

Uncertainty Quantification of MIR-PIV Measurements for Pebble-Bed Gap Flow Based on the ISO-GUM Framework

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1. Introduction

A pebble-bed reactor (PBR) is a reactor concept in which tens of thousands of spherical pebbles containing TRISO fuel particles are packed into the core, and it offers high-temperature operation and inherent safety. In the irregular pore structure formed by the pebbles, however, the local coolant flow differs greatly from gap to gap, and this flow non-uniformity directly affects local heat transfer and hot-spot formation.

Such local flow cannot be represented by one-dimensional system codes or porous-media models, and direct measurement under prototypic conditions is limited by cost and instrumentation access. The prediction of the local flow therefore relies on CFD that resolves the pore scale, but the CFD results still require experimental validation. Because the flow physics is governed by dimensionless similarity (Re), such validation is feasible in laboratory-scale experiments under simulated conditions. The pebble bed, however, is opaque and hinders optical measurement. When the refractive indices of a transparent test section and the working fluid are matched (matched index of refraction, MIR), particle image velocimetry (PIV) provides distortion-free visualization of the interior flow [1]. The MIR-PIV experiment is thus suitable for validating pebble-bed flow fields.

For a flow-visualization experiment to serve as a validation reference, the reliability of the measured results is essential. In the nuclear field, where safety is the highest priority among all considerations, this reliability must be established with data precisely quantified within standardized frameworks. The ISO-GUM provides the standard procedure for quantifying the measurement uncertainty, and ASME V&V 20 provides the standard for judging CFD validity with this uncertainty (u_D) as a necessary input [2]. Data evaluated through these two frameworks can therefore serve as a reliable reference for nuclear applications. However, MIR-PIV studies of packed structures have focused mainly on measuring and understanding the flow structure [1, 3], and studies that report both component-level uncertainty quantification and statistical-convergence verification remain rare, especially for nuclear pebble-bed geometries.

In this study, all error sources of the MIR-PIV measurement of the gap flow in a face-centered cubic (FCC) pebble bed were classified and quantified. The

classification and estimation strategy followed the review of Sciacchitano [4], and the conversion of each component into a velocity uncertainty and their combination followed the uncertainty-propagation methodology of the ISO-GUM [5]. By quantifying the component-level uncertainties at three Reynolds numbers and verifying the convergence of the mean and RMS statistics, we assess whether the present data are valid as a reference for CFD turbulence-model validation.

2. Methods

2.1 Experimental Facility and Test Section

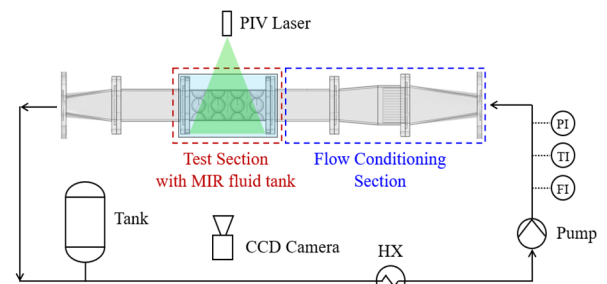


Fig. 1. Schematic of the experimental loop and the PIV measurement configuration

Fig. 1 shows a schematic of the experimental loop including the test section. The loop is a closed circuit consisting of a centrifugal pump, a reservoir tank, a heat exchanger, instrumentation (thermocouple, flowmeter, and pressure gauge), and the test section. The flow rate was adjusted by controlling the pump speed with an inverter. The test section, 940 mm in total length, consists of transition regions at both ends connecting the circular loop pipe to the square channel, a flow-conditioning region composed of a honeycomb straightener and a flow-development region (about 5D based on the pebble diameter, $D = 30$ mm), the pebble-bed region serving as the PIV measurement region, and a downstream flow-development region; all parts were 3D-printed from a photocurable transparent resin.

The refractive index of the 3D-printed test section is $n = 1.546$, and the working fluid, a benzyl benzoate–isopropyl myristate mixture (volume fraction ≈ 0.84), was tuned to the same value using an Abbe refractometer [6]. Since the fluid index varies with temperature, the loop temperature was held constant by the heat exchanger and recorded for each experimental condition

(29–37 °C). A quantitative tolerance for the residual index mismatch has not been established. Instead, the absence of visible distortion of a background grid was used as the acceptance criterion for the matching condition, which secured distortion-free optical access through the curved walls (Fig. 2).

