Feasibility Study on a Hybrid Singular Value Decomposition–Gaussian Process Regression Approach for Reactor 3D Power Reconstruction

방민혁 (Minhyeok Bang)

Department of Nuclear and Quantum Engineering
Korea Advanced Institute of Science and Technology (KAIST)

KNS Autumn Meeting 2026

28th October - 30th October

Table of contents

1. **Backgrounds of the study**

- Target system for replacement– COLSS/CPCS
- A Korean successor – ICOPS
- Research proposal
- New approach – temperature integration & basis extraction & GPR

2. **Study Design**

- SVD-based compact power distribution representation
- Target reactor: ATOM
- Training dataset generation
- GPR inputs: ex-core detector and CETs
- 3 steps of Gaussian process regression (GPR) model
- A comparison: ANN model structure

3. **Numerical results**

- Performance benchmark on LFO scenario
- Performance benchmark on extreme scenario – whole CR malfunction

4. **Conclusions and Future works**

What are COLSS and CPCS?

COLSS (Core Operating Limit Supervisory System) & CPCS (Core Protection Calculator System)

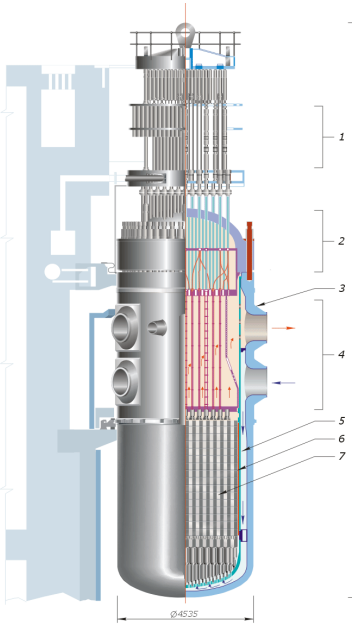
- Digital computer system for core monitoring and protection of 'System 80' plants, **still used in APR-1400**
- Developed by Combustion Engineering Inc., **patented by WEC.**



Core monitoring system (COLSS)

Aids the plant operator in monitoring selected Limiting Conditions for Operation (LCOs).

Does not provide any direct protection functions.





Core protection system (CPCS)

In the case of **design basis events, automatically initiate the trip** sequence to protect the reactor.

More conservative in DNBR/LPD calculations.[1)]

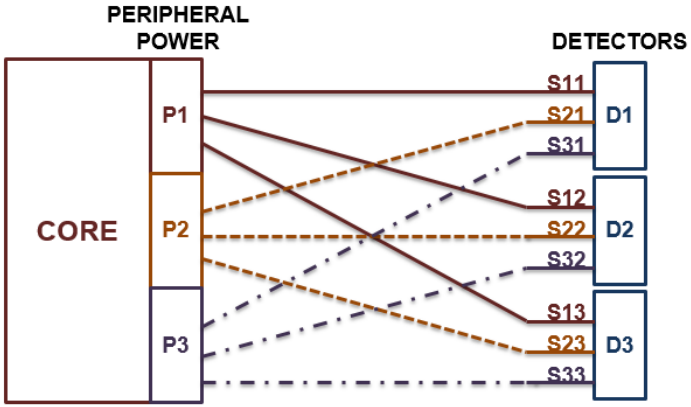
What are the limitations of COLSS and CPCS?

Limitations of COLSS (Core Operating Limit Supervisory System)

- It relies on rhodium-based in-core self-powered-neutron-detectors (SPNDs).
- *Rhodium-based SPNDs response may be delayed up to 5 mins[2] → **unsuitable for protection uses.**
  * Cobalt-based prompt SPNDs suffer γ noise, lifetime is also limited by the insulation integrity.[3]
- SPND's are costly and require regular maintenance/replacements.

Limitations of CPCS (Core Protection Calculator System)

- CPCS doesn't use in-core instrumentations, **lacks radial information (pre-calculated radial factors).**
- CPCS only calculates the **1-dimensional** pseudo-hot-rod axial power distribution.
- In calculating the axial power, it simply uses **a 3x3 matrix** and cubic spline basis functions.



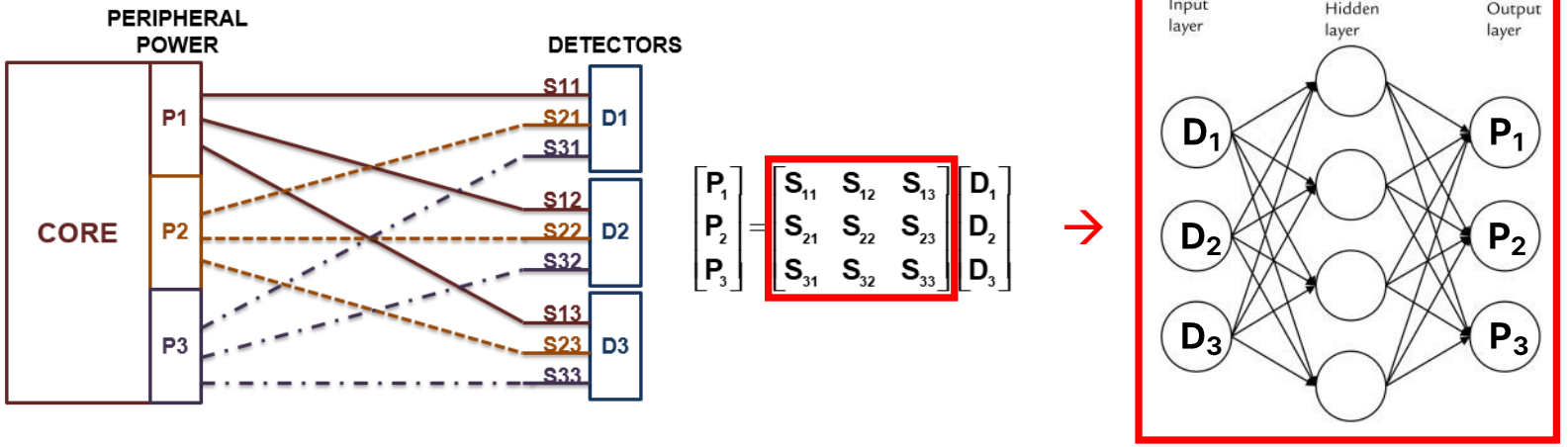
$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix}$$

- To compensate the inaccuracies, it introduces many conservative penalty factors → **less thermal margin!**

