

Single-Phase Flow and Heat Transfer Analysis of a Helical Coil Tube Using CUPID

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1. Introduction

Helical coil steam generators are used in many small modular reactor designs, including SMART, i-SMR, and NuScale [1]. In an integral layout the steam generator has to fit in between the core and the reactor vessel wall, and a coiled tube bundle provides a large heat transfer area within that space. The coil also accommodates thermal expansion without additional supports, and the once-through secondary side produces superheated steam without a steam drum. Many of these designs operate under natural circulation, so the pressure drop across the steam generator directly affects the primary flow rate.

The curvature of the tube creates a centrifugal force changing the flow cross-section, forming pair of counter rotating vortices in the plane normal to the flow. This secondary motion, known as Dean flow, increases the wall shear stress, thins the thermal boundary layer near the outer wall, and delays transition to turbulence well beyond the straight tube value.

Two of the affected quantities are the Darcy friction factor and the Nusselt number. Both are higher in a coiled tube compared to a straight tube of the same diameter, and the increase depends on the Dean number, which combines the flow rate with the curvature ratio. Separate correlations have been developed for helical tubes in both cases. A code used for helical coil steam generator analysis needs to reproduce both. In this work CUPID code is used [2]. It is a component-scale thermal-hydraulic code developed at KAERI for three-dimensional two-phase flow in reactor components such as the reactor vessel, the steam generator and the containment. The code solves two-fluid three-field conservation equations using a cell-centered finite volume method with a semi-implicit time advancement.

This paper reports a single-phase analysis of CUPID for a helical tube based on the steam generator geometry of the SMART integral test loop. An O-type grid mesh was generated for the coiled tube and a grid sensitivity study was carried out. Fifteen cases were then computed for the friction factor and for the Nusselt number with isothermal wall boundary conditions, covering the laminar and the turbulent regime. Both quantities are compared with correlations developed for helical tubes with straight-tube correlations included as a baseline to show that the curvature effect is reproduced.

2. Methodology

2.1. Geometry and computational domain

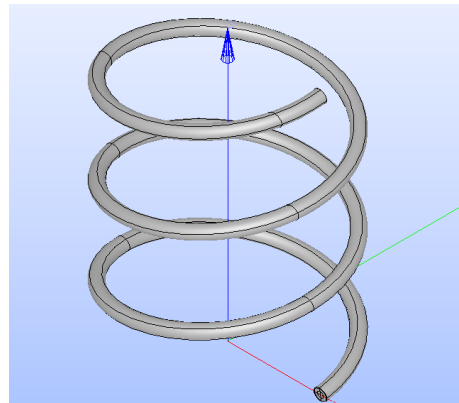


Fig. 1. Helical coil geometry.

The computational domain shown in the Fig. 1. is a single helically coiled tube based on the steam generator of the SMART integral test loop [1]. The tube inner diameter is 14.62 mm and the coil diameter is 215 mm, which gives a curvature ratio d/D of 0.068. The domain covers approximately three turns, for a total arc length of 2.05 m.

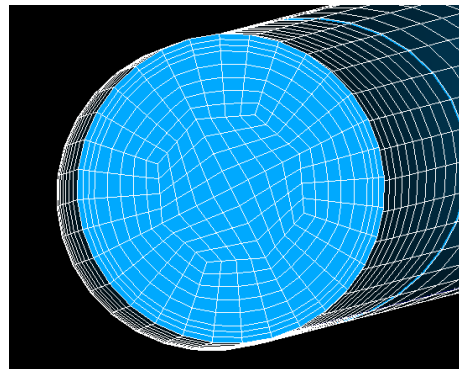


Fig. 2. O-type grid cross-section

The tube was meshed with an O-type grid, which is shown in the Fig. 2. A block of hexahedral cells occupies the core of the cross-section, and the region between that block and the tube wall is filled with rings of cells

that follow the perimeter. The cells next to the wall are aligned with it and are graded in the radial direction. This arrangement avoids the cell degeneracy that occurs at the center of a polar mesh. The cross-section pattern was then extruded along the helical tube axis, so that every cell in the domain is a hexahedron.

2.2. Numerical model

All calculations were run with the liquid only, so the two-fluid formulation reduces to a single-phase calculation. Water properties were evaluated from steam tables at a system pressure of 10 MPa.

