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Online Quantification of Delayed Neutron Precursor Drift Reactivity in Molten Salt Reactor Flow and Power Transients with a Shadow-Precursor Observer

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Background: MSR System Analysis with Dymola/TRANSFORM

▪ Development of Marine MSR System Technologies (2023–2026)

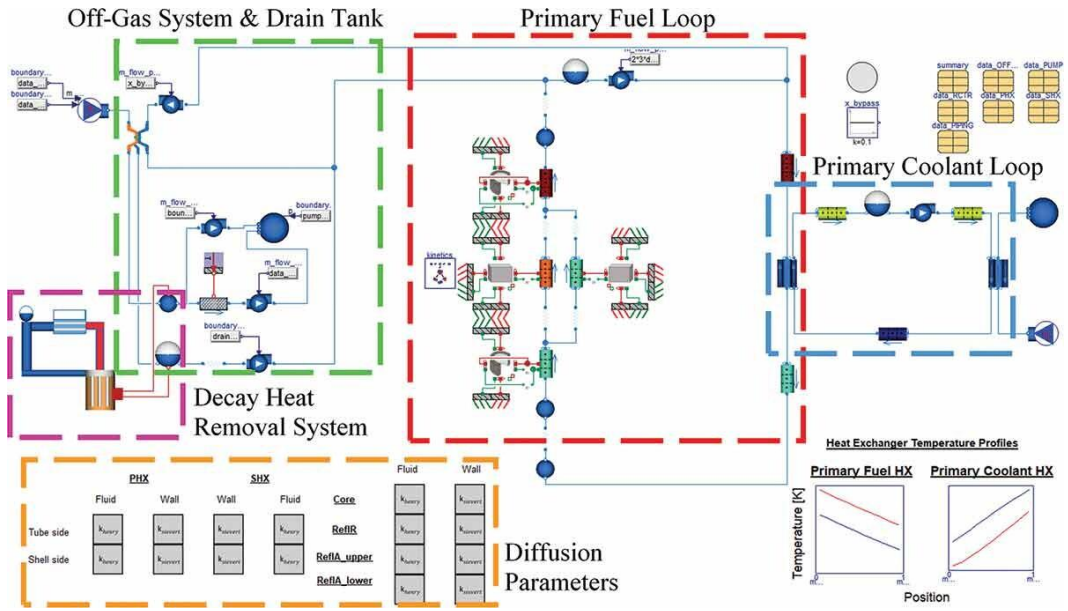
- **Hyundai E&C**: Dymola/TRANSFORM system models of an MSR for performance analysis and control-logic development
- **Kyung Hee University**: coupling of flow and neutron kinetics in these models, and its verification

▪ Dymola

- Modelica-based commercial environment for multi-domain modeling and simulation (Dassault Systèmes)

▪ TRANSFORM

- Open-source Modelica library for advanced reactor system dynamics
- Developed at ORNL
- Applied to a full-plant MSR model

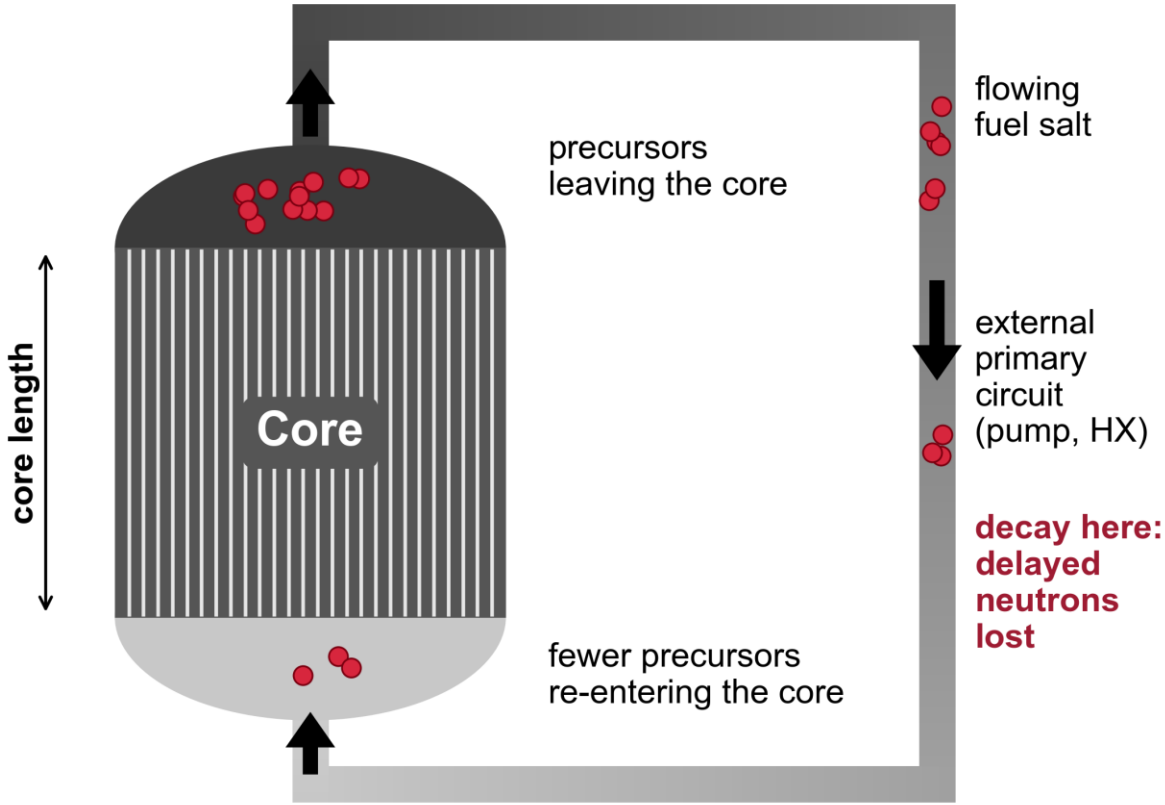


System model of a fluoride salt–fueled thermal reactor based on the MSDR (Greenwood et al., 2020, Fig. 9)