A previous ANN approach – ICOPS

Key ideas of ICOPS (In-COre Protection System) – replacing CPCS

- Replacing the 3x3 matrix, called **SAM (shape annealing matrix)** with **a single-layered neural network.**



ICOPS achievements

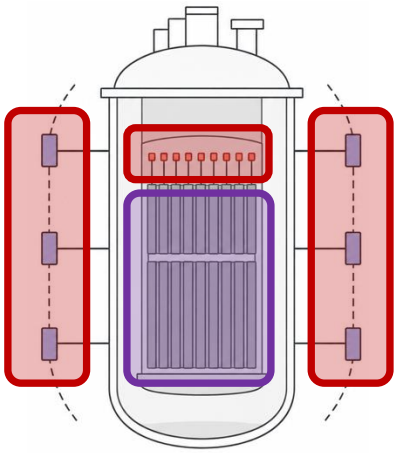
- Compared to the previous method (SAM), axial power prediction error was reduced in *all scenarios.
- ICOPS demonstrates **the viability of machine-learning-based safety features.**
- ICOPS is designed to meet the **latest global safety standards,** required by IAEA, WENRA, EUR, etc.
- ANN-based ICOPS **has been adopted in the APR-1000[4)]** and is being evaluated as part of its licensing review.

* Hanul unit 4, cycle 8 & 10 / Shin-Kori unit 1, cycle 1 / Shin-Wolsong unit 1, cycle 1

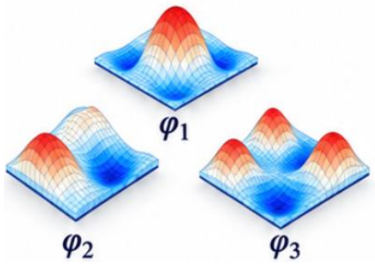
Research proposal

Problem definition

- Using minimal sensor signals, reconstruct 3D core power reconstruction. → **radial power information!**
- Default sensor set: **ex-core detector + core-exit thermocouple (CET).**
- Optional extra sensor (for large reactors): **in-core neutron flux detector (SPND).**

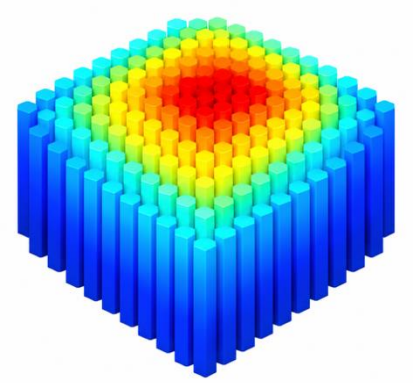


Input: sensor signals
ex-core detector / CET
***optional in-core flux detector for large reactors**



$$\hat{\mathbf{a}} = \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \end{bmatrix}$$

Method: linear combination
(coefficient found by GPR)



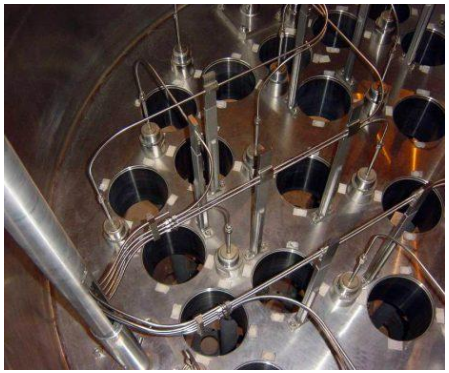
$$\hat{\mathbf{P}} = \sum_{i=1}^{k} \hat{a}_i \boldsymbol{\varphi}_i$$

Result: 3D power distribution

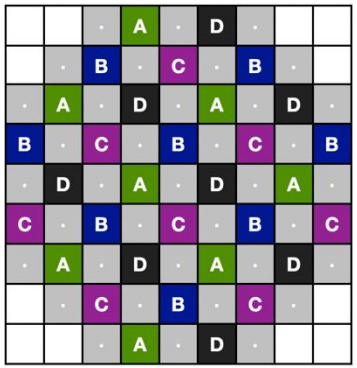
Proposed approaches (1) – temperature integration

Utilisation of core-exit thermocouples (CETs) with ex-core neutron detectors

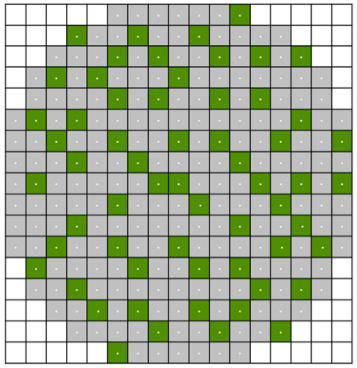
- CETs has fast response[5] (~0.7s) and provides radial information → **viable for protection use!**



CETs viewed from the top



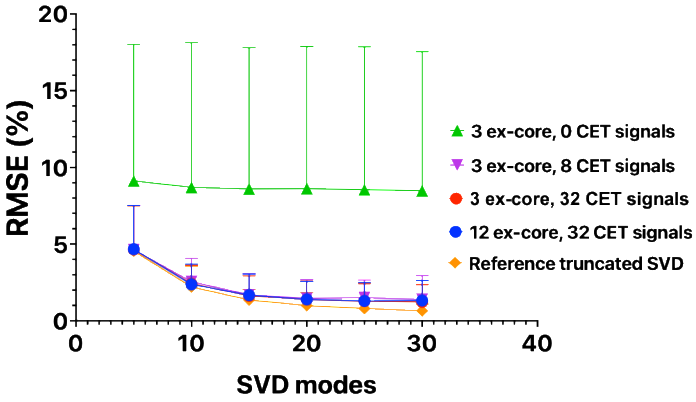
Proposed SMR CET locations

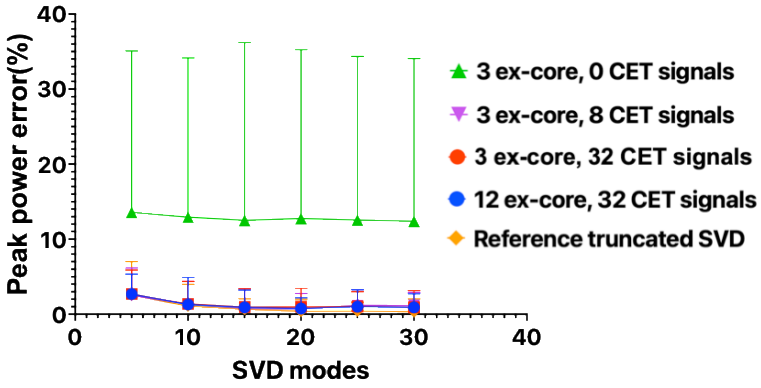


APR-1400's CET locations

CETs vastly improve the 3D power prediction performance, combined with ex-core neutron detectors

