:

$$\langle \Sigma_{\mathrm{d}}\psi \rangle = \frac{\langle \Sigma_{\mathrm{d}}\psi_{\mathrm{s}} \rangle}{1-k_{\mathrm{det}}}. \qquad (5)$$

Let us suppose that neutron count rates are measured at both reference and target subcritical states. If the effective multiplication factor k_{eff} at the reference state is known beforehand, the effective multiplication factor k_{eff} at the target is estimated as follows:

$$f_{\mathrm{c,target}}\, k_{\mathrm{eff,target}} = 1 - f_{\mathrm{s}}\left(1 - f_{\mathrm{c,ref}}\, k_{\mathrm{eff,ref}}\right)\frac{\langle \Sigma_{\mathrm{d}}\psi_{\mathrm{ref}} \rangle}{\langle \Sigma_{\mathrm{d}}\psi_{\mathrm{target}} \rangle}, \qquad (6)$$

where the subscripts *ref* and *target* mean the values at reference and target subcritical states, respectively; f_{s} is a source-flux correction factor,

$$f_{\mathrm{s}} \equiv \langle \Sigma_{\mathrm{d}}\psi_{\mathrm{s,target}} \rangle \big/ \langle \Sigma_{\mathrm{d}}\psi_{\mathrm{s,ref}} \rangle; \qquad (7)$$

f_{c} is conversion factor from k_{det} to k_{eff},

$$f_{\mathrm{c}} = k_{\mathrm{det}} / k_{\mathrm{eff}}. \qquad (8)$$

It is noted that these factors f_{s} and f_{c} can be evaluated by only forward neutron flux calculations without adjoint calculations.

2.2 Search Strategy of Detector Position for NSM

If the neutron detector is set at an appropriate position where $k_{\mathrm{det}} \approx k_{\mathrm{eff}}$, the conversion factor f_{c} is nearly equal to unity, thus it is expected that f_{c} can be negligible. Based on this idea, the appropriate detector position is predicted from the numerical analysis at the reference state.

(1) k_{eff}-eigenvalue calculation is carried out to estimate the value of $k_{\mathrm{eff,ref}}$ at the reference state.
(2) External source problem is carried out under the actual fission-neutron production condition to estimate spatial distribution of ψ
(3) External source problem is carried out under the fictitious non-production conditions condition ($\nu\Sigma_{\mathrm{f}} = 0$) to estimate spatial distribution of source-flux ψ_{s}
(4) Fission-flux ψ_{f} is evaluated by subtracting ψ_{s} from ψ
(5) By supposing a point-wise neutron detector, count rates $\langle \Sigma_{\mathrm{d}}\psi \rangle$ and $\langle \Sigma_{\mathrm{d}}\psi_{\mathrm{f}} \rangle$ are evaluated at the detector position.
(6) The spatial distribution of $k_{\mathrm{det,ref}}$ is evaluated by Eq. (4)

(7) Appropriate detector position where $k_{\mathrm{det,ref}} \approx k_{\mathrm{eff,ref}}$ can be searched.

2.3 Calculation Model and Condition

The numerical analysis was carried out by using THREEDANT, which is a three-dimensional multi-group discrete ordinates transport code [4]. The calculation model and condition of numerical analysis are as follows:

(1) Homogeneous rectangular parallelepiped core. The dimension is 41 cm in *x*-direction, 33 cm in *y*-direction, and 49 cm in *z*-direction, respectively. The core is surrounded by a 20 cm thick reflector.
(2) Two energy group constants are quoted from reference [5]. Three subcritical cores (C30, C35, C40) are analyzed by changing the group constants of core region.
(3) Point-wise source at $(x, y, z) = (-31, 0, 0)$ in the reflector region. In addition, the energy spectrum of external source is the same as fission spectrum.
(4) Point-wise detector for only thermal neutron.
(5) Total number of spatial meshes is 81 × 73 × 89 for *x*-, *y*-, and *z*-directions, respectively.
(6) EO8 quadrature set is used for SN solid angle quadrature set [6].
(7) Convergence criteria is 1.0×10^{-6} for inner iteration.

