Spatial Variation of Hydrodynamic Mass Coefficients for Tube Bundle in a Cylindrical Shell

Keum-Hee Yang [a], Ki-Wahn Ryu [a*], Chi-Yong Park [b]
[a]*Department of Aerospace Engr., Chonbuk National Univ.,664-14 Deokjin 1-Ga, Jeonju, Jeonbuk 561-756, Korea*
[b]*KEPCO Research Institute, 103-16 Munji-Dong, Yuseong-Gu, Daejeon 305-380, Korea*
[*]*Corresponding Author: kwryu@chonbuk.ac.kr*

1. Introduction

Wear of the steam generator (SG) tubes affects the performance of nuclear power plants. Generally, the problem is caused by excessive flow-induced vibration (FIV). In analyzing the FIV, many researchers have used a uniform added mass coefficient for all of the SG tubes. However, the outermost SG tubes have more structural problems than inside tubes. The purpose of this study is to find out the added mass coefficients of each tube in a cylindrical shell.

2. Methods and Results

2.1 Formulation of added mass coefficients

The fluid is assumed to be incompressible, inviscid and irrotational; thus, the potential flow theory is applied. The cylinders are assumed to be infinitely long and their axes are parallel to one another; i.e., the two dimensional problem is solved [1]. Laplace's equation can be used to accurately describe the behavior of fluid potential. The velocity potential is expressed in terms of a series with unknown coefficients [2, 3]. Unknown expansion coefficients of the series formulation are determined by matrix inversion of a truncated set of infinite equations obtained by imposing the prescribed boundary conditions. These coefficients are then used to calculate fluid pressure and hydrodynamic forces acting on each cylinder.

Motions of a group of J circular cylinders vibration in an ideal incompressible fluid are considered, as shown in Fig. 1. The axes of the cylinders are perpendicular to the x - y plane. The hydrodynamic force acting i -cylinder can be written as follow.

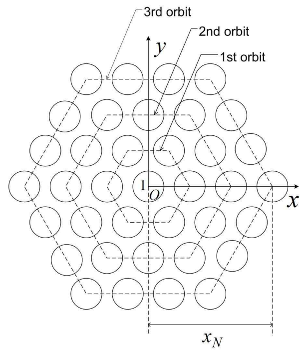
$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \begin{bmatrix} m_{xx} & m_{xy} \\ m_{yx} & m_{yy} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial^2 x}{\partial t^2} \\ \dfrac{\partial^2 y}{\partial t^2} \end{Bmatrix} \tag{1}$$

Where, bracket [ ] denotes the added mass matrix. Briefly, Eq. (1) can be written $F_i = m_{ij}\ddot{x}_j$. Let R be the radius of cylinder i . The added mass is proportional to cross-section area of cylinder. The added mass coefficients can be combined into a single added mass matrix m_{ij} .

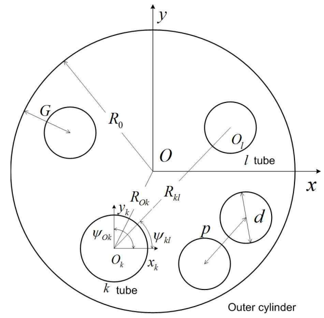
$$m_{ij} = \rho\pi R^2 \begin{bmatrix} \alpha & \sigma \\ \tau & \beta \end{bmatrix} \tag{2}$$

Considering the motion of a group of J circular cylinders in a outer cylindrical shell. A added mass matrix associated with the motion of cylinder k due to the acceleration of cylinder l, assuming all other cylinders are stationary, can be written.

$$m_{ij} = \rho\pi R^2 \begin{bmatrix} \alpha_{kl} & \sigma_{kl} \\ \tau_{kl} & \beta_{kl} \end{bmatrix} \tag{3}$$



(a) Triangular tube bundle (3-orbit around the No. 1 tube)



(b) View of SG tubes and outer cylinder

Fig. 1 Schematic diagram for tube bundle

In Eq. (3), added mass coefficients can be written as Eq. (4). α_{ii}, β_{ii}, σ_{ii}, and τ_{ii} are self-added mass coefficients, which are proportional to the hydrodynamic force acting on cylinder i due to its own acceleration, while the others are mutual-added mass coefficients, which are proportional to the hydrodynamic force acting on a cylinder due to the acceleration of another cylinder.

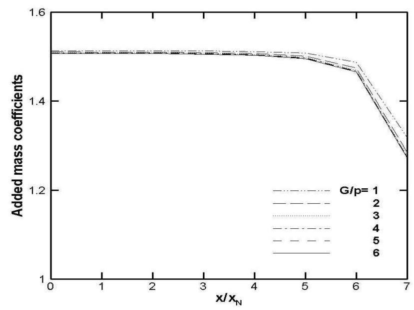
Because of a general SG tube behavior on in-plane or out-of-plane mode, actually σ_{kl} and τ_{kl} are in no use. Then α_{kl} and β_{kl} are similar at inside SG tubes, but different at outermost tubes.