- **w/o CETs**: maximum **RMSE ~18%, peak power error ~34%**
- **with CETs**: maximum **RMSE ~3%, peak power error ~3%**





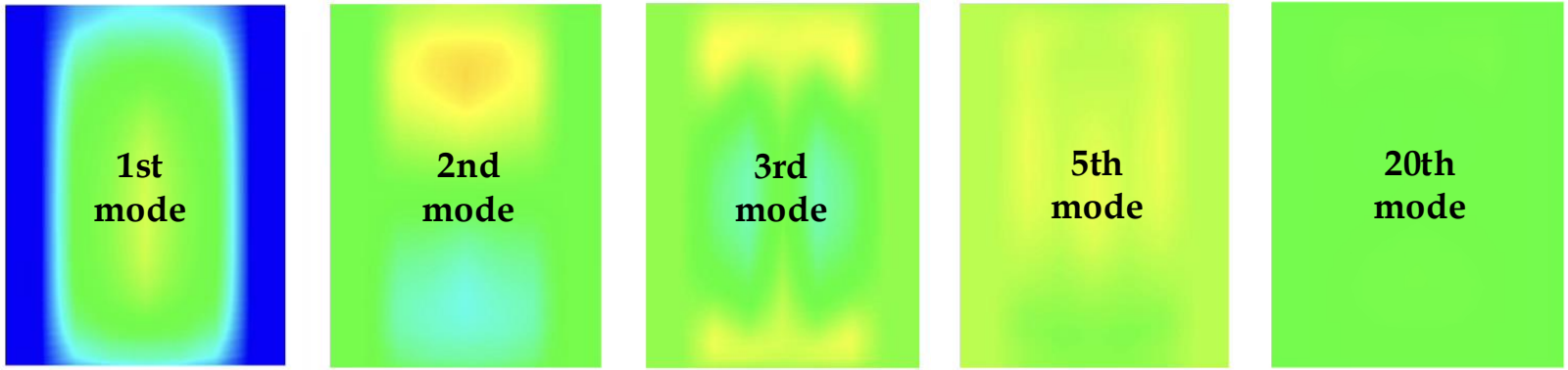
A single CET group out of 4 enhances the accuracy a lot!

New approach (2) – basis extraction using SVD

The two roles of SVD:

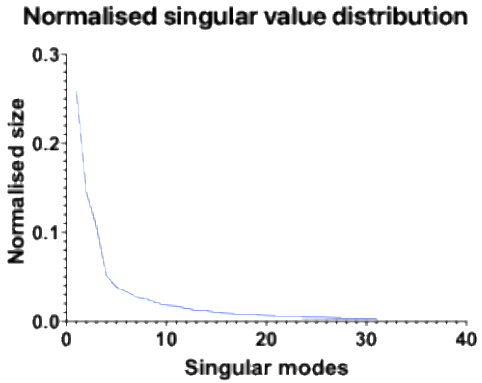
Role (1): extract dominant spatial modes from power distributions.

- **Multiple power distributions** are decomposed as a **linear combination of multiple orthonormal vectors.**
- The power prediction problem is then **reduced to simply finding a few scalar coefficients.**



Role (2): reduce the dimension of the higher-dimensional 3D power **(low-rank approximation).**

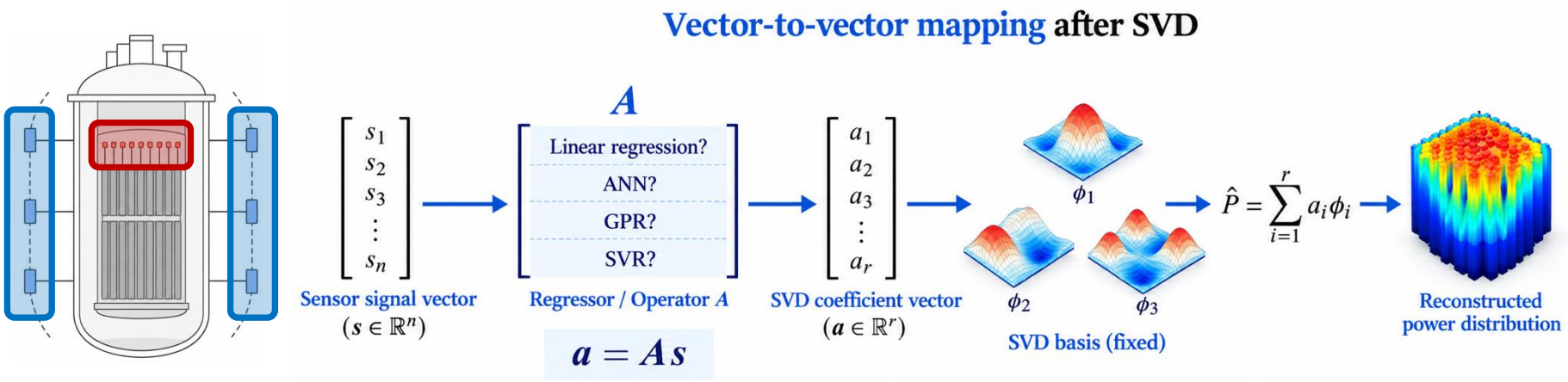
- Each spatial mode has a corresponding **singular value → a good index for rank-k approximation!**



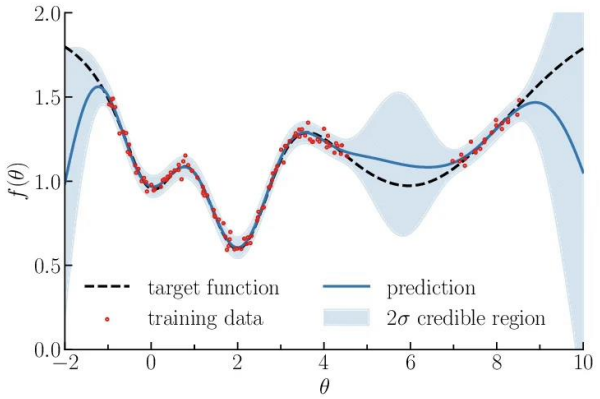
New approach (3) – Gaussian process regression (GPR)

With SVD, the problem is reduced to finding **vector – vector mapping** functions.

