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Comparison of Uncertainty According to Covariance Group Structure for VHTR

Tae Young Han, Seungsu Yuk, Chan Soo Kim
Korea Atomic Energy Research Institute, 989-111, Daedeok-daero, Yuseong-gu, Daejeon, Korea
Corresponding author: tyhan@kaeri.re.kr

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1. Introduction

The Korea Atomic Energy Research Institute (KAERI) has developed a sensitivity and uncertainty analysis code system for VHTRs, designated as DeCART2D/MUSAD [1]. This code system employs a two-step uncertainty analysis procedure corresponding to the two-step nuclear design process. In the lattice physics step, DeCART2D [2] generates multi-group constants in a 190-group energy structure. MUSAD then calculates the uncertainties using these constants and covariance data, which are processed from ENDF/B-VIII.0 in the same 190-group structure via the ERRORR(J) module of NJOY. However, the 190-group structure requires a heavy computational burden and high memory overhead, as MUSAD must generate correlated random samples [1] while considering relevant nuclides, cross-section types, energy groups, and spatial regions based on the covariance matrix. In particular, the multi-group constants generated during 2D core depletion calculations demand enormous amounts of memory, making uncertainty analysis practically infeasible on a standard workstation.

In this study, uncertainty analysis using a condensed 47-group structure was examined to improve computational efficiency. The results from the two group structures were compared and evaluated in terms of accuracy. In the following sections, the analysis methodology and computational tools are described in detail, followed by the presentation and discussion of the comparison results.

2. Methods and Results

2.1 Analysis code system

DeCART2D (Deterministic Core Analysis based on Ray Tracing for 2-Dimensional core) was developed by KAERI for lattice physics analysis, solving the 2D neutron transport equation. For uncertainty analysis, DeCART2D solves the generalized adjoint equation by treating the few-group cross sections as generalized adjoint source terms.

MUSAD (Modules of Uncertainty and Sensitivity Analysis for DeCART2D), also developed by KAERI, evaluates sensitivities and uncertainties using covariance data and multi-group cross sections. It utilizes Generalized Perturbation Theory (GPT) [3] to assess the uncertainties in the multiplication factor and few-group cross sections. Furthermore, MUSAD accounts for implicit uncertainty—defined as the change in uncertainty resulting from self-shielding effects—by applying a sensitivity correction method.

2.2 47 Group Covariance and Constants

Conventionally, DeCART2D uses a 47-group structure for PWR analysis which is subset of the 190-group structure and a 190-group structure for VHTR analysis. In this study, the 190-group structure was used for the lattice calculation of DeCART2D and the 47-group structure was adopted to accelerate the uncertainty analysis in MUSAD.

The 47-group covariance data was directly generated using ERRORR(J) of NJOY based on ENDF/B-VIII.0. DeCART2D produced the 47-group cross-sections from the 190-group cross-sections using the group editing modules. Additionally, the group collapsing for the adjoint flux consistent to the 47-group structure was performed via following formula:

$$\phi_G^* = \frac{\sum_{g\in G} \sigma_{tg}\phi_g^*\phi_g}{\sum_{g\in G} \sigma_{tg}\phi_g} \tag{1.1}$$

where, G and g are the indexes for the 47-group and 190-group structure and ϕ_g^* and ϕ_g are the adjoint flux and forward flux, respectively. In addition, σ_{tg} is the total cross section.

Furthermore, group collapsing for the correction factor of the implicit effect [1]—which accounts for uncertainty variations occurring during effective cross-section generation under self-shielding—was performed as follows:

$$f_G = \frac{\sigma'_G}{\sigma_G} \approx \frac{\sum_{g\in G} \sigma'_g\phi_g}{\sum_{g\in G} \sigma_g\phi_g} = \frac{\sum_{g\in G} \sigma'_g\phi_g}{\sum_{g\in G} \frac{\sigma'_g\phi_g}{f_g}} \tag{1.2}$$

where, the correction factor for the implicit effect is defined as $f = \frac{\sigma'}{\sigma}$ and σ' is the self-shielded cross-section and σ is the unshielded cross-section.

2.3 Analysis Results

MHTGR benchmark [4] proposed the Exercise I-1b problem for a single fuel pin model based on the MHTGR design. This model incorporates a double heterogeneous fuel compact with randomly dispersed UCO TRISO particles, as depicted in Figure 1.

