

A Fracture Energy Regularized Damage Model for Tensile Cracking Modeling in Reinforced Concrete Structure

Habeun Choi ^{a*}, Tae-Hyun Kwon ^a

^aKorea Atomic Energy Research Institute, 989-111 Daedeok-daero, Yuseong-gu, Daejeon 305-353, Republic of Korea

*Corresponding author: hbchoi@kaeri.re.kr

***Keywords :** containment building, Mazars damage model, fracture energy regularization, crack width evaluation

1. Introduction

The containment building is the final barrier against the release of radioactive material to the environment. Under severe accident conditions, the internal pressure puts the reinforced concrete wall in tension, and the cracks that open through the wall thickness may act as leakage paths. Because the leakage rate through such a crack path is governed by the crack opening width, a quantitative leakage assessment calls for the crack pattern and the crack width, not for the load-carrying capacity alone.

Continuum damage models describe cracking as a distributed degradation of the material stiffness. Since no crack surface has to be tracked and no separate initiation or propagation criterion is needed, they are well suited to large three-dimensional models and are easily combined with commercial finite element software. Two shortcomings, however, limit their use for crack propagation modeling and the corresponding leakage assessment. First, once the material softening behavior starts, the deformation localizes into a band whose width is dictated by the mesh, so that the dissipated energy is mesh dependent unless the constitutive law is regularized by a characteristic length. However, the definitions of the characteristic length in common use depend on the element size alone and ignore the orientation of the crack. Second, a smeared formulation returns a damage variable rather than a displacement jump, so the crack opening, which is the quantity of interest for leakage, is not directly available.

This study addresses both points. A Mazars-type isotropic damage model with fracture energy regularization [1] is implemented in an Abaqus user element subroutine (UEL) in which the characteristic length is measured across the element along the principal strain direction, and the crack opening is recovered from the same length scale and reported together with the damage variable. The model is then applied to a reinforced concrete member for which measured crack patterns and crack widths are available.

2. Regularized damage model

Cracking behavior is represented by a single scalar variable d that degrades the elastic stiffness, so that the stress follows from $\sigma = (1 - d)E_0\varepsilon$ and no permanent strain remains upon unloading. Damage growth is driven

by an equivalent strain built from the positive principal strains,

$$\tilde{\varepsilon} = \sqrt{(\varepsilon_1^+)^2 + (\varepsilon_2^+)^2 + (\varepsilon_3^+)^2}. \quad (1)$$

together with a loading function $f = \tilde{\varepsilon} - \kappa$, in which the history variable κ stores the largest equivalent strain attained so far and starts from the damage threshold strain $\varepsilon_{d0} = f_t/E_0$. Damage evolves only when $f \geq 0$, and the tensile and compressive contributions are combined as

$$d = \alpha_T d_T + \alpha_C d_C \quad (2)$$

where the weights α_T and α_C are obtained from the parts of the principal strains that are induced by the tensile and compressive stress components, respectively.

To remove the mesh dependency of the softening branch, the specific fracture energy is rescaled by the crack bandwidth, $g_F = G_F/l_{ch}$, so that the area under the stress-strain curve reproduces the fracture energy of the material irrespective of the element size. A bilinear traction-separation law is adopted in tension and the MC2010 relation in compression, and the crack opening w_n of the cohesive law is mapped onto the equivalent strain through

$$\tilde{\varepsilon} = \varepsilon_{d0} + \frac{w_n}{l_{ch}} \quad (3)$$

The tensile damage variable is then evaluated on each branch of the bilinear law from the ratio between the current secant modulus and the undamaged one, $d_T = 1 - E/E_0$.

3. Direction-dependent characteristic length

Equation (3) is objective only if l_{ch} represents the width of the localization band measured normal to the crack. Conventional definitions use the element size alone, namely $(2A_e)^{1/2}$ for triangular and $A_e^{1/2}$ for quadrilateral elements, and $(12V_e)^{1/3}$ for tetrahedral and $V_e^{1/3}$ for hexahedral elements; the geometric measure supplied by Abaqus through the variable CELENT is of the same kind. None of them depends on the crack orientation, so the energy released per unit crack area is over- or underestimated for elongated or distorted elements and whenever the crack is inclined with respect to the element edges.

In the present formulation the crack normal $\mathbf{n}$ is taken as the eigenvector associated with the maximum positive principal strain, and l_{ch} is computed at every material point as the distance across the element measured along

that direction (Fig. 1). Because the nodal coordinates of the element are required, the constitutive model is implemented in the user element subroutine. Inverting Eq. (3) with the same length scale gives the crack opening

$$w_n = l_{ch} \left(\bar{\varepsilon} - \frac{f_t}{E_0} \right) \quad (4)$$

which is stored as a solution-dependent variable so that the crack width can be contoured together with the damage variable.