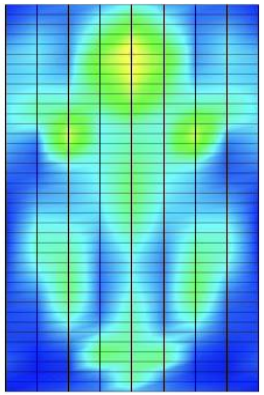
- Proper choice of **signal – SVD coefficient mapping relationship** regressor.



- GPR is a non-linear function, that provides the **SVD coefficient for each sensor signal** and **its uncertainty.**



A preliminary power prediction
3D uncertainty map.

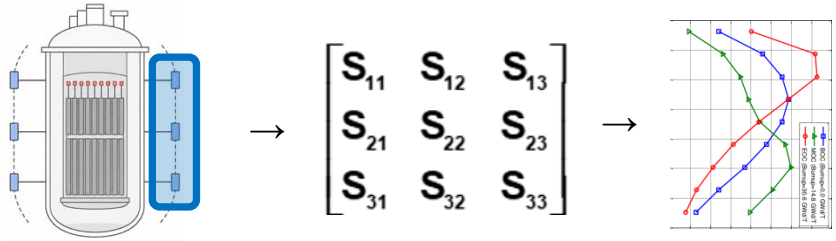


Uncertainty in each SVD coefficient is easily propagated to the whole 3D power, since it is a linear modelling.

Proposed approaches (4) – in summary

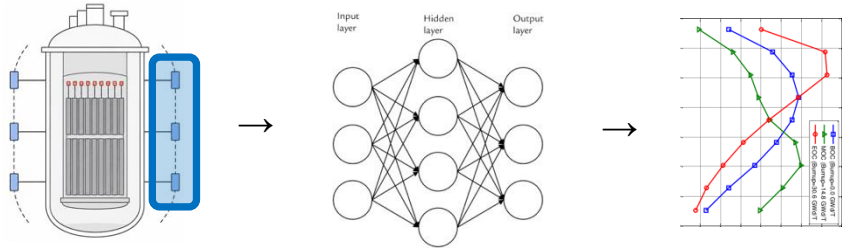
Traditional CPCS

- Ex-core signals ($\mathbb{R}^3$) → 3x3 matrix → 1D axial power ($\mathbb{R}^3$)



Korean ICOPS

- Ex-core signals ($\mathbb{R}^3$) → Neural network → 1D axial power ($\mathbb{R}^{20}$)



Proposed approach

- Ex-core/CET signals ($\mathbb{R}^{44}$) → GPR regression → SVD coefficients → Linear comb. → 3D power ($\mathbb{R}^{2760}$)

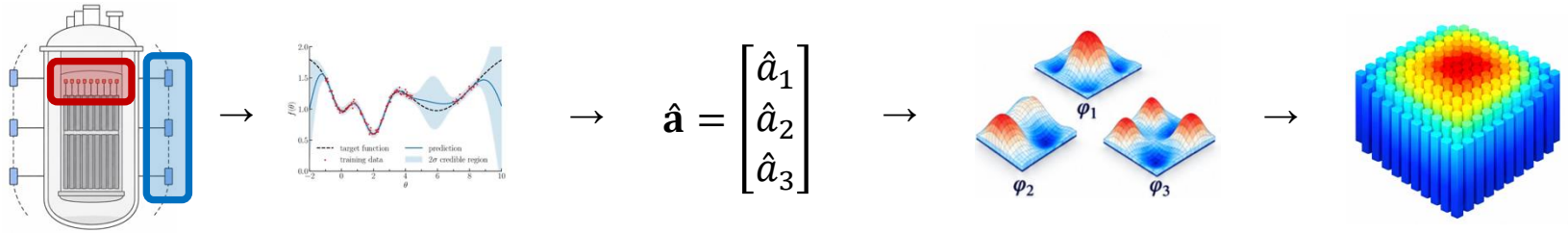


Table of contents

1. **Backgrounds of the study**
- Target system for replacement– COLSS/CPCS
- A Korean successor – ICOPS
- Research proposal
- New approach – temperature integration & basis extraction & GPR

2. **Study Design**
- SVD-based compact power distribution representation
- Target reactor: ATOM
- Training dataset generation
- GPR inputs: ex-core detector and CETs
- 3 steps of Gaussian process regression (GPR) model
- A comparison: ANN model structure

3. **Numerical results**
- Performance benchmark on LFO scenario
- Performance benchmark on extreme scenario – whole CR malfunction

4. **Conclusions and Future works**

SVD-based compact power distribution representation

SVD-based orthogonal basis-wise power reconstruction

- **Enhanced interpretability, compact representation of 3D power.**
- Only **a few detectable modes used** for full power reconstruction.

Step 1. SVD Basis Construction

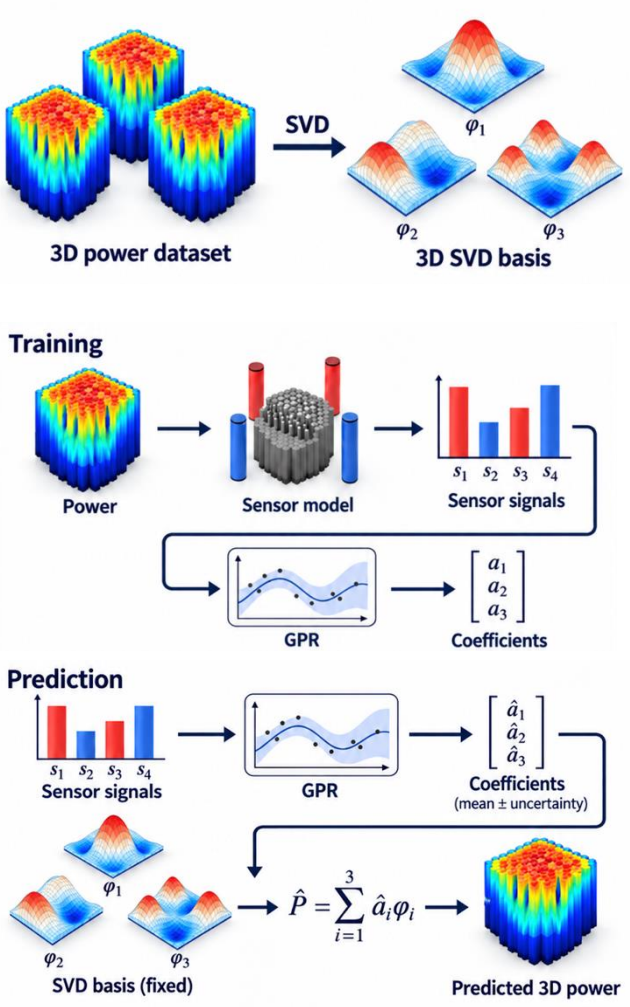
- Generate a dataset of a wide range of 3D power distributions
- Extract dominant spatial modes using SVD