MSR: Circulating Fuel Couples Flow and Criticality

▪ Flow as a Reactivity Channel

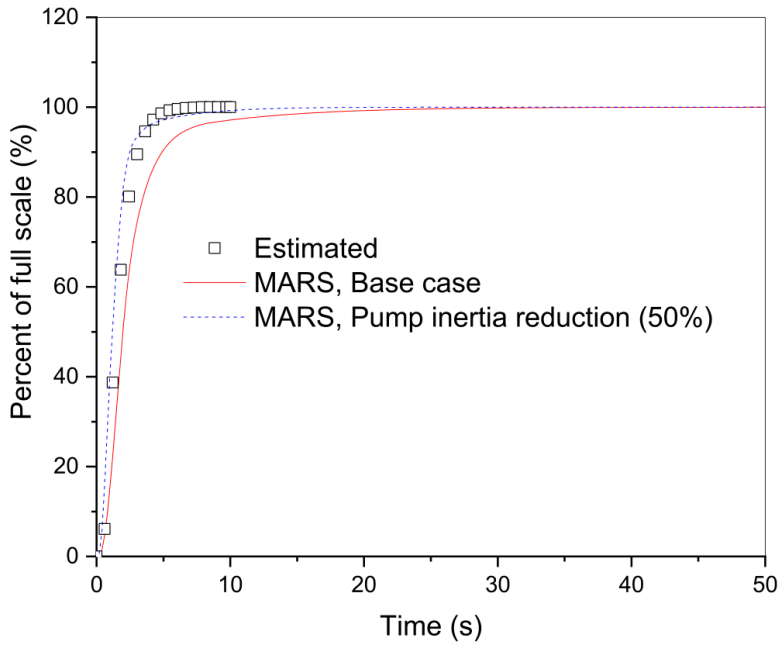
- **DNP advection**: precursors move with the fuel salt (fission products too; not covered here)
- **Loss in the loop**: DNPs decaying outside the core do not contribute; $\beta_{eff} < \beta_0$
- **Consequence**: flow change → delayed neutron source change → reactivity change

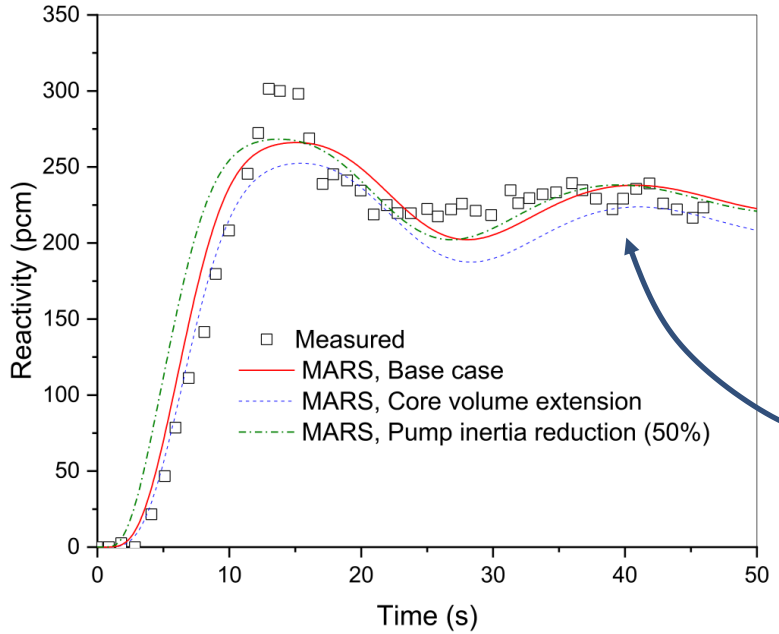


Simplified scheme of a circulating-fuel reactor, redrawn (Cammi et al., 2011, Fig. 1)

Known: Flow-Dependent β_{eff} Loss, Reproduced for MSRE

- Steady-state loss for ^{235}U: ~200 pcm class
- 212 pcm: measured, MSRE zero-power pump tests (Prince et al., 1968)
- MSRE pump startup reproduced with $\rho_{loss}(t)$ (MARS; Jeong et al., 2026)





Test Condition

Flux servo held the reactor critical at a constant flux demand

$$\frac{dN}{dt} \approx 0 = \frac{\rho - \beta_0}{\Lambda} N(t) + \sum_i \lambda_i C_i^{core}(t)$$

$$\Downarrow$$

$$\rho_{ext} = \beta_0 - \beta_{eff} \equiv \rho_{loss}$$

where $\beta_{eff} = \frac{\Lambda}{N} \sum_i \lambda_i C_i^{core}$

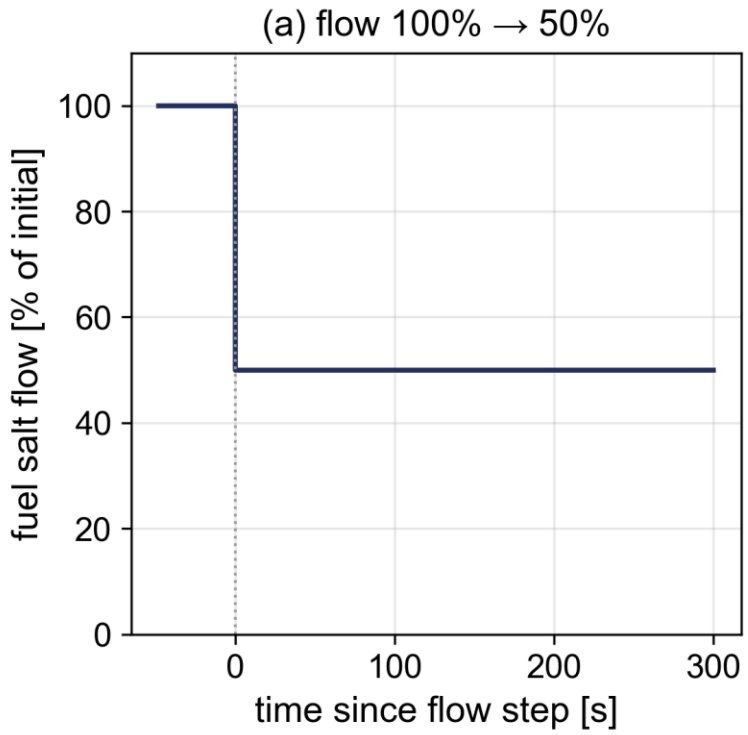
MSRE pump startup: flow (left), rod reactivity (right) (Jeong et al., 2026, Figs. 4–5)