2.4 Numerical Results

In order to search appropriate detector positions in two dimensions, Fig. 1 plots the relative difference between k_{det} and k_{eff} at $z = 0$ plane within the range of –0.5 % to 0.5%. In Fig.1, green colored regions can be regard as the detector positions of $k_{\mathrm{det}} \approx k_{\mathrm{eff}}$. As shown in Fig.1, the detector positions of $k_{\mathrm{det}} \approx k_{\mathrm{eff}}$ can be find out not only core regions but also reflector regions. Furthermore, these detector positions are quite stationary for subcritical states. For example, a point-wise detector is set at $(x, y, z) = (-9, 0, 0)$ (Fig. 1). As a numerical simulation of NSM, C35 subcritical state was regarded as the reference state, target multiplication factors $k_{\mathrm{eff,\ target}}$ for C30 and C45 states were estimated by NSM without correction factor f_s and f_c (Table I). By virtue of appropriate detector positions of $k_{\mathrm{det}} \approx k_{\mathrm{eff}}$, the neutron multiplication factors are well estimated even without corrections. In other word, by setting detector at the position where $k_{\mathrm{det}} \approx k_{\mathrm{eff}}$, the impact of correction factor can be reduced. Of course, if the conversion factors f_s and f_c at both reference and target states can be taken into account, the estimated multiplication factors are nearly equal to effective neutron multiplication factors $k_{\mathrm{eff,\ target}}$ at the target states.

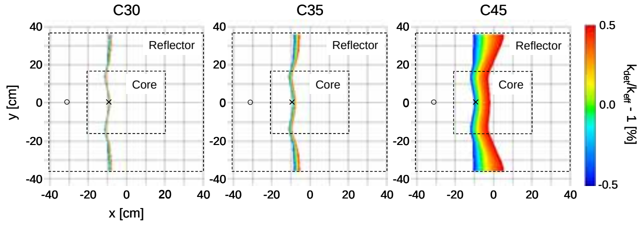

Fig. 1: Point-wise detector positions of $k_{\mathrm{det}} \approx k_{\mathrm{eff}}$ at $z = 0$ plane (circle and cross mean a point-wise source and a selected detector position)

Table I: Estimated multiplication factor by NSM

	C45	C35 (ref.)	C30
k_{eff} [-]	0.99273	0.96639	0.92311
k_{det} [-]	0.99121	0.96347	0.92452
f_s [-]	1.27162	1.00000	0.84264
f_c [-]	0.99847	0.99698	1.00152
estimated k by NSM without correction[-]	0.99364	0.96639	0.91758
rel. dif. [%dk/k]	0.09		-0.60

3. Conclusions

In this paper, based on the idea of k_{det}, it is clear that correlation factors of the NSM method consist of the sourc-flux correction factor f_s to fix the difference of non-fission component of neutron count rate, and the conversion factor f_c to convert from k_{det} to the effective multiplication factor k_{eff}. Through the numerical analysis of three-dimensional two-group transport calculations for simple geometry, appropriate detector positions where $k_{\mathrm{det}} \approx k_{\mathrm{eff}}$ could be find out for only one certain subcritical state, *e.g.*, the reference state. By putting a neutron detector at such a particular position, the target neutron multiplication factors can be well estimated even without any corrections.

REFERENCES

[1] T. Mukaiyama, *et al.*, Reactivity Measurement in a Far-Subcritical Sast System [II] Neutron source multiplication method, JAERI-M 6067, Japan Atomic Energy Research Institute (JAERI), 1975.
[2] N. Mizoo, Reliability of a New Modified Neutron Source Multiplication Method, JAERI-M 7135, Japan Atomic Energy Research Institute (JAERI), 1977.
[3] D. G. Cacuci, Handbook of Nuclear Engineering: Vol.3 Reactor Analysis, Springer, pp. 2147-2152, 2010.
[4] R. E. Alcouffe, *et al.*, DANTSYS: A Diffusion Accelerated Neutral Particle Transport Code Systemm, Los Alamos National Laboratory report LA-12969-M, 1995.
[5] T. Misawa, H. Unesaki, C. Pyeon, Nuclear Reactor Physics Experiments, Kyoto University Press, 2010.
[6] T. Endo, A. Yamamoto, Development of New Solid Angle Quadrature Sets to Satisfy Even- and Odd-moment Conditions, *J. Nucl. Sci. Technol.*, **44**, pp. 1249-1258, 2007.