Step 2. Coefficient Prediction Model Training

- Generate corresponding sensor responses using sensor models
- Train GPR models for sensor-to-coefficient mapping

Step 3. Performance Validation

- Predict SVD mode coefficients from sensor signals
- Reconstruct the 3D power distribution
- Evaluate coefficient prediction and reconstruction accuracy

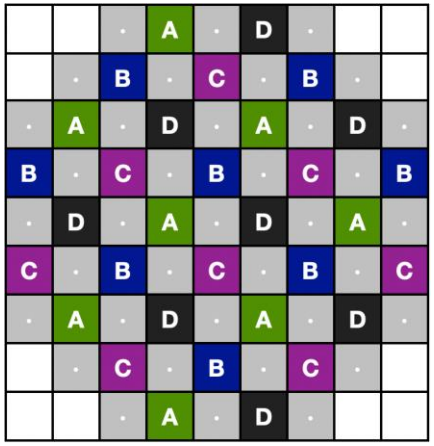


Target reactor: ATOM

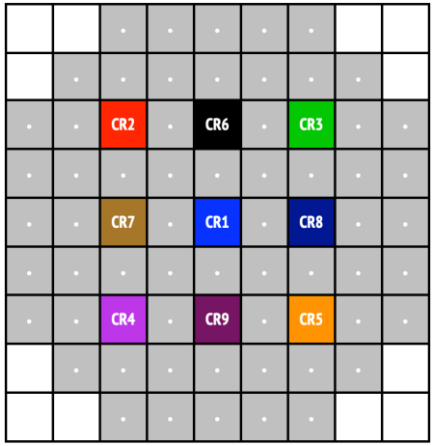
Reactor model for the proof-of-concept study – ATOM[6)]

- A KAIST-designed **soluble-boron free** SMR reactor, used as a surrogate model for i-SMR.
- 450 MWth nominal power, 69 assemblies with 2m active core height.
- 4 ex-core detectors, 32/69 assemblies equipped with CETs.

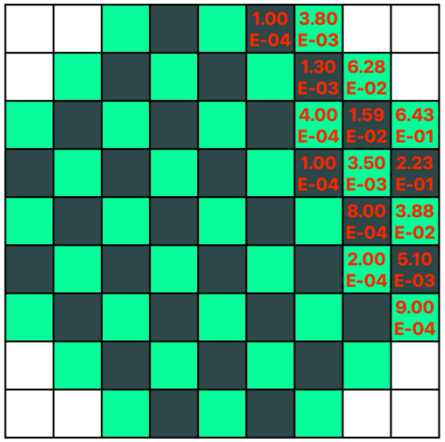
Thermal output	450 MWth
System pressure	15.5 MPa
FA type/ number	17 x 17, 69
Fuel material	UO2, 4.95w/o
Avg. fuel temp.	900.00 K



32 CETs divided into 4 groups (8 for each)

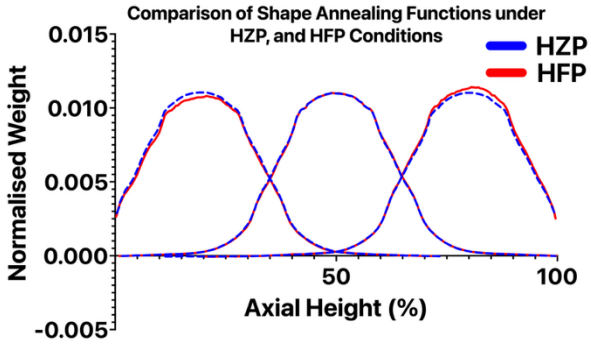


9 control rods



*Ex-core detector radial[7)] weights

*Ex-core detector[7)] axial weights

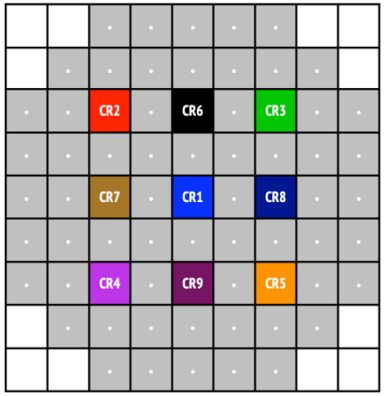


*Adopted from SMART-660;
both reactors share 69 assemblies, 17x17 lattice,
2m active core height.

Training dataset generation

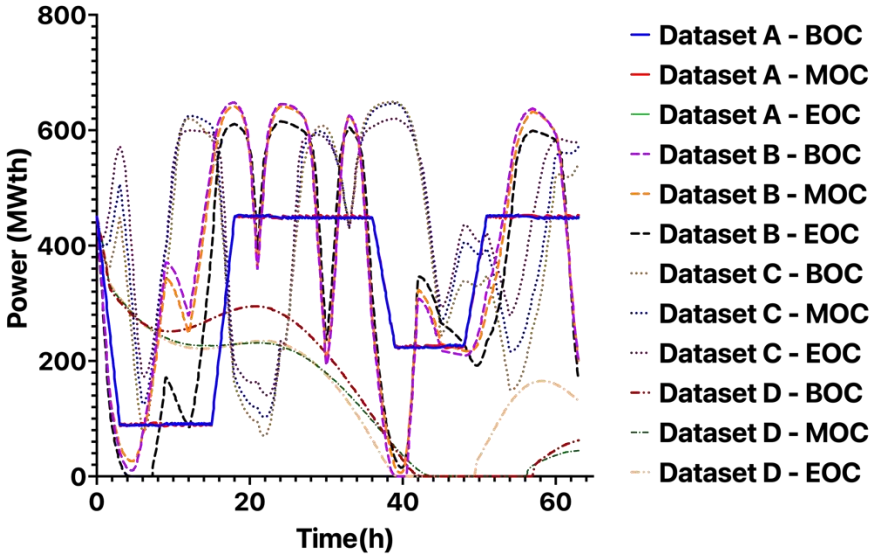
Datasets generation method: KAIST in-house PWR analysis code, KANT[8].