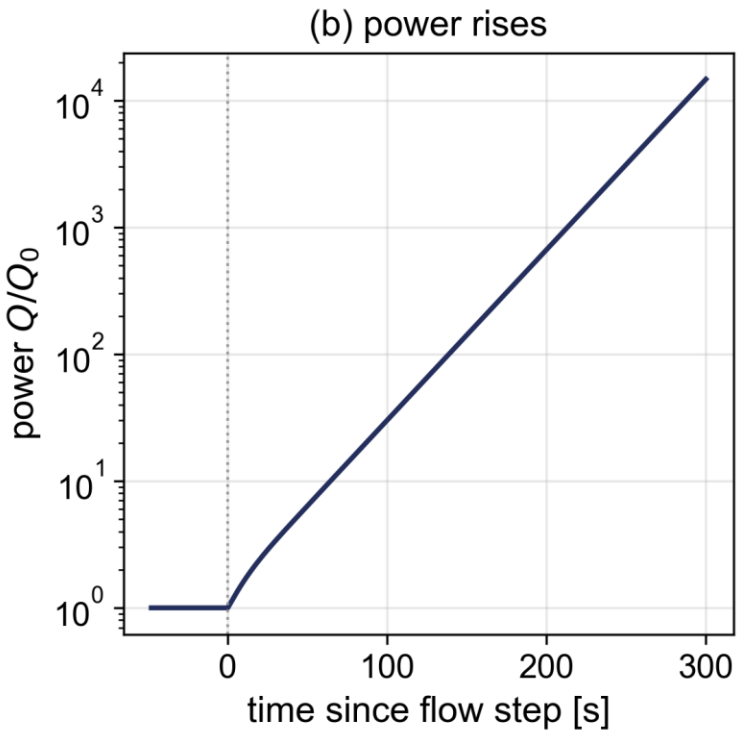
Is the Problem Solved? A Free-Power Example

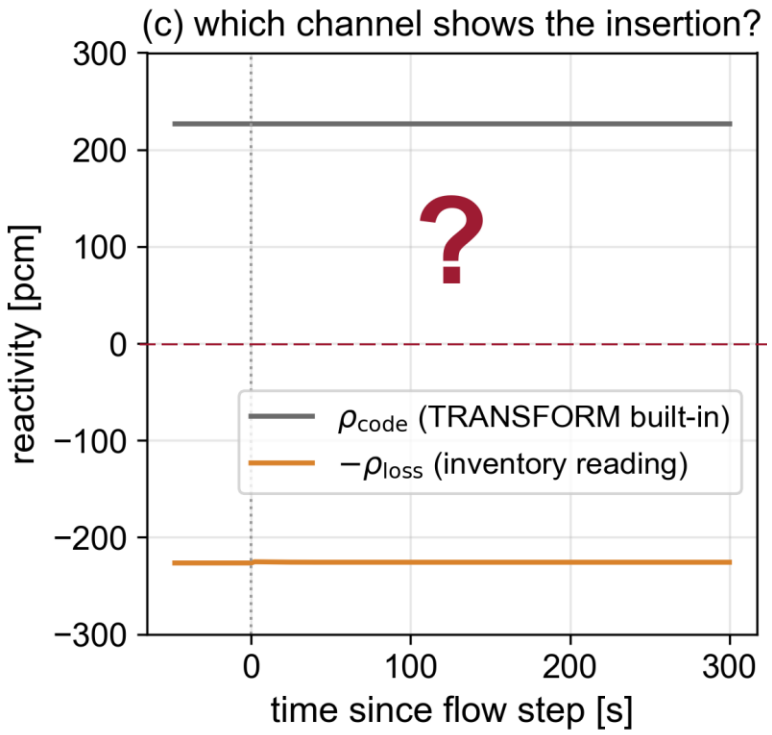
$$\frac{dN}{dt} = \frac{\rho - \beta_0}{\Lambda} N(t) + \sum_i \lambda_i C_i^{core}(t)$$

$$\rho_{code} = \rho_{ext} + \rho_{Tfuel} + \rho_{Xe} + \cdots$$

- **Flow Reduction without Feedback**







Power ×1.47 × 10^4 in 300 s, while ρ_{code} stays at +226.4 pcm and $-\rho_{loss}$ moves only from −226.4 to −225.7 pcm

Purpose and Objectives of This Research

▪ Purpose

- Derive a reactivity parameter that represents the **DNP drift reactivity in general transients**, and verify it numerically on a **set of benchmark problems** designed for this purpose.

▪ Objectives

- **Derive** the **shadow-precursor observer** ρ_{drift}

$$\rho_{drift} = -\frac{\Lambda}{N}\sum_i \lambda_i (C_i^s - C_i^{core})$$

- **Implement** it in Dymola/TRANSFORM as a one-way monitor and report the **drift-inclusive total reactivity**

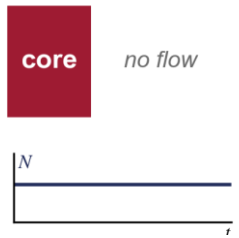
$$\rho_{tot} = \rho_{code} + \rho_{drift}$$

- **Verify** it on **three benchmark problems** against independent references

Case	Power	Flow	External ρ	Feedback	Checked against
1	Constant	100% → 50%	-	No	$-\rho_{loss}$
2	Free	100% → 50%	-	No / FTC	Period → inhour $\rho_{tot} \rightarrow 0$ (FTC)
3	Free	Fixed (100%)	+20/50/100 pcm	No / FTC	Period → inhour $\rho_{tot} \rightarrow 0$ (FTC)

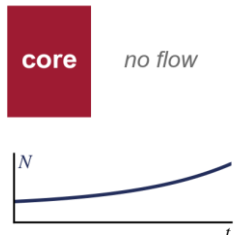
Theory ①: Three Reference States

▪ Three Precursor States under the Same Power History N(t)



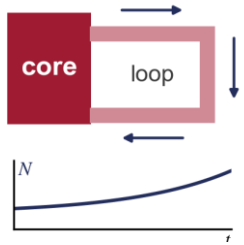
- ① **Equilibrium (eq)**: stationary fuel, at equilibrium

$$\lambda_i C_i^{\mathrm{eq}}(t) = \frac{\beta_i}{\Lambda} N(t)$$



- ② **Shadow (s)**: stationary fuel, same N(t)

$$\frac{dC_i^{\mathrm{s}}}{dt} = \frac{\beta_i}{\Lambda} N(t) - \lambda_i C_i^{\mathrm{s}}$$



- ③ **Circulating (core)**: advection and decay

$$\frac{\partial c_i}{\partial t} + v\frac{\partial c_i}{\partial z} = \frac{\beta_i}{\Lambda} N(t)\, q(z) - \lambda_i c_i, \quad C_i^{\mathrm{core}} = \int_{\mathrm{core}} c_i\, dz$$

q(z): fission power shape, zero in the loop; core cells only in C^{core}

Parameters from eq and core

$$\beta_0 = \frac{\Lambda}{N}\sum_i \lambda_i C_i^{\mathrm{eq}}, \qquad \beta_{\mathrm{eff}}(t) \equiv \frac{\Lambda}{N}\sum_i \lambda_i C_i^{\mathrm{core}}$$

$$\rho_{\mathrm{loss}} = \beta_0 - \beta_{\mathrm{eff}} = \frac{\Lambda}{N}\sum_i \lambda_i \left(C_i^{\mathrm{eq}} - C_i^{\mathrm{core}}\right) \qquad \text{③ vs ①}$$