The standard k-epsilon model with wall functions was used for the turbulent cases. The laminar cases were run with the turbulence model switched off. Convective terms are discretized to second order, and the Rhie-Chow scheme is used for the interpolation of the velocity at the cell faces.

The domain was initialized at 10 MPa and 473 K. A uniform velocity and a fixed temperature of 473 K were imposed at the inlet. At the outlet the pressure was fixed at 10 MPa and a Neumann condition was applied to the remaining variables. The friction cases were run without heat transfer. The heat transfer cases use an isothermal wall boundary condition set to 493 K, which gives a wall to inlet temperature difference of 20 K. Each case was run until the pressure drop and the outlet temperature stopped changing and simulations reached the steady state.

2.3. Analyzed parameters and reference correlations

Two quantities were selected for the validation, the Darcy friction factor and the Nusselt number. They are the standard measures of pressure drop and heat transfer in a tube, and correlations for coiled tubes are available for both.

Each quantity is compared with a different correlation in the laminar and in the turbulent regime, so the regime boundary has to be fixed first. In a coil the secondary flow suppresses the growth of disturbances and transition occurs later than in a straight tube. The critical Reynolds number is taken from Ito [3]

$$Re_{crit} = 2 \times 10^4 \delta^{0.32} \quad (1)$$

which gives 8460 for the present curvature ratio, against about 2300 for a straight tube.

The Darcy friction factor measures the hydraulic resistance of the tube. It expresses the pressure lost per unit length as a fraction of the dynamic pressure of the flow, scaled by the tube diameter, so that tubes of different size carrying different flow rates can be compared on the same basis. For fully developed flow it is defined as:

$$f = \frac{2d(dp/ds)}{\rho v^2} \quad (2)$$

where s is the coordinate along the tube axis, v is the bulk velocity and dp/ds is the pressure gradient along

the tube. An equivalent form in terms of the wall shear stress is $f = 8 \tau_w / (\rho v^2)$.

The calculated friction factor is compared with two correlations for helically coiled tubes. In the laminar regime the correlation of White [4] is used, written as a ratio to the straight-tube value f_s :

$$\frac{f}{f_s} = \left\{ 1 - \left[1 - \left(\frac{11.6}{De} \right)^{0.45} \right]^{1/0.45} \right\}^{-1} \quad (3)$$

which applies for $De > 11.6$ and returns the straight-tube value below that. In the turbulent regime the correlation of Mishra and Gupta [5] is used:

$$f = 0.316 Re^{-0.25} + 0.03 \left(\frac{d}{D} \right)^{1/2} \quad (4)$$

which is validated for $4.5 \times 10^3 < Re < 10^5$.

The Nusselt number measures the heat transfer at the wall. It is the ratio of the heat carried away by the moving fluid to the heat that conduction alone would carry across a stationary layer of the same thickness. For a given heat load and temperature difference it fixes the tube length required. It is defined as:

$$Nu = \frac{hd}{k} \quad (5)$$

where h is the heat transfer coefficient at the wall and k is the thermal conductivity of the fluid evaluated at the bulk temperature.

The calculated Nusselt number is compared with the correlation of Mori and Nakayama [6] for turbulent flow in a coiled tube,

$$Nu = \frac{Pr^{0.441} Re^{5/6} \delta^{1/12}}{41} \left[1 + \frac{0.061}{(Re \delta^{2.5})^{1/6}} \right] \quad (6)$$

In the laminar regime the calculated Nusselt number is compared with the correlation of Manlapaz and Churchill [7] for a constant wall temperature:

$$Nu = \left[\left(3.657 + \frac{4.343}{a} \right)^3 + 1.158 \left(\frac{De}{b} \right)^{3/2} \right]^{1/3} \quad (7)$$

with $a = [1 + 957/(De^2 Pr)]^2$ and $b = 1 + 0.477/Pr$. As De approaches zero this expression returns 3.657, which is the straight-tube laminar value under the same boundary condition.

Straight-tube correlations are used as a baseline, so that the curvature enhancement can be separated from the resistance and the heat transfer that any tube would produce. In the laminar regime the straight-tube friction factor is $f_s = 64/Re$. In the turbulent regime the correlation of Blasius is used:

$$f_{Blasius} = 0.3164 Re^{-0.25} \quad (8)$$

For the Nusselt number the turbulent baseline is the correlation of Gnielinski [8]:

$$Nu = \frac{(f/8)(Re - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)} \quad (9)$$

with $f = (0.79 \ln Re - 1.64)^{-2}$, stated for $3000 \leq Re \leq 5 \times 10^6$ and $0.5 \leq Pr \leq 2000$.