- A total of 4 scenarios, with **8:2** training/validation partitioning.
- **All transients induced by control rods; soluble-boron free reactor.**



Datasets generated:

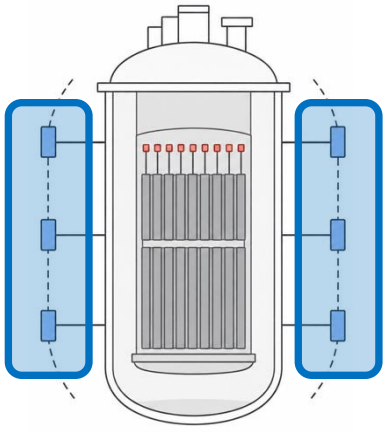
- 4 scenarios included, each of which has 2,280 datapoints in total.
- Dataset A: 100 – 20 - 50 load-follow. **(baseline)**
- Dataset B: central CR withdrawn, other CRs moving randomly. **(peripheral flux oscillation)**
- Dataset C: All CRs moving randomly. **(whole-core flux oscillation)**
- Dataset D: All CRs gradually inserted from 0% to 100%. **(supplementary)**
- 2,280 * 4 = 9,120 datapoints.

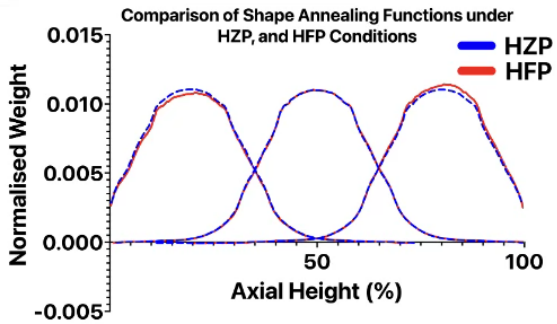


GPR inputs: ex-core detector and CET

GPR's 1st input: 12-channel (4 radial locations, 3 axial locations) ex-core detector signals

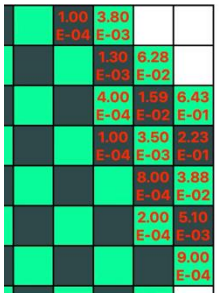
- ATOM's 3D ex-core detector assembly weights.





Axial weighting function[7] (AWF)

X



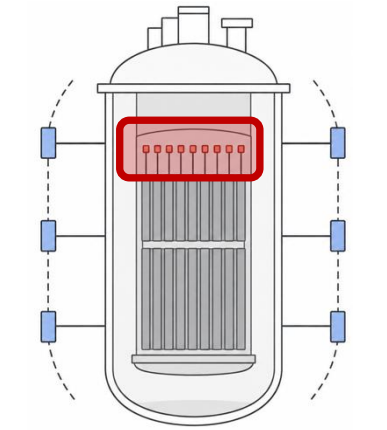
Radial weighting function[7] (RWF)

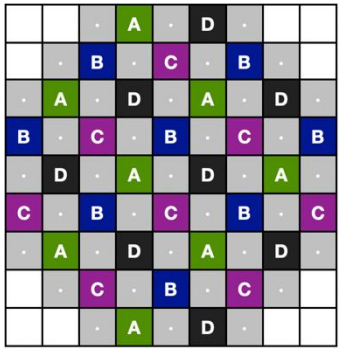
=

3D node-wise weights map for ex-core detectors

GPR's 2nd input: 32-channel CET signal

- Assembly-wise axial integral power used as surrogate, no time delay, no TH effects.



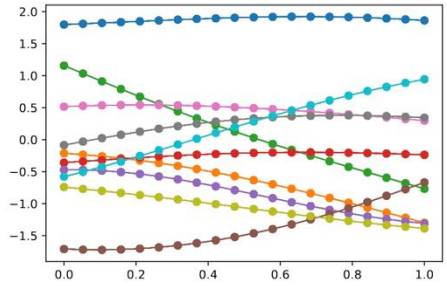


Core exit coolant temperature obtained from 32/69 assemblies (**8 for each of group A, B, C, D**)

3 steps of GPR

1st step – a blank distribution of MVN function curves.

$$^{*}k(x_i, x_j) = \sigma_f^2 \exp\left(-\frac{\|x_i - x_j\|^2}{2l^2}\right)$$



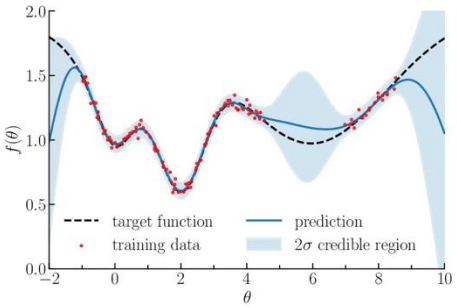
An example of a 20-variate MVN.

20 points are sampled simultaneously,

And each **point is correlated** based on their distances*

- why the curve is 'smooth'.

2nd step – fix the shape of MVN distribution using training data.

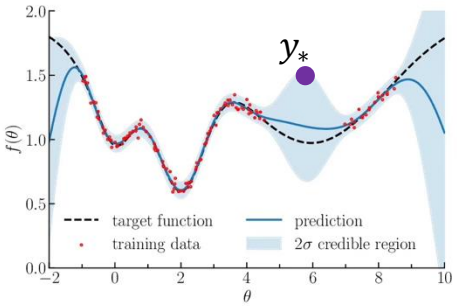


Training data fix the shape:

y decides where the curve goes, density of x decides how certain it is.

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \sim \mathcal{N}\left(\mathbf{0}, \begin{bmatrix} k(x_1,x_1) & k(x_1,x_2) & \cdots & k(x_1,x_n) \\ k(x_2,x_1) & k(x_2,x_2) & \cdots & k(x_2,x_n) \\ \vdots & \vdots & \ddots & \vdots \\ k(x_n,x_1) & k(x_n,x_2) & \cdots & k(x_n,x_n) \end{bmatrix}\right) = \mathcal{N}(\mathbf{0},\ K)$$

3rd step – make prediction on a new point.