Same N(t) and dN/dt in the two kinetics equations

s: $$\frac{dN}{dt} = \frac{\rho_{\mathrm{code}} + \rho_{\mathrm{drift}} - \beta_0}{\Lambda} N + \sum_i \lambda_i C_i^{\mathrm{s}}$$

core: $$\frac{dN}{dt} = \frac{\rho_{\mathrm{code}} - \beta_0}{\Lambda} N + \sum_i \lambda_i C_i^{\mathrm{core}}$$

↓ subtract

$$-\rho_{\mathrm{drift}} = \frac{\Lambda}{N}\sum_i \lambda_i \left(C_i^{\mathrm{s}} - C_i^{\mathrm{core}}\right) \qquad \text{③ vs ②}$$

J. Lee, S. Paek, Nucl. Eng. Technol. 59 (2027) 104741

Theory ②: Interpretation of ρ_{drift} and ρ_{tot}

Decomposition of ρ_{loss}

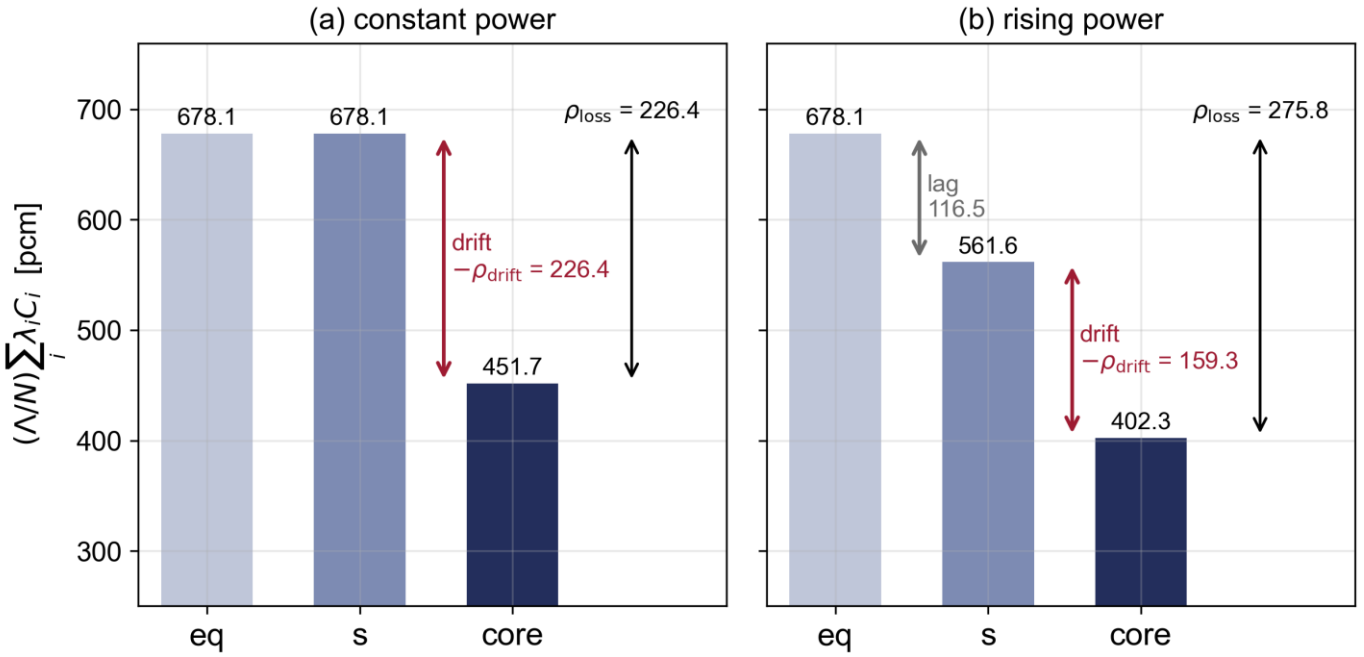
$$\rho_{\text{loss}} = \beta_0 - \beta_{\text{eff}} = \underbrace{\frac{\Lambda}{N}\sum_i \lambda_i (C_i^{\text{eq}} - C_i^{\text{s}})}_{\text{precursor lag}} + \underbrace{\frac{\Lambda}{N}\sum_i \lambda_i (C_i^{\text{s}} - C_i^{\text{core}})}_{\text{drift } (= -\rho_{\text{drift}})}$$

Limiting Cases

- **Zero flow**: $C^{\text{s}} = C^{\text{core}} \rightarrow \rho_{\text{drift}} \equiv 0$ for any $N(t)$
- **Constant power** (a): lag = 0 $\rightarrow \rho_{\text{drift}} = -\rho_{\text{loss}}$
- **Rising power** (b): lag > 0 $\rightarrow -\rho_{\text{loss}}$ overstates the drift loss by the lag