2.4. Calculation of parameters

All parameters were taken from when the simulation reached steady case in a converged case. Density, viscosity and thermal conductivity were taken from the calculated output fields, so the Reynolds number and the correlations are evaluated with the same properties as the calculation itself.

The friction factor was obtained from the axial pressure gradient. The pressure was area-averaged on cross-sections distributed through the tube and a straight line was fitted to these values against the axial coordinate. The gradient along the tube follows from the slope of that line as:

$$\frac{dp}{ds} = \left(\frac{dp}{dz} \right) \sin \alpha \quad (10)$$

where α is the helix angle, and f follows from (2). The coefficient of determination of the fit is recorded for each case, because a developed flow gives a linear pressure profile and any departure from a straight line indicates that the window still contains entrance effects.

For heat transfer, the bulk temperature at a cross-section was obtained as a mass-flux weighted average of the liquid temperature, and the mass flow rate as the integral of the mass flux over the same section. The bulk conductivity was averaged with the same weight and is the value used in (5).

The heat transfer coefficient was obtained by two independent methods. The first is an energy balance over the whole tube. The heat gained by the fluid was compared to the heat transferred through the wall using the logarithmic mean temperature difference,

$$\Delta T_{lm} = \frac{(T_w - T_{b,in}) - (T_w - T_{b,out})}{\ln [(T_w - T_{b,in}) / (T_w - T_{b,out})]} \quad (11)$$

so that $h = \dot{m} c_p (T_{b,out} - T_{b,in}) / (A_w \Delta T_{lm})$. The wall area A_w was computed as $\pi d \Delta s$ from the arc length.

The second method uses the temperature wall function of Kader [9]. The wall cells were selected from the three-dimensional field using the boundary condition written by the code, and were then ordered by arc length along the helix. The wall temperature profile is expressed as:

$$T^+ = Pr y^+ e^{-\Gamma} + \left(\frac{Pr_t \ln y^+}{\kappa} + \beta \right) e^{-1/\Gamma} \quad (12)$$

with $\beta = (3.85 Pr^{1/3} - 1.3)^2 + 2.12 \ln Pr$ and $\Gamma = 0.01 (Pr y^+)^4 / (1 + 5 Pr^3 y^+)$, taking $Pr_t = 0.9$ and $\kappa = 0.41$. The Prandtl number in (12) is evaluated at the wall. The wall heat flux follows as:

$$q_w = \frac{\rho c_p u_\tau (T_w - T_P)}{T^+} \quad (13)$$

where T_P is the liquid temperature in the wall cell, and the heat transfer coefficient is referred to the local bulk temperature, $h = q_w / (T_w - T_b)$.

Those two methods are independent. The energy balance uses the temperature rise of the fluid and the wall function uses the flux at the wall.

2.5. Mesh sensitivity study

Mesh study was performed on three grids of the same O-type grid topology, with 90 900, 181 440 and 443 520 elements. The refinement ratio is 1.26 between the first two levels and 1.35 between the second and the third. A case at 1 m/s, corresponding to $Re = 9.8 \times 10^4$, was re-computed on each grid, and the Nusselt number obtained from the calculations was used as the indicator. Results are shown in Table I.

Table I: Mesh sensitivity

Mesh	Elements	Refinement ratio	y^+ range	Nu	Change [%]
Coarse	90 900	-	42 - 78	306.3	-
Medium	181 440	1.26	47 - 78	300.1	-2.03
Fine	443 520	1.35	44 - 78	301.4	+0.41

The Nusselt number obtained on the coarse grid is 306.3. Increasing the number of elements to 181 440 lowers it to 300.1, a change of -2.03 %. Increasing the count further to 443 520 gives 301.4, a change of only +0.41 % compared to the medium grid. The second refinement changes the result by a factor of five less than the first, even though it adds more elements in absolute terms than the first refinement did. The wall-adjacent cell remains in the logarithmic layer on all three grids, with y^+ between 42 and 78, so the near-wall treatment is the same in every case and the differences reflect the resolution of the interior of the tube rather than a change in the wall model.

The solution achieved grid independent at the medium level. The medium grid resolves the cross-section and the streamwise development of the flow within half a percent of the fine grid, at less than half of its cost, and it was used for all remaining cases.