Extend the block matrix by one point

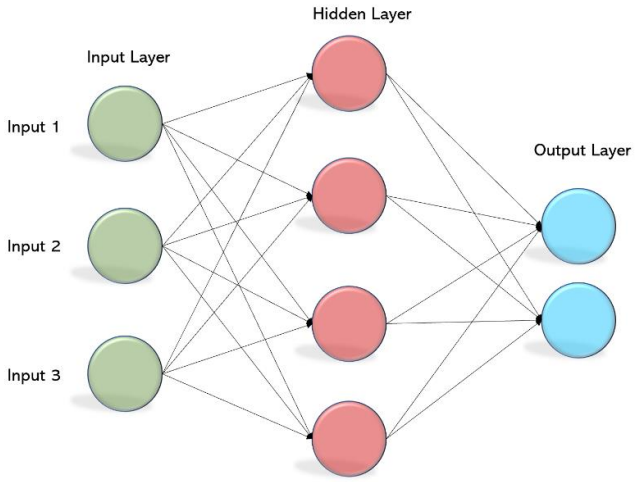
mean & variance of the new point are computed analytically.

$$\begin{bmatrix} \mathbf{y} \\ y_* \end{bmatrix} \sim \mathcal{N}\left(\mathbf{0}, \begin{bmatrix} K & \mathbf{k}_* \\ \mathbf{k}_*^T & k_{**} \end{bmatrix}\right)$$

ANN model structure

A three-hidden-layer ANN

- **Input:** ex-core detector & CET signals.
- **Output:** predicted SVD coefficients.



Parameter	Value / Specification
Input Shape	(4,)
Hidden Layer 1	Dense (4096)
Hidden Layer 2	Dense (1024)
Hidden Layer 3	Dense (256)
Output Shape	(5) – (30)
Optimiser	Adam
Epochs	300 (with early stopping)

Model name	1st layer	2nd layer	3rd layer	SVD modes	Loss
Funnel-M30	4096	1024	256	30	Singular value
Funnel-M25	4096	1024	256	25	Singular value
Funnel-M20	4096	1024	256	20	Singular value
Funnel-M15	4096	1024	256	15	Singular value
Funnel-M10	4096	1024	256	10	Singular value
Funnel-M5	4096	1024	256	5	Singular value

Table of contents

1. **Backgrounds of the study**
- Target system for replacement– COLSS/CPCS
- A Korean successor – ICOPS
- Research proposal
- New approach – temperature integration & basis extraction & GPR

2. **Study Design**
- SVD-based compact power distribution representation
- Target reactor: ATOM
- Training dataset generation
- GPR inputs: ex-core detector and CETs
- 3 steps of Gaussian process regression (GPR) model
- A comparison: ANN model structure

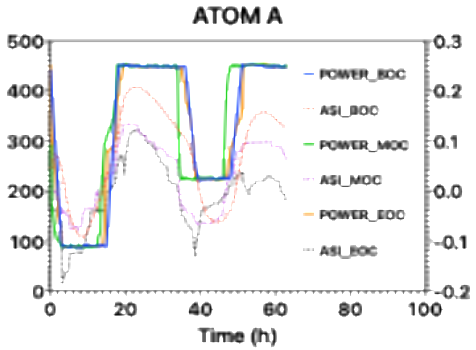
3. **Numerical results**
- Performance benchmark on LFO scenario
- Performance benchmark on extreme scenario – whole CR malfunction

4. **Conclusions and Future works**

Performance benchmark on LFO scenario

Load-follow operation dataset

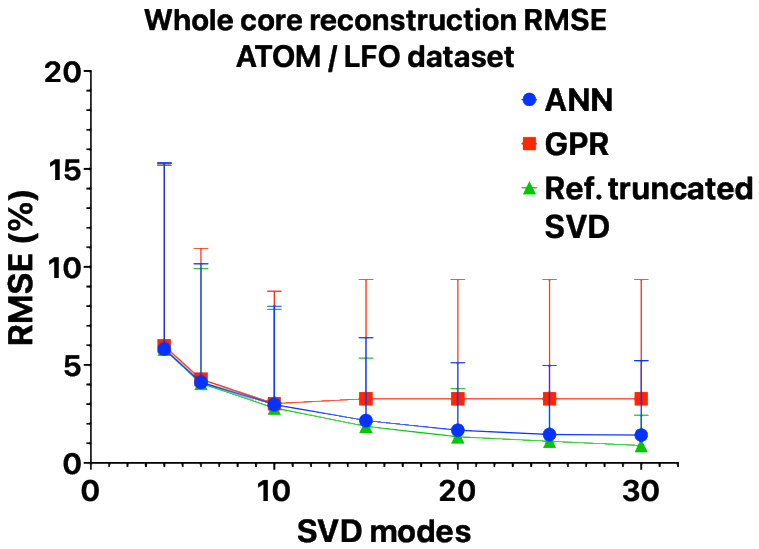
- Performance validation on LFO dataset.
- ANN slightly outperforms GPR.
- Performance saturation of GPR after 15 modes.