Total Reactivity

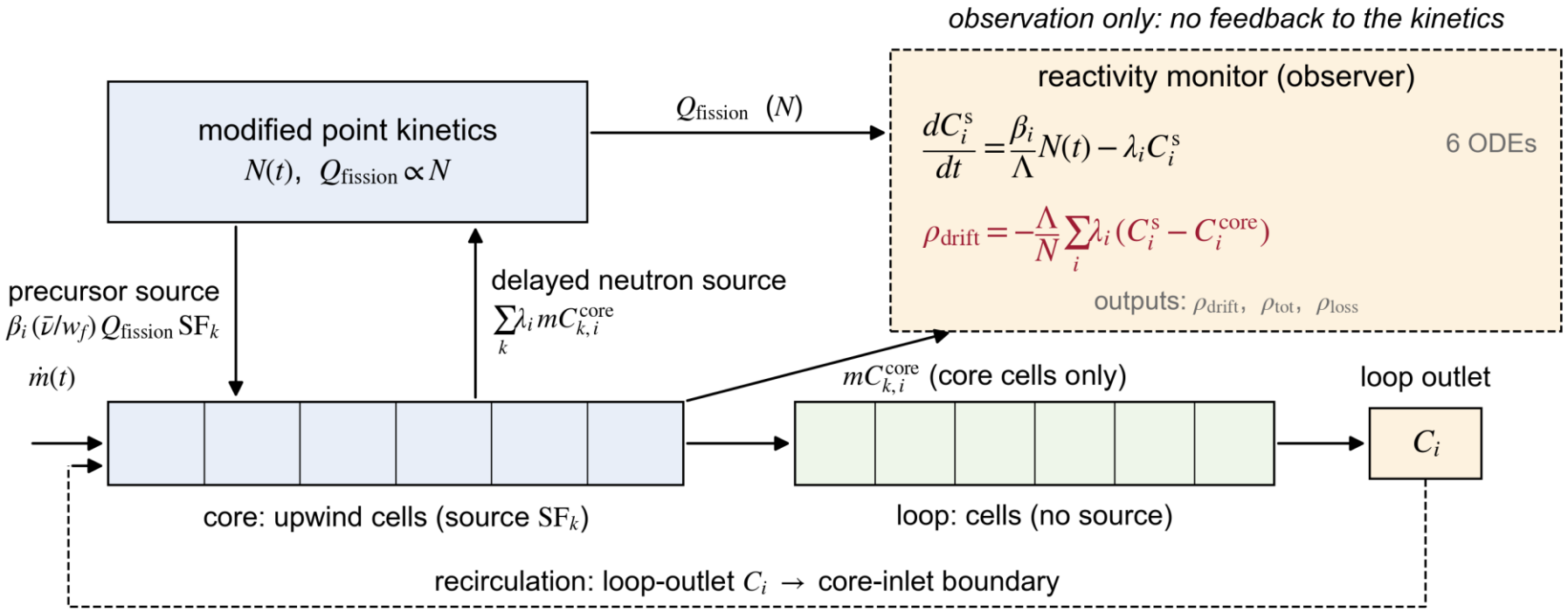
- $\rho_{\text{tot}} = \rho_{\text{code}} + \rho_{\text{drift}}$: s-state reactivity (Theory ①)
- Stationary-fuel equivalent for any $N(t)$



Example: fixed flow, +50 pcm step, t = t_{end}; 10-cell model; bars in pcm

Verification Model in Dymola/TRANSFORM

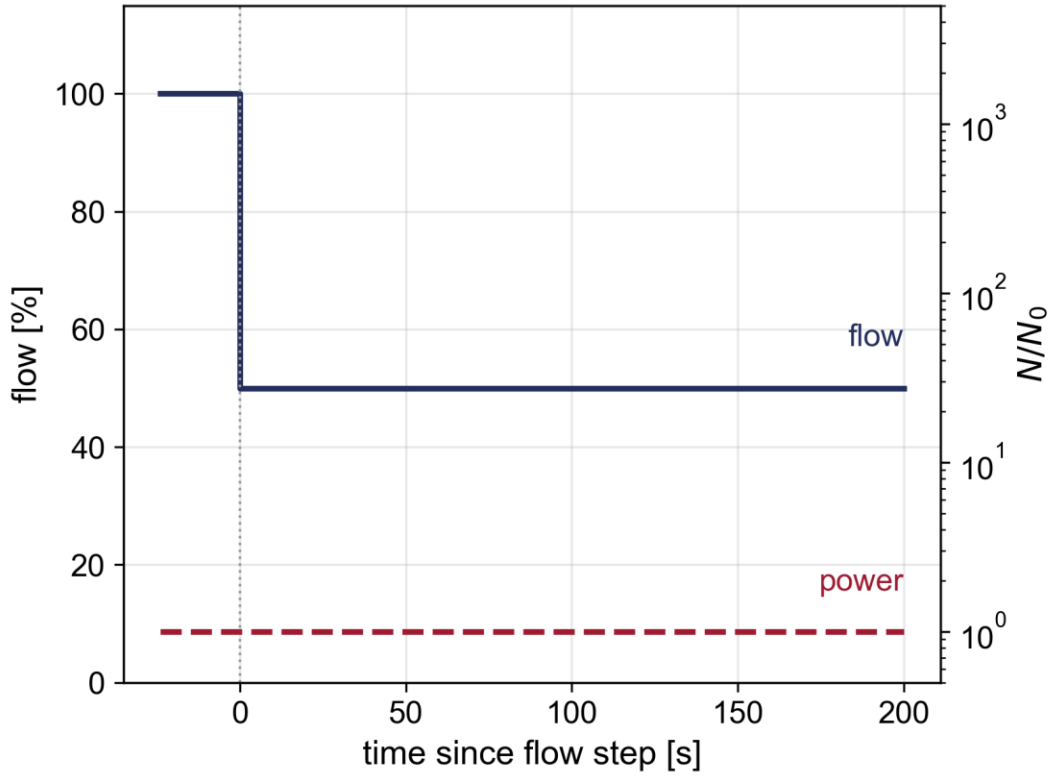
- **Observer Implementation: Six One-Way ODEs**