3. Results and discussion

Twenty-seven cases were computed using the same geometry and operating conditions. Fifteen of them are adiabatic and provide the friction factor, seven in the laminar regime and eight in the turbulent regime, covering

Reynolds numbers from 188 to 4.7×10^5 and Dean numbers from 49 to 1.2×10^5 . The remaining twelve carry the isothermal wall boundary condition and provide the Nusselt number, five in the laminar regime and seven in the turbulent regime, covering Reynolds numbers from 1.0×10^3 to 4.9×10^5 . The velocity was varied between 0.002 and 5 m/s.

3.1. Friction factor results

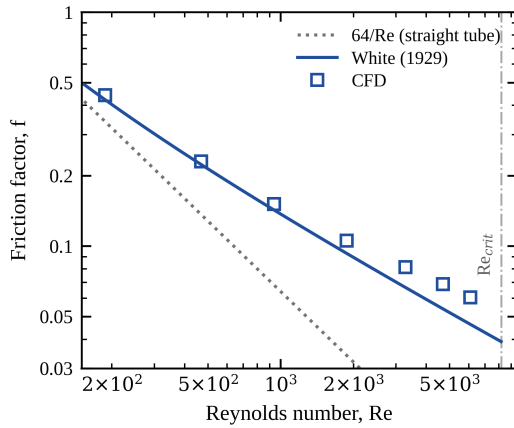


Fig. 3. Darcy friction factor against Reynolds number in the laminar regime.

The laminar results are shown in Fig. 3. Every case lies above the straight-tube value of $64/Re$, so the additional resistance produced by the secondary flow is reproduced. The enhancement grows from 1.29 at FL1 to 5.75 at FL7. The match with White depends on the Dean number. The three lowest cases, FL1 to FL3 at De below 250, vary by 4.6 % on average. Above that the calculated values rise above the correlation, reaching +13.5 % at FL4, +22.1 % at FL5 and +30.4 % at FL7. The departure is monotonic in the Dean number rather than scattered between cases, and it grows with the increase of the curvature effect. This points to a systematic cause rather than to the numerical uncertainty of individual runs.

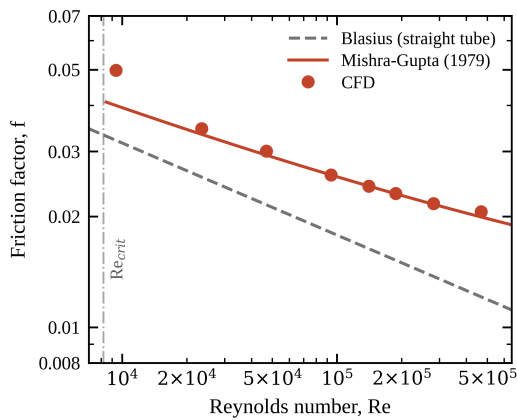


Fig. 4. Darcy friction factor against Reynolds number in the turbulent regime.

The turbulent results are shown in Fig. 4. The agreement with Mishra and Gupta is close over most of the range. Cases FT4 to FT7, spanning Re from 9.4×10^4 to 2.8×10^5 , deviate by less than 0.6 %, and FT2 and FT3 by less than 3.7 %. The enhancement over the Blasius value grows from 1.35 to 1.70 across the range.

Both ends of the range are worse, and both fall outside the conditions for which the correlation was stated. FT1 at $Re = 9377$ over-predicts by 24.6 %. It sits at 1.14 times the critical Reynolds number, where the flow is still developing towards fully turbulent behavior and neither branch of the correlation applies cleanly. FT8 at $Re = 4.7 \times 10^5$ over-predicts by 3.3 % and lies almost five times above the upper Reynolds number limit of 10^5 , so the comparison there tests the extrapolation of the correlation as much as the calculation. The turbulent branch is reproduced to 4.4 % over the full range and to 2.1 % within the stated range of the correlation. The laminar branch is reproduced to 9.9 % up to $De = 1000$ and to 15.3 % over the full range, the difference between the two figures coming entirely from the three highest Dean numbers.