$$
\begin{aligned}
\alpha_{kl} &= -\alpha_{k1l} - \sum_{J=1}^{\infty} J\left(\frac{R_{0k}}{R_0}\right)^{J-1}\left\{\alpha_{0Jl}\cos(J-1)\psi_{0k} + \tau_{0Jl}\sin(J-1)\psi_{0k}\right\} - \sum_{J=1}^{\infty}\sum_{j=1}^{k}{}^{*}(-1)^{j} J\left(\frac{R_j}{R_{kj}}\right)^{J+1}\left\{\alpha_{jJl}\cos(J+1)\psi_{kj} + \tau_{jJl}\sin(J+1)\psi_{kj}\right\} \\
\beta_{kl} &= -\beta_{k1l} - \sum_{J=1}^{\infty} J\left(\frac{R_{0k}}{R_0}\right)^{J-1}\left\{-\sigma_{0Jl}\sin(J-1)\psi_{0k} + \beta_{0Jl}\cos(J-1)\psi_{0k}\right\} - \sum_{J=1}^{\infty}\sum_{j=1}^{k}{}^{*}(-1)^{J} J\left(\frac{R_j}{R_{kj}}\right)^{J+1}\left\{\sigma_{jJl}\sin(J+1)\psi_{kj} - \beta_{jJl}\cos(J+1)\psi_{kj}\right\} \\
\sigma_{kl} &= -\sigma_{k1l} - \sum_{J=1}^{\infty} J\left(\frac{R_{0k}}{R_0}\right)^{J-1}\left\{\sigma_{0Jl}\cos(J-1)\psi_{0k} + \beta_{0Jl}\sin(J-1)\psi_{0k}\right\} - \sum_{J=1}^{\infty}\sum_{j=1}^{k}{}^{*}(-1)^{J} J\left(\frac{R_j}{R_{kj}}\right)^{J+1}\left\{\sigma_{jJl}\cos(J+1)\psi_{kj} + \beta_{jJl}\sin(J+1)\psi_{kj}\right\} \\
\tau_{kl} &= -\tau_{k1l} - \sum_{J=1}^{\infty} J\left(\frac{R_{0k}}{R_0}\right)^{J-1}\left\{-\alpha_{0Jl}\sin(J-1)\psi_{0k} + \tau_{0Jl}\cos(J-1)\psi_{0k}\right\} - \sum_{J=1}^{\infty}\sum_{j=1}^{k}{}^{*}(-1)^{j} J\left(\frac{R_j}{R_{kj}}\right)^{J+1}\left\{\alpha_{jJl}\sin(J+1)\psi_{kj} - \tau_{jJl}\cos(J+1)\psi_{kj}\right\}
\end{aligned}
\quad (4)
$$

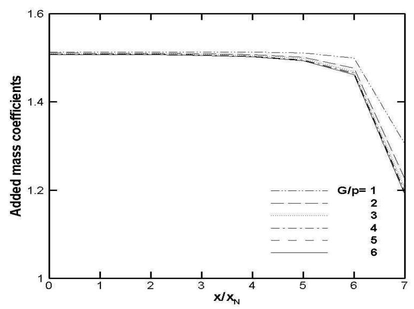
2.2 Numerical results

Assuming that 169 tubes are in a fluid-containing cylindrical shell, added mass coefficients according to gap changes between outermost tubes and cylindrical shell are considered. Let G be the difference between the radius of cylindrical shell (R_0) and the center location of outermost tube along x-direction (x_N) as shown in Fig. 1.

Fig. 2 shows the added mass coefficients as G changes ($p/d = 1.33$). All added mass coefficients are decreased with increasing G/p. Added mass coefficients (α_{11}, β_{11}) of central tubes do not differ significantly with increasing G/p. On the other hand added mass coefficients of outermost tubes differ more. It is required to use the specific added mass coefficient according to mode shape.



(a) α_{kk}



(b) β_{kk}

Fig. 2 Added mass coefficients for $p/d = 1.33$

Relative error are listed in Table 1 according to G/p in order to compare added mass coefficients of the central tube with those of the outermost tube. Relative error can be expressed as follows:

$$\varepsilon_{\alpha} = \frac{\alpha_{11} - \alpha_{NN}}{\alpha_{11}} \times 100\ (\%) \quad (5)$$

$$\varepsilon_{\beta} = \frac{\beta_{11} - \beta_{NN}}{\beta_{11}} \times 100\ (\%) \quad (6)$$

As G/p is increased, relative error is increased. When G/p has very large value, ε_{α} and ε_{β} are respectively 15.7 % and 21.3 %. It shows that the added mass coefficient is asymptotically converged to the value of the tube array in a free fluid field.

Table 1 Relative errors according to G/p

G/p	1	2	3	4	5	6	∞
ε_{α}	12.9	14.9	15.4	15.5	15.6	15.6	15.7
ε_{β}	13.7	18.9	20.2	20.7	20.9	21.0	21.3

3. Conclusions

As potential flow theory is assumed, a numerical study was performed to analyze the distribution of added mass coefficients of SG tubes, which are arranged for triangular type, according to gap changes between outermost tubes and cylindrical shell. The following conclusions are obtained.

From the study of 169 cylinders in a fluid-containing cylindrical shell, it is seen that all added mass coefficients are decreased with increasing G/p, which is gap between outermost tubes and cylindrical shell. Relative errors between central tube and outermost tube are generally converged with increasing G/p. Besides, added mass coefficients of the central tube are equal in x and y directions. But the outermost tube has different added mass coefficients in x and y directions.

ACKNOWLEGEMENTS

This work was supported by the Power Generation & Electricity Delivery of the Korea Institute of Energy Technology Evaluation and Planning (KETEP) grant funded by the Korea Government Ministry of Knowledge Economy (No. 2009T100100644).

REFERENCES

[1] S. S. Chen and Ho Chung, “Design Guide for Calculating Hydrodynamic Mass Part Ⅰ: Circular Cylindrical Structure”, Argonne National Lab., Ill. (USA), 1976
[2] S. S. Chen, “Vibration of a Row of Circular Cylinders in a Liquid”, *Journal of Engineering for Industry*, *Trans. ASME*, Vol. 97, 1975, pp. 1212-1218
[3] S. S. Chen, “Vibration of Nuclear Fuel Bundles”, *Nucl. Eng. Des*., Vol. 35, 1975, pp. 399-422