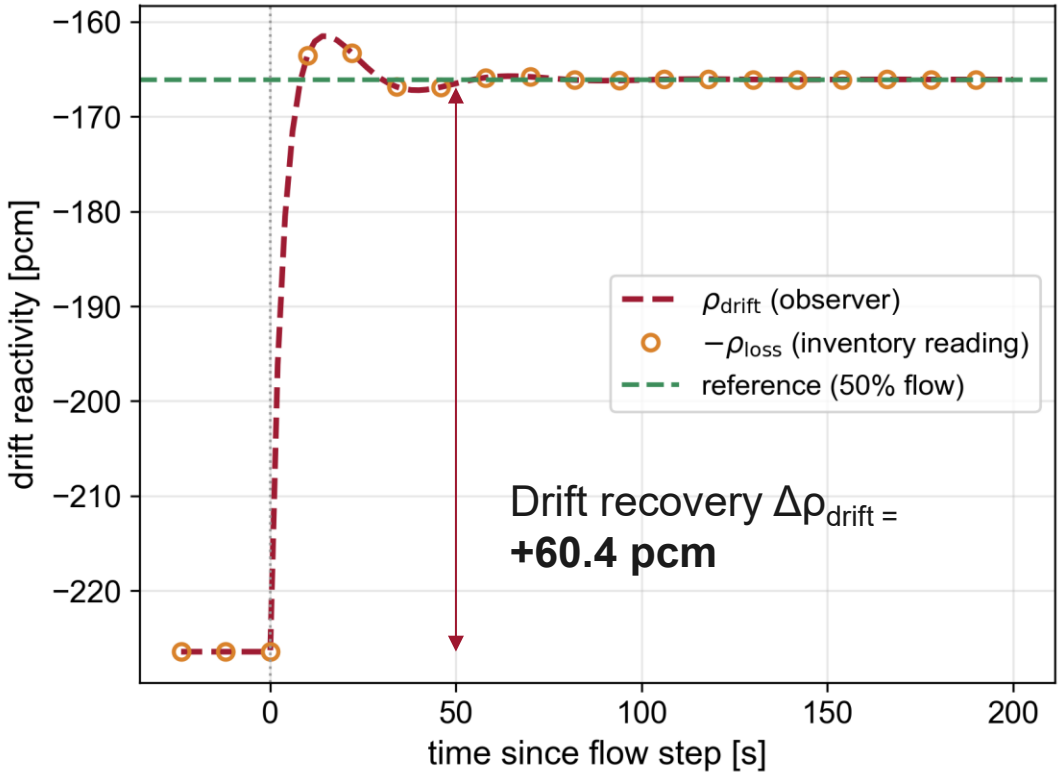


6-group ^{235}U (Jeong and Cho, 2026); modified point kinetics (Greenwood and Betzler, 2019); $\beta_0 = 0.006781$; $\Lambda = 2.4 \times 10^{-4}$ s; $\tau_C/\tau_L = 10.3/14.9$ s; 10 core cells; uniform power; isothermal (FTC: $\alpha = -1 \times 10^{-4}$ K^{-1})

Benchmark 1: Constant Power, Flow 100% → 50%

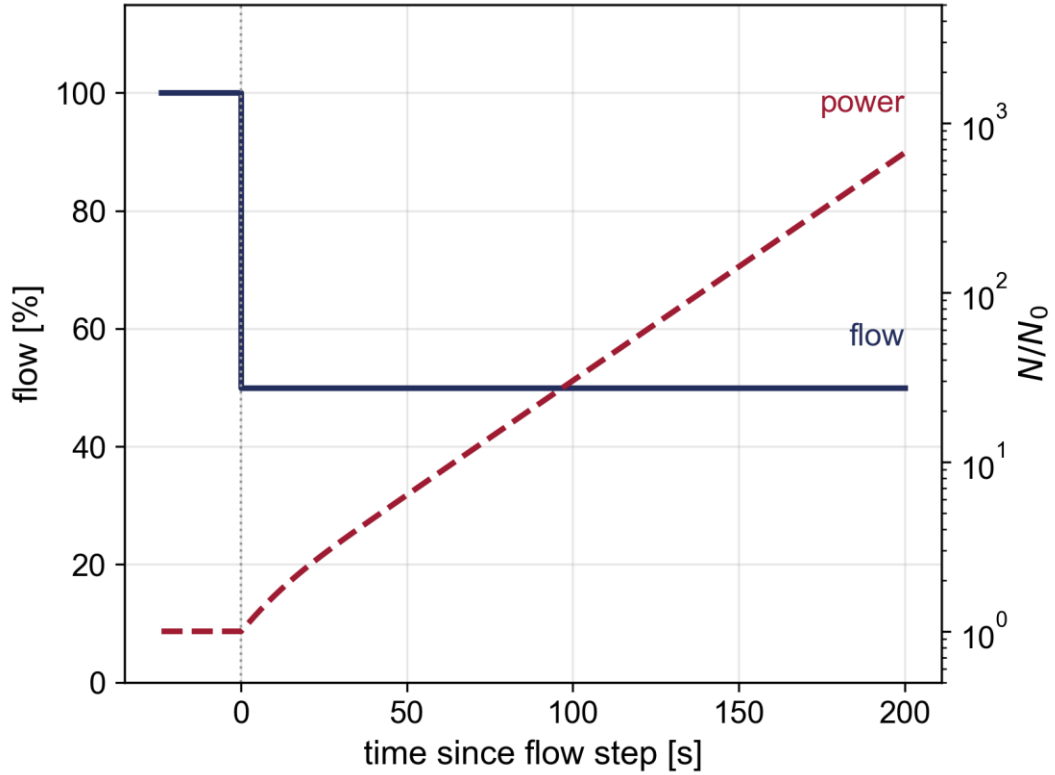
- **Comparison with the Inventory Reading at Constant Power**
  - $\rho_{drift} = -\rho_{loss}$ at all times, within machine precision ($\max|\rho_{drift} + \rho_{loss}| = 2.6\times10^{-18}$)

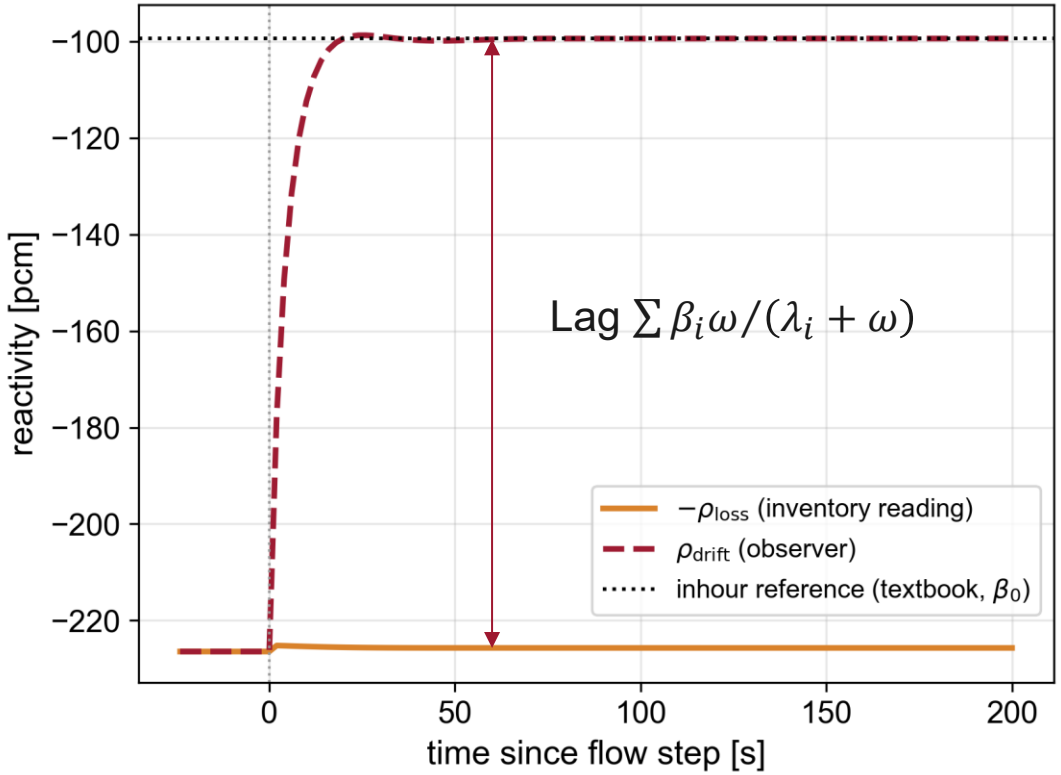




Benchmark 2: Free Power, Flow 100% → 50%, No Feedback

- **Textbook Inhour Reference and the Hidden Drift Insertion**
  - Answers the free-power example: the drift inserts **+127.0 pcm**, hidden from $-\rho_{loss}$ by the lag
  - ρ_{drift} matches the textbook inhour reference (diff. 5.1 × 10^{-11})