The pebble-bed region is a stack of pebbles in an FCC arrangement of seven layers (3×3 in the odd layers and 2×2 in the even layers) inside a square channel (60×60 mm). To eliminate the wall effect and to reproduce a packing equivalent to an infinite pebble bed (porosity $\varepsilon \approx 0.2595$), the outer hemispheres of the outermost pebbles in the odd layers were removed, and the flat cut faces were in full contact with the channel walls (Fig. 2). The point contacts between pebbles and between pebbles and walls were relieved with cylindrical bridges (bridge diameter d , $d/D = 0.15$), a value within the range recommended by the authors' previous study [7] ($d/D < 0.2$) that can be fabricated reliably by 3D printing.

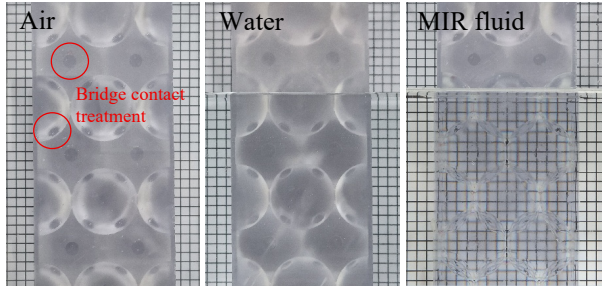


Fig. 2. Verification of refractive index matching

2.2 PIV Configuration and Test Conditions

The PIV measurements used a dual-pulse Nd:YAG laser and a CCD camera, operated at 10 Hz with a pulse interval (Δt) of 50–75 μ s, and the images were processed by multi-pass cross-correlation with 64×64 px interrogation windows (50% overlap). The vertical measurement plane passes through the pebble centers at one quarter of the channel width ($W_{ch}/4 = 15$ mm, $W_{ch} = 60$ mm) from the side wall. Fluorescent Rhodamine B particles ($dp = 15$ μ m, $pp = 1,100$ kg/m³) were used as tracers, and a 550 nm long-pass filter on the camera blocked the pebble-surface laser reflections and passed only the fluorescence signal (580 nm). The particle Stokes number ($\approx 5.7 \times 10^{-4}$) is small enough for faithful flow tracking.

The flow rate Q was measured with a clamp-on ultrasonic flowmeter installed on the loop piping (Fig. 1), and the superficial inlet velocity U_{inlet} was determined from the measured flow rate as

$$U_{inlet} = \frac{Q}{A_{ch}} \quad (1)$$

where A_{ch} is the cross-sectional area of the square channel (60×60 mm). The Reynolds number is defined with U_{inlet} as the reference velocity and the pebble diameter D as the length scale:

$$Re = \frac{\rho U_{inlet} D}{\mu} \quad (2)$$

where ρ and μ are the density and dynamic viscosity of the working fluid at the measured temperature of each condition. The tests were conducted at $Re = 1,879, 2,252$, and $2,689$, well above the packed-bed turbulent-transition threshold [8], with 250 image pairs (25 s) acquired per condition. The velocity results are normalized by the interstitial velocity U_{int} , which accounts for the FCC bed porosity ($\varepsilon \approx 0.2595$):

$$U_{int} = \frac{U_{inlet}}{\varepsilon} \quad (3)$$

2.3 Uncertainty Quantification Method

The uncertainty was quantified within the ISO-GUM framework [5], following the error-source classification adopted in the review of Sciacchitano [4]; the classification and estimation methods are summarized in Table I in Section 3.2. The PIV velocity U is computed from the following data-reduction equation.

$$U = \frac{S \cdot \Delta X}{\Delta t} \quad (4)$$

Here S is the magnification (mm/px), ΔX is the particle displacement obtained from the interrogation window (px), and Δt is the pulse interval. Every error source acts on one of the three variables of Eq. (4): calibration on S , timing and synchronization on Δt , and particle tracing, peak locking, and the random component on ΔX . The combined standard uncertainty u_c and the expanded uncertainty u_e are then obtained as follows.

$$u_c = \sqrt{\sum_{i=1}^n [c_i \cdot u(x_i)]^2} \quad (5)$$

$$u_e = k \cdot u_c \quad (6)$$

Here $u(x_i)$ is the standard uncertainty of the error source x_i , $c_i = \partial U / \partial x_i$ is the sensitivity coefficient converting it into velocity units, n is the number of error sources, and k is the coverage factor, taken as 1.96 for a 95% confidence level under the Gaussian assumption.

The systematic components that cannot be separated from the image records (calibration, timing and synchronization, particle tracing, and peak locking) were estimated individually from a-priori information, while the random components contained in the images (illumination variation, out-of-plane loss, correlation noise, etc.) were estimated collectively as a statistical term $\sigma/\sqrt{N}$, where σ is the temporal standard deviation at each grid point and N is the number of independent samples. The acquisition interval (100 ms) is much longer than the flow characteristic time (about 4 ms), so the 250 samples are statistically independent.