3.2. Nusselt number results

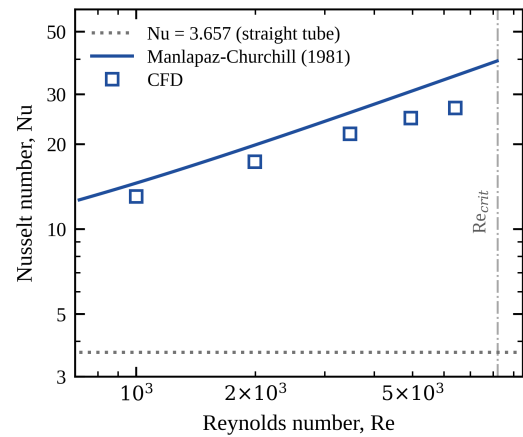


Fig. 5. Nusselt number against Reynolds number in the laminar regime.

The laminar results are shown in Fig. 5. Every case lies well above the straight-tube value of 3.657, by a factor of 3.56 at L1 rising to 7.34 at L5, so the heat transfer enhancement produced by the secondary flow is reproduced. All five cases underpredict Manlapaz and Churchill, and the deviation grows steadily with the Dean number, from -10.3 % at L1 to -23.1 % at L5. The mean absolute error is 16.2 %. As in the laminar friction branch the departure is monotonic in the Dean number rather than scattered between cases. The calculation overpredicts the resistance and underpredicts the heat transfer, with both errors growing as the curvature effect increases.

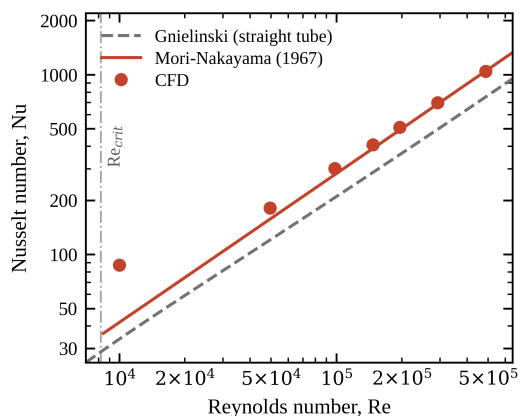


Fig. 6. Nusselt number against Reynolds number in the turbulent regime.

The turbulent results are shown in Fig. 6. The calculated values approach Mori and Nakayama as the Reynolds number increases, from +15.2 % at T2 through +5.2 % at T4 to -0.3 % at T7. The mean absolute error is 5.7 % over the six cases above the transition region and 2.7 % for the four cases at Re above 10^5 . The enhancement over the Gnielinski value is between 1.39 and 1.51 across the cases. T1 is not included in these figures. At $Re = 10\,004$ it is very close to the critical Reynolds number and it is in the near transition period, its wall cells are in the buffer layer rather than in the logarithmic layer where the wall function applies, and the fluid leaves the domain within 1.3 % of the wall temperature, so the driving temperature difference at the outlet is too small to resolve. It overpredicts the correlation by 108 % and is excluded from the statistics.

4. Conclusion

The CUPID code was used for analysis a helically coiled tube based on the SMART integral test loop steam generator geometry. The single-phase friction factor and Nusselt number were compared with correlations for coiled tubes over Reynolds numbers from 188 to 4.9×10^5 .

The calculation reproduces the enhancement caused by the curvature in both regimes and for both quantities. Every case lies above the corresponding straight-tube value, and the enhancement grows with the Dean number, reaching a factor of 5.75 in friction and 7.34 in heat transfer at the highest laminar Dean number.

In the turbulent regime the friction factor agrees with Mishra and Gupta to 2.1 % within the stated range of the correlation and to 4.4 % over all eight cases. The Nusselt number agrees with Mori and Nakayama to 5.7 %, improving to 2.7 % above $Re = 10^5$. The calculated Nusselt number follows $Re^{0.77}$ against $Re^{0.83}$ in the correlation, so the two converge as the Reynolds number rises and the close agreement at the top of the range follows from that convergence rather than from uniform accuracy.

In the laminar regime the friction factor agrees with

White to 9.9 % up to $De = 1000$ and to 15.3 % over the full range, and the Nusselt number agrees with Manlapaz and Churchill to 16.2 %. Both deviations grow monotonically with the Dean number, the calculation overpredicts the resistance and underpredicts the heat transfer. Both deviations are closer to the curvature term rather than the straight-tube baseline. The validity of the two correlations at high Dean number and the extent of flow development within the sampled length are both to be improved.

The laminar heat transfer cases are limited by their long flow time. The fluid approaches the wall temperature before the outlet, which leaves a small temperature difference and makes these values less resolved than the turbulent ones. A shorter geometry or a smaller temperature difference would remove this constraint.

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