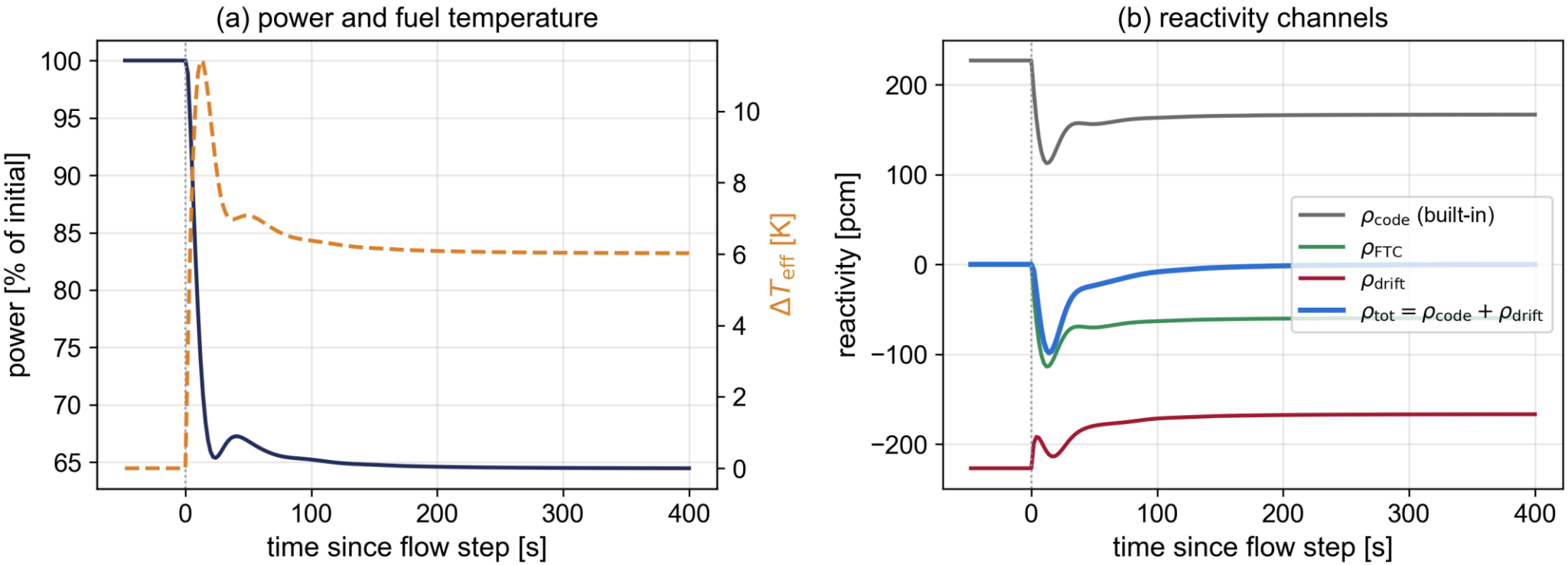




Benchmark 2: Free Power, Flow 100% → 50%, with Fuel Temperature Feedback

▪ Reactivity Balance at the New Steady State

- Only $\rho_{tot} = \rho_{code} + \rho_{drift}$ returns to **zero at the new steady state**



New steady state: power 64.5%, ΔT_{eff} = +6.0 K, $\Delta\rho_{drift}$ = +60.20 pcm; ρ_{code} = +166.7 pcm, while max $|\rho_{ext} + \rho_{FTC} + \rho_{drift}| = 1.4 \times 10^{-13}$

Benchmark 3: Fixed Flow, External Reactivity Step

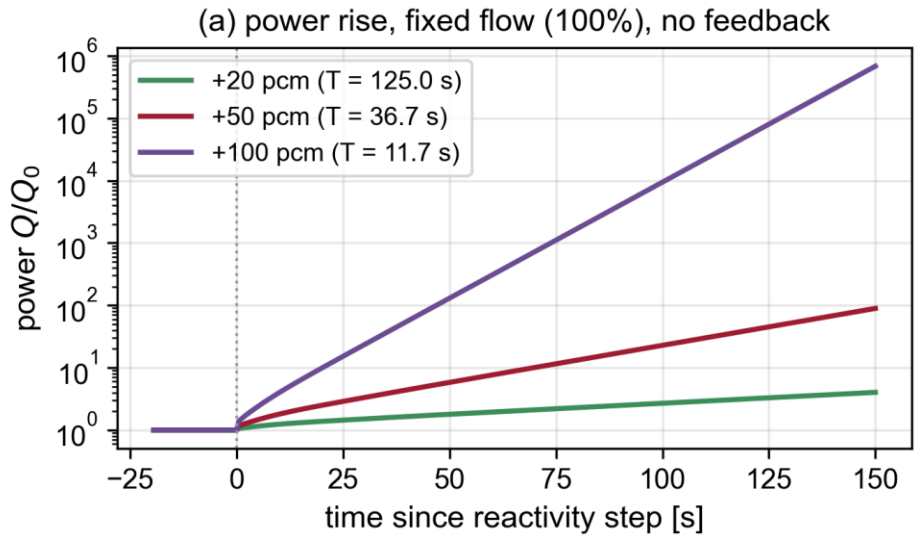
▪ Drift Recovery at Fixed Flow

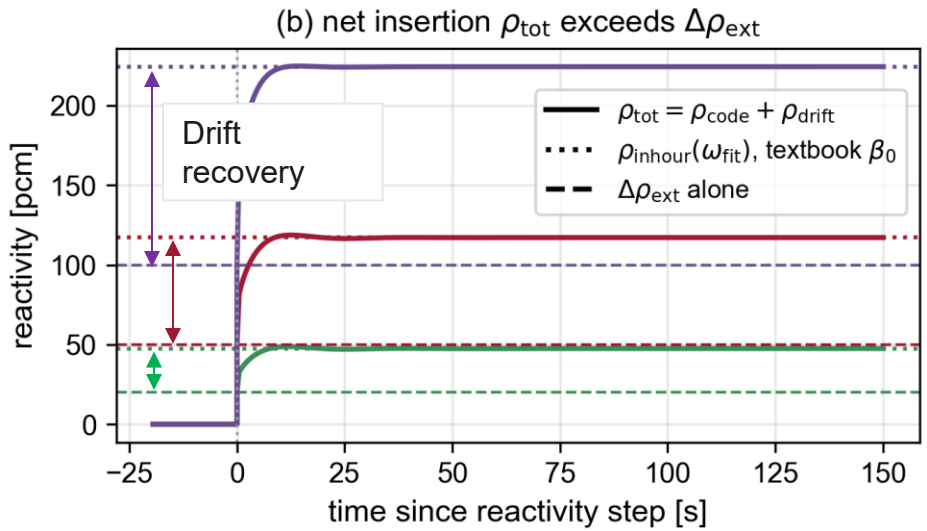
- Flow is fixed, yet **drift loss shrinks as the power rises**: $\rho_{tot} = \Delta\rho_{ext} + \Delta\rho_{drift}$
- The period is **shorter by a factor of 2.7–4.0** than the textbook prediction for $\Delta\rho_{ext}$ alone