3. Results and Discussion

3.1 Measured Mean Velocity Field

From the 2D-2C PIV measurement, the time-averaged streamwise and transverse velocity fields (U , V) and the corresponding turbulence intensities (u'_{rms} , v'_{rms}) are obtained on the measurement plane. Fig. 3(a) shows the time-averaged velocity field, normalized by U_{int} , at the representative condition ($Re = 2,689$). The flow accelerates into local jets through the gaps between the pebbles, and the peak velocity reaches several times U_{int} , while low-velocity regions form behind the pebbles. Fig. 3(b) presents the U profile along line P with the expanded uncertainty of Section 3.2 as error bars, which shows how the present dataset can be used for profile-level CFD validation. The strong acceleration and shear of the gap jets are the source of the large turbulent fluctuations discussed in the next section.

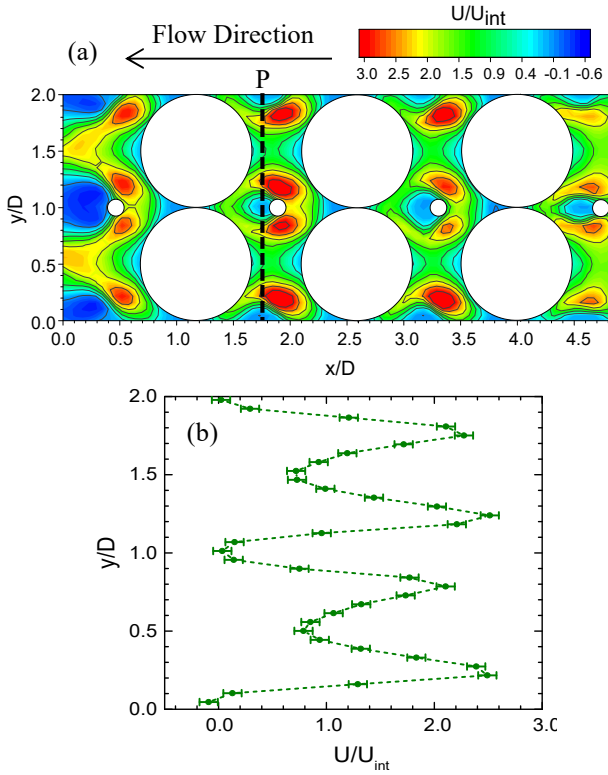


Fig. 3. (a) U/U_{int} contour at $Re = 2,689$ with profile line P. (b) U/U_{int} profile along P with u_e (95%) error bars.

3.2 Component-Level Uncertainty Analysis

Table I summarizes the uncertainty of each error source acting on the variables of Eq. (4). The estimation of each component is described below.

The calibration uncertainty acts on the magnification S . The present calibration uses the channel width W_{ch} as the reference length because W_{ch} is directly visible in the measurement plane (in-situ calibration). Therefore, no separate calibration target or parallax correction is required, and the remaining error sources are the

machining tolerance of the channel ($\pm 0.2\%$, ± 0.12 mm for 60 mm) and the wall-detection precision in the mask image (± 10 px). Their RSS showed that wall detection is dominant, and the velocity-converted value was 0.90 – 0.91% of U_{int} .

The timing and synchronization uncertainty acts on Δt . Combining the laser pulse jitter (~ 0.5 ns) and the delay-generator jitter (~ 50 ps) from the manufacturers' specifications gives a level of 10^{-5} of Δt (50 – 75 μ s), which is practically negligible.

Particle tracing is the error in ΔX caused by particles failing to follow the flow. The slip velocity estimated from the Stokes number of Section 2.2 ($\approx 5.7 \times 10^{-4}$) remained at 0.16 – 0.24% even under the conservative condition of using the peak gap velocity as the characteristic velocity.

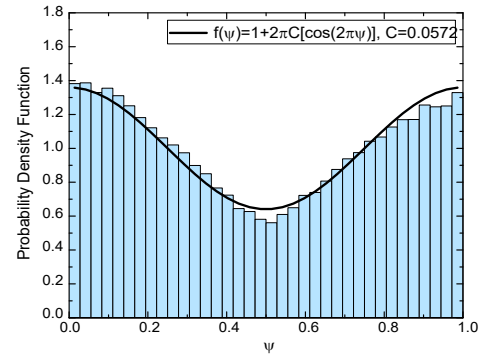


Fig. 4. Measured histogram of the fractional displacement ψ with the fit of Eq. (7) (U , $Re = 2,689$).