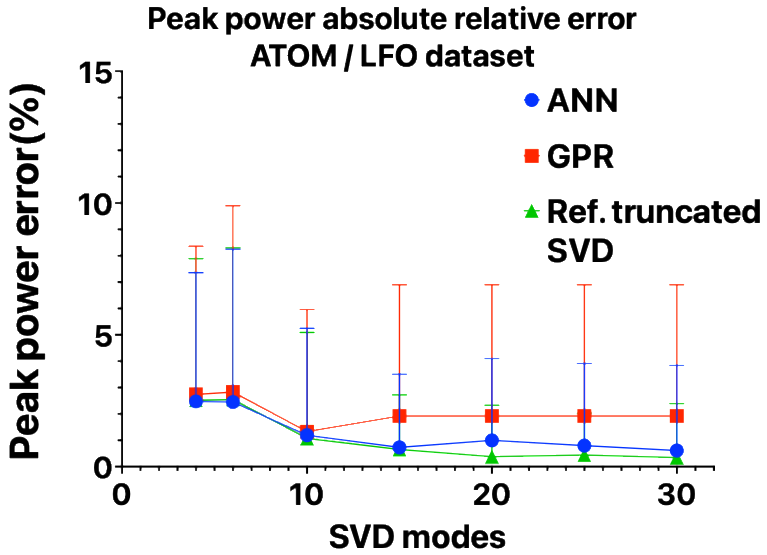


ANN > **GPR**

Max. RMSE: ~4%
Max peak error: ~3%

Max. RMSE: ~6%
Max peak error: ~5%

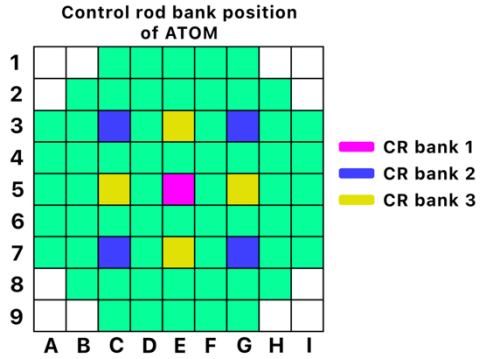




Performance benchmark on extreme scenario

Extreme operation dataset

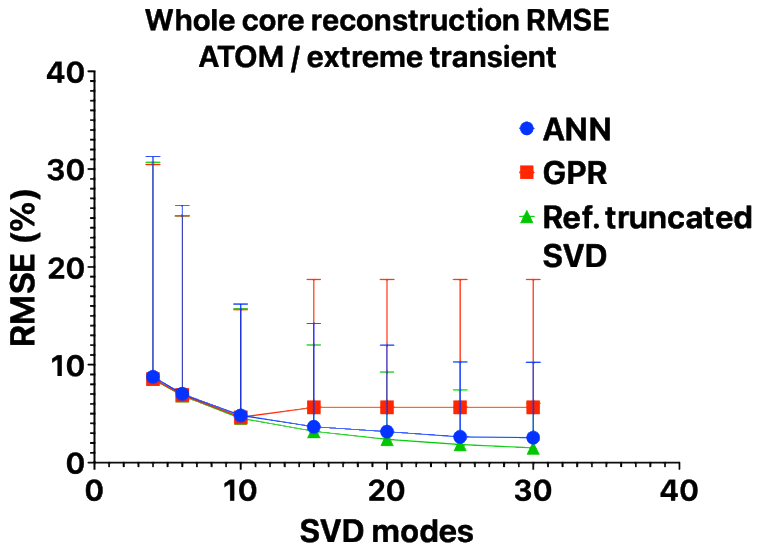
- Performance validation on **all CR failure** dataset.
- ANN slightly outperforms GPR.
- Performance saturation of GPR after 15 modes.

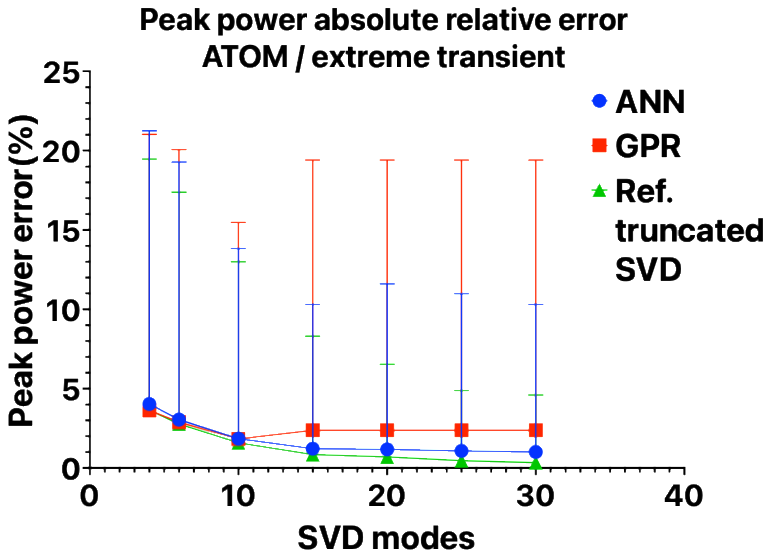


ANN > **GPR**

Max. RMSE: ~8%
Max peak error: ~9%

Max. RMSE: ~13%
Max peak error: ~17%





Conclusions

Conclusions

- GPR successfully regressed SVD coefficients from ex-core detector and CET signals, demonstrating its feasibility for 3D power reconstruction.
- The linear SVD reconstruction enables straightforward analytical propagation of coefficient covariance into a 3D power uncertainty map.
- GPR tended to underperform ANN and showed performance saturation with 15 or more SVD modes, whereas ANN continued to improve.

Future Work

- Investigate the causes of GPR performance saturation, particularly the prediction of higher-order modal coefficients.
- Propagate GPR uncertainty to LPD and DNBR and assess whether the resulting uncertainty estimates adequately represent actual prediction errors against reference calculations.

References

1) Na, Man Gyun, Dong Won Jung, Sun Mi Lee, Sun Ho Shin, Kibog Lee, and Yoon Joon Lee. Estimation of the Power Peaking Factor in a Nuclear Reactor Using Fuzzy Neural Networks. n.d.
2) U.S. Nuclear Regulatory Commission. (2011, September 8). CE technology cross training R325C: Chapter 09.2 — Incoreinstrumentation system [Training material]. https://www.nrc.gov/docs/ML1125/ML11251A026.pdf
3) AREVA NP Inc., U.S. EPR Nuclear Incore Instrumentation Systems Report, Technical Report ANP-10271NP, Rev. 0, Dec. 2006.
4) 한국원자력안전기술원, "APR1000 표준설계인가 심사계획(안)," 제210회 원자력안전위원회 보고안건 제1호, 2025년 3월 27일.
5) Sweeney, F. J. (1984). In-core coolant velocity measurements in a pressurized water reactor using temperature-neutron noise cross correlation [Conference paper, CONF-840614-74]. American Nuclear Society Annual Meeting, New Orleans, LA, United States. Oak Ridge National Laboratory.
6) X. H. Nguyen, C. H. Kim, and Y. Kim, "An advanced core design for a soluble-boron-free small modular reactor ATOM with centrally-shielded burnable absorber," Nuclear Engineering and Technology, vol. 51, no. 2, pp. 369–376, 2019. https://doi.org/10.1016/j.net.2018.10.016
7) G. Roh, K.-S. Kim, B.-S. Koo, C.-C. Lee, and K.-Y. Kim, "Ex-core detector response evaluation of the SMART reactor by using the DORT code," Journal of Nuclear Science and Technology, vol. 45, no. sup5, pp. 78–81, 2008. https://doi.org/10.1080/00223131.2008.10875790
8) T. Oh, Y. Jeong, H. Khalefih, and Y. Kim, "Development and validation of multiphysics PWR core simulator KANT," Nuclear Engineering and Technology, vol. 55, no. 6, pp. 2230–2245, 2023. https://doi.org/10.1016/j.net.2023.02.025

Any questions?