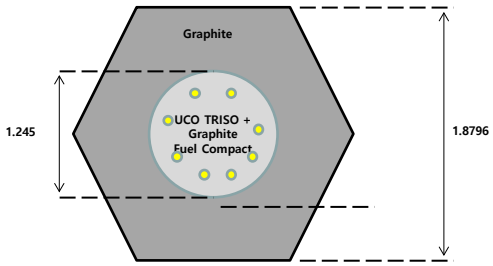


Fig. 1. MHTGR I-1b fuel pin problem

Table I compares the sensitivities of the multiplication factor for the I-1b pin problem. The sensitivity can be defined with respect to cross sections, forward flux, and adjoint flux. The comparison demonstrates that changing the group structure alters the sensitivity by up to 8.9% for the C-12 scattering cross section. It could be attributed to the error of group collapsing for scattering matrix.

Table I : Sensitivity for I-1b problem

Contributor	k_{inf} Sensitivity		Diff.(%)
	190G	47G	
^{235}U ν	9.930E-01	9.930E-01	0.0
^{235}U χ	9.930E-01	9.930E-01	0.0
^{238}U cap[1)]	-3.258E-01	-3.089E-01	-5.2
^{12}C sct[2)]	**3.042E-01**	**3.315E-01**	**8.9**
^{235}U fis[3)]	2.502E-01	2.484E-01	-0.7
^{235}U cap	-2.047E-01	-2.031E-01	-0.8
^{12}C cap	-8.391E-03	-8.497E-03	1.3
^{238}U ν	6.991E-03	6.992E-03	0.0
^{238}U χ	6.991E-03	6.992E-03	0.0
^{238}U fis	3.796E-03	3.805E-03	0.3

1) cap : capture cross section
2) sct : scattering cross section
3) fis : fission cross section

Table II summarizes the multiplication factor uncertainties evaluated using the two group structures. The uncertainty is expressed as the product of the sensitivity vector and the covariance matrix based on the sandwich rule [3]. Therefore, discrepancies between the two group structures stem from both sensitivity variations (Table I) and differences in the covariance matrices. As shown in Table II, the uncertainty difference induced by C-12 scattering is 8.8%. This case aligns well with the sensitivity trend observed in Table I. Conversely, while the sensitivity difference for U-238 capture cross section is 5.2%, its corresponding uncertainty difference reaches up to 7.1%. This indicates that variations in the covariance data structure itself account for the discrepancy.

Overall, the difference in total uncertainty between the 190-group and 47-group calculations was confirmed to be approximately 3.0%.

Table II : Uncertainty for I-1b problem

Contributor	k_{inf} Uncertainty (%)			
	190G Explicit	47G Explicit	190G Explicit +Implicit	47G Explicit +Implicit
^{235}U ν - ^{235}U ν	0.453	0.453	0.453	0.453
^{238}U cap - ^{238}U cap	**0.305**	**0.336**	**0.266**	**0.285**
^{12}C sct - ^{12}C sct	**0.216**	**0.235**	**0.216**	**0.235**
^{235}U cap - ^{235}U cap	**0.093**	**0.102**	**0.093**	**0.101**
^{235}U fis - ^{235}U fis	0.078	0.081	0.078	0.081
^{235}U χ - ^{235}U χ	0.076	0.069	0.076	0.069
^{235}U cap - ^{235}U fis	0.037	0.040	0.037	0.040
^{12}C cap - ^{12}C cap	0.023	0.023	0.023	0.023
^{238}U ν - ^{238}U ν	0.009	0.009	0.009	0.009
^{235}U sct - ^{235}U fis	-0.006	-0.006	-0.006	-0.006
^{238}U fis - ^{238}U fis	0.005	0.005	0.005	0.005
^{235}U sct - ^{235}U cap	0.004	0.003	0.004	0.003
^{238}U sct - ^{238}U cap	-0.003	-0.009	-0.004	-0.008
^{12}C sct - ^{12}C cap	0.003	0.003	0.003	0.003
^{238}U sct - ^{238}U sct	0.002	0.000	0.002	0.000
Total Uncertainty	**0.607**	**0.631**	**0.588**	**0.605**

3. Conclusions

In this study, an uncertainty analysis was performed for the MHTGR Exercise I-1b benchmark using both 190-group and 47-group covariance data. DeCART2D generated the 47-group cross sections and collapsed adjoint fluxes, while MUSAD evaluated the sensitivities and uncertainties associated with the multiplication factor. The numerical results indicate that reducing the group structure affects the C-12 scattering-induced sensitivity by 8.9% and leads to a 7.1% difference in the U-238 capture-induced uncertainty. Nevertheless, the resulting difference in the total uncertainty between the two group structures was constrained to approximately 3.0%.

It is expected that this investigation could provide valuable insights into the impact of energy group condensation for efficient S/U analysis in a current HTGR design project.

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