Peak locking is a systematic bias in ΔX , in which the sub-pixel fitting of the correlation peak is pulled toward integer-pixel displacements. In the absence of bias, the fractional part of the displacement (ψ) should be uniformly distributed. The measured histogram, however, shows a clear distortion (Fig. 4), which indicates the presence of peak locking. The bias $B(\psi)$ was modeled as a sine function, the lowest-order odd periodic mode that vanishes at integer and half-integer pixels, and this bias distorts the histogram of ψ into the following form.

$$f(\psi) = 1 + 2\pi C \cdot \cos 2\pi\psi \quad (7)$$

$$B(\psi) = -C \cdot \sin 2\pi\psi \quad (8)$$

Here C is the amplitude of the bias. C was quantified by fitting Eq. (7) to the ψ histogram measured for each condition, and the RMS of the bias ($= C/\sqrt{2}$) was converted into velocity and included in the systematic term. Although the bias may partially cancel in the time-averaging process, this cancellation was not assumed and the full RMS was conservatively included. The results, 0.93 – 2.30% , differed by more than a factor of two among the conditions because the particle-image diameter and the histogram distortion vary with the imaging optics, showing that peak locking must be quantified by condition-specific measurement rather than by assumed specifications.

Table I. Uncertainty components of the PIV measurement (U , expressed as $\%U_{\text{int}}$)

Parameter	Acts on	Type	Estimation Method	$Re\ 1,879$	$Re\ 2,252$	$Re\ 2,689$
Calibration	S	Systematic	Machining tolerance + wall detection (RSS)	0.90%	0.91%	0.91%
Timing & Synchronization	Δt	Systematic	Manufacturer jitter specs	< 0.01%	< 0.01%	< 0.01%
Particle tracing	ΔX	Systematic	Stokes number	0.16%	0.19%	0.24%
Peak locking	ΔX	Systematic	Measured histogram fit, Eqs. (7)–(8)	0.93%	2.19%	2.30%
In-plane velocity gradient	ΔX	Systematic	Mitigated by iterative window deformation	negligible	negligible	negligible
Random	ΔX	Random	$\sigma/\sqrt{N}$	3.15%	3.67%	3.53%
Combined uncertainty	-	-	RSS, Eq. (5)	3.42%	4.38%	4.32%
Expanded uncertainty	-	-	$k = 1.96$, Eq. (6)	6.69%	8.58%	8.47%

In this study, the iterative window deformation was applied in the multi-pass processing, which substantially mitigates the errors arising from in-plane velocity gradients [4]. Although the residual error may persist in the narrow gaps and in the gap-jet shear layers, it is expected to remain below the dominant error components. Therefore, this error was neglected and not separately quantified.

The random component, 3.15–3.67%, was the largest under all conditions, and the expanded uncertainty u_e was 6.7–8.6% for U and 5.3–6.8% for V . Because the random term is estimated from the temporal standard deviation σ at each grid point, it contains both measurement noise and the turbulent fluctuation of the flow itself. The measured fluctuation is large ($u'_{\text{rms}}/U_{\text{int}} \approx 30\text{--}56\%$). This is a characteristic of gap flow, in which the strong shear and lateral oscillation of the gap jets generate turbulence throughout the flow, and similar levels were reported for a randomly packed bed [3]. Even after reduction by $\sqrt{N} \approx 15.8$ by averaging 250 pairs, the fluctuation term remains the largest component. The expanded uncertainty is of the same order as, but higher than, the 1–5% velocity uncertainties reported in typical free-stream PIV applications [4]. This difference reflects the flow physics rather than a measurement deficiency, and the estimate is a conservative upper bound because the noise is included. The random uncertainty is thus governed by the sample size N , while the systematic components set the lower bound, so the improvement with increasing N is not unlimited.

In ASME V&V 20, a model is regarded as validated when the comparison error E , the difference between the simulation and the experimental data, is smaller than the validation uncertainty u_{val} .

$$u_{\text{val}} = \sqrt{u_{\text{num}}^2 + u_D^2 + u_{\text{input}}^2} \quad (9)$$

Here u_{num} is the numerical uncertainty of the simulation, u_{input} is the uncertainty from the simulation input conditions, and u_D is the measurement uncertainty of the experimental data. The expanded uncertainty quantified in this study enters directly as u_D , while u_{num}

and u_{input} are evaluated on the CFD side. The present results therefore provide the measurement side of the quantitative threshold for pebble-bed CFD validation.