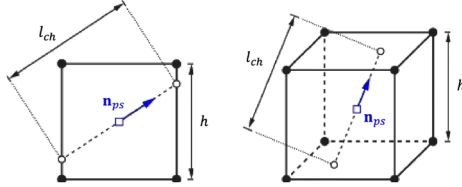


Fig. 1. Characteristic length measured across the element along the principal strain direction.

4. Numerical example

The proposed methodology is first verified on a single element under uniaxial tension, for which the computed stress–displacement and stress–crack width responses reproduce the prescribed traction–separation law. A reinforced concrete hollow cylinder tested under uniaxial tension [2] is then analyzed. The specimen is 730 mm high with outer and inner diameters of 253 mm and 181 mm, giving a wall thickness of 72 mm, and is reinforced with $\phi 11$ bars and threaded rods corresponding to a reinforcement ratio of 0.82%. The concrete properties are estimated from the measured compressive strength $f_c = 31.9$ MPa as $E_0 = 26.6$ GPa and $f_t = 3.16$ MPa following ACI 318, and $G_F = 75.7$ N/m following CEB-FIP MC90. The yield strength and elastic modulus are 500 MPa and 200 GPa for the bars and 800 MPa and 210 GPa for the threaded rods. Concrete and reinforcement are discretized with 566,176 and 4,672 C3D8R elements, respectively, giving 621,744 nodes at an element size of 5 to 5.5 mm, and a perfect bond is assumed at the concrete–reinforcement interface.

Because the specimen is nominally axisymmetric and uniformly loaded, a homogeneous description leaves the concrete stress uniform except close to the reinforcement, where the bond transfer perturbs it. Damage therefore starts only there and, since every other point is equally close to the strength limit, spreads over a wide area instead of localizing, giving broad diffuse bands rather than distinct cracks (Fig. 2a). The tensile strength is therefore prescribed as a spatially varying random field with a mean of 3.16 MPa and a coefficient of variation of about 7 %. The weakest sections then dictate where cracking starts, and the damage collapses into narrow transverse cracks distributed along the height (Fig. 2b). The computational results demonstrate that the concrete heterogeneity should be accounted for explicitly to model such cracking behavior in members without a geometric concentration.

The crack width recovered from Eq. (4) is confined to the same narrow bands and reaches about 0.1 mm at an applied elongation of 0.11 mm (Fig. 2c), for which the computed reaction is approximately 237 kN. This value lies within the range of 216 to 256 kN over which cracks were recorded in the test, so both the location and the magnitude of the predicted crack opening are consistent with the measurement.

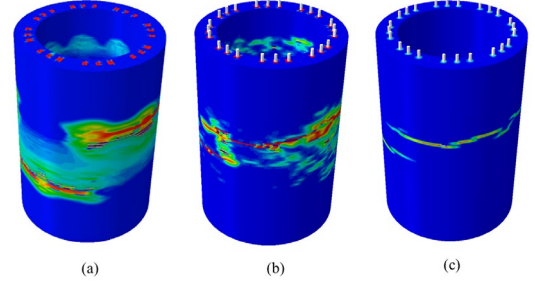


Fig. 2. Damage distribution with (a) a uniform and (b) a spatially varying tensile strength field, and (c) the crack width distribution corresponding to (b).

5. Conclusions

In this study, a fracture energy regularized continuum damage model is developed for tensile cracking of reinforced concrete structures and implemented in the Abaqus user element subroutine. Measuring the characteristic length along the principal strain direction removes the orientation bias of the conventional element-size measures, and the same length scale directly yields the crack width, which governs the leakage rate of the radioactive material and is not available from the damage variable alone. By considering the concrete heterogeneity via a spatially varying tensile strength, the model accurately reproduces the discrete cracking pattern and the measured crack width in the hollow cylinder example. Crack path prediction remains less accurate under biaxial stress states; refining the equivalent strain definition and the damage combination rule, and coupling the predicted crack widths with a leakage rate evaluation, are left for future work.

ACKNOWLEDGEMENT

This work was supported by the National Research Foundation of Korea (NRF) funded by the Ministry of Science and ICT (RS-2022-00144409) and the Korea Institute of Energy Technology Evaluation and Planning (KETEP) funded by the Ministry of Climate, Energy & Environment (MCEE) of the Republic of Korea (20224B10300010).

REFERENCES

- [1] M. Arruda, J. Pacheco, L.M. Castro, E. Júlio, A modified Mazars damage model with energy regularization, *Engineering Fracture Mechanics*, Vol. 259, p. 108129, 2022.
- [2] L. Ma, O.S. Kwon, E. Bentz, J. Tchermer, Laboratory Leakage Rate Test Results for Cracked Reinforced Concrete Elements, *Proceedings of SMiRT-28*, 2025.