3.3 Statistical Convergence and Turbulence-Statistics Uncertainty

The random-uncertainty estimate ($\sigma/\sqrt{N}$) of Section 3.2 and the reported turbulence intensities are valid only when the statistics have converged within the number of samples. This convergence was confirmed by the convergence histories at two representative interior points per velocity component, selected where the mean velocity is sufficiently large (Fig. 5).

The time-averaged velocity converges into the 95% confidence envelope within $N = 250$. This shows that 250 image pairs are sufficient for the mean velocity field. The turbulence intensities (u'_{rms} , v'_{rms}) also remain within the corresponding 95% confidence envelope for all three conditions. However, RMS statistics converge more slowly than the mean, and the finite-sample uncertainty of the turbulence intensity at $N = 250$ is about 10% at the 95% confidence level. This value is large compared with conventional turbulence-statistics measurements using thousands of images.

3.4 Limitations

The present measurement resolves only two in-plane velocity components (2D-2C) of a three-dimensional flow, and the data are therefore restricted to the in-plane statistics on a single plane. Near the pebble surfaces, the out-of-plane motion can raise the local noise level. This noise is included in the random uncertainty but is not separately resolved.

The 250 image pairs are sufficient for the mean velocity but are below the thousands of samples typically required for converged turbulence statistics, and the turbulence intensities therefore have a finite-sample uncertainty of about 10% (Section 3.3). Nevertheless, the data remain valid as a validation reference within the quantified uncertainties.

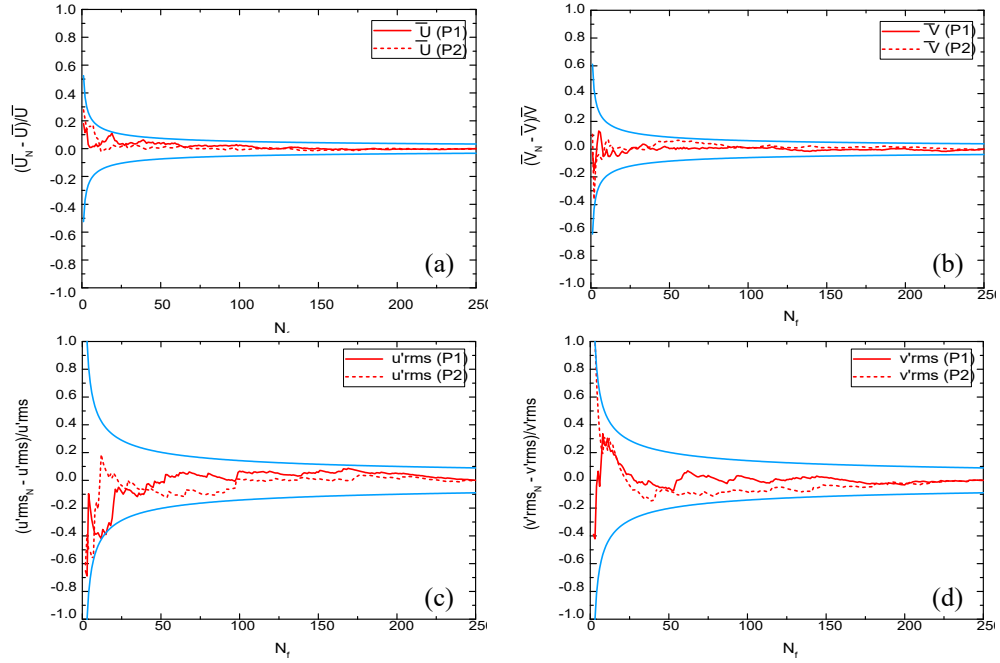


Fig. 5. Convergence histories with 95% envelopes: (a, b) mean U , V ; (c, d) u'_{rms} , v'_{rms} .

4. Conclusions

The uncertainty of the MIR-PIV measurement of the gap flow in an FCC pebble bed was quantified component by component within the ISO-GUM framework. The expanded uncertainty was 6.7–8.6% for the streamwise component, and the convergence of the mean and RMS statistics was confirmed, so the present data can serve as a reference for CFD turbulence-model validation. The quantified uncertainty can be used directly as u_D in ASME V&V 20, providing the measurement basis for the validation judgment.

The component-level results showed that the random term dominates, and this term originates from the turbulent fluctuation of the flow itself. The number of samples is therefore the practical determinant of the uncertainty.

The contribution of this work lies on the measurement side. The in-situ calibration, the condition-specific quantification of peak locking, and the convergence verification together establish a documented uncertainty-quantification procedure for pebble-bed MIR-PIV, and this procedure can also be applied to other packed-structure measurements with curved optical interfaces. As future work, a quantitative comparison with CFD turbulence models will be performed based on the present dataset.

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