ρ in pcm

$\Delta\rho_{ext}$ (input)	$\Delta\rho_{drift}$ (observer)	$\rho_{tot} = \Delta\rho_{ext} + \Delta\rho_{drift}$	T_{sim} [s]	$T_{textbook}$ [s] ($\Delta\rho_{ext}$ only)
+20	+27.6	47.6	125.0	338.4
+50	+67.1	117.1	36.7	117.6
+100	+124.3	224.3	11.7	46.5

Check: $\rho_{inhour}(1/T_{sim}) = \rho_{tot}$ within 1.5×10^{-9} (dimensionless) for all three steps

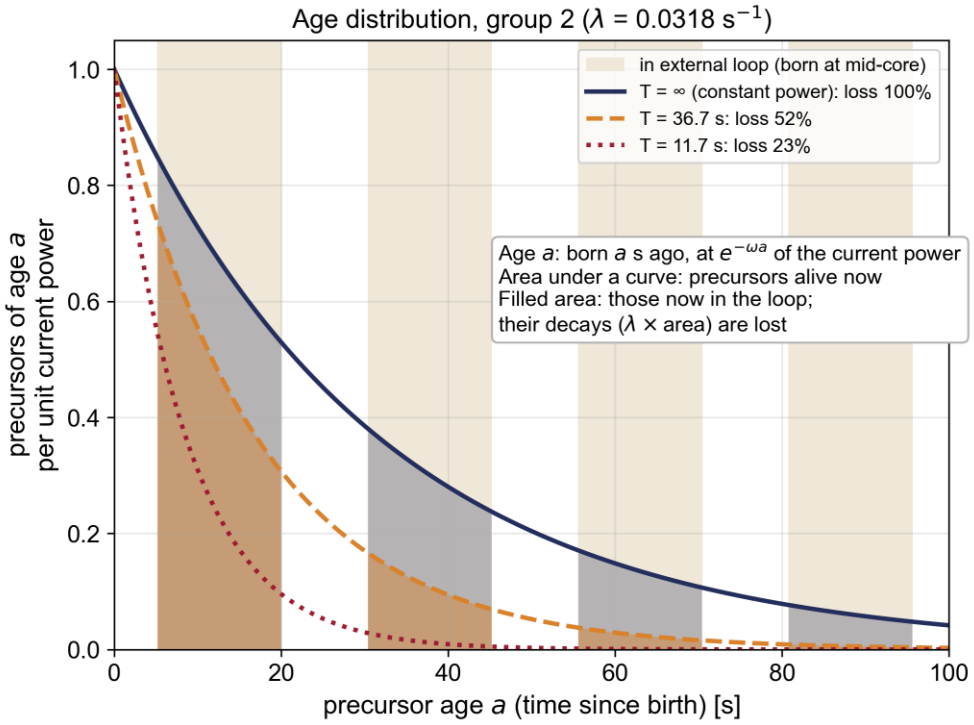


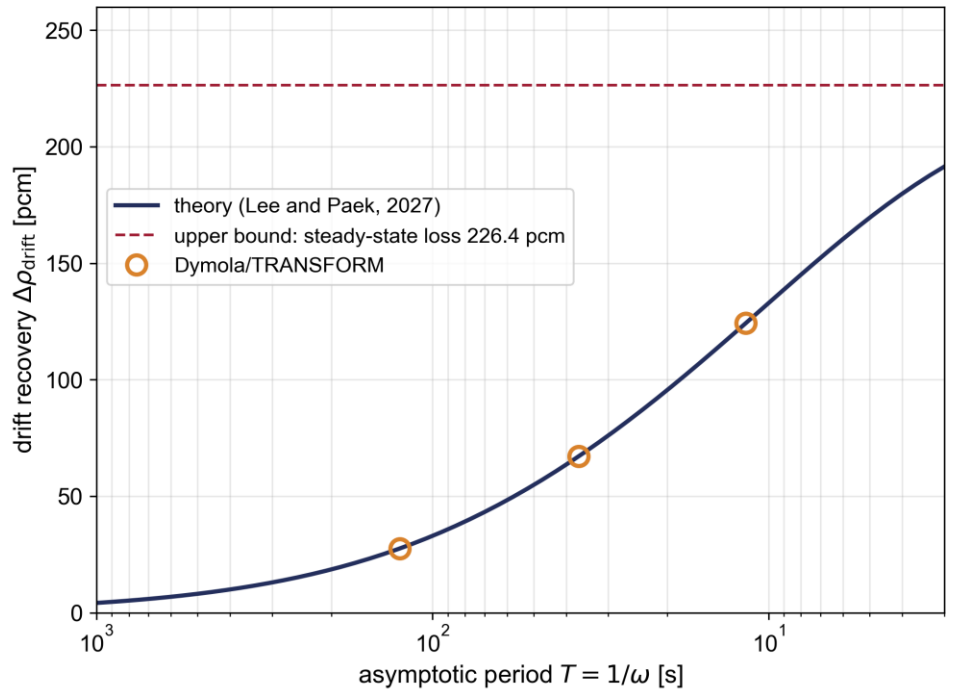


Benchmark 3: Why the Drift Loss Shrinks on a Rising Power

- **Precursor Age Distribution and Drift Recovery**

$$\frac{\partial c_i}{\partial t} + v\frac{\partial c_i}{\partial z} = \frac{\beta_i}{\Lambda} N\, q(z) - \lambda_i c_i \qquad c_i = \tilde{c}_i\, e^{\omega t}, \quad N = N_0\, e^{\omega t} \quad \Rightarrow \quad v\frac{d\tilde{c}_i}{dz} = \frac{\beta_i}{\Lambda} N_0\, q(z) - (\lambda_i + \omega)\,\tilde{c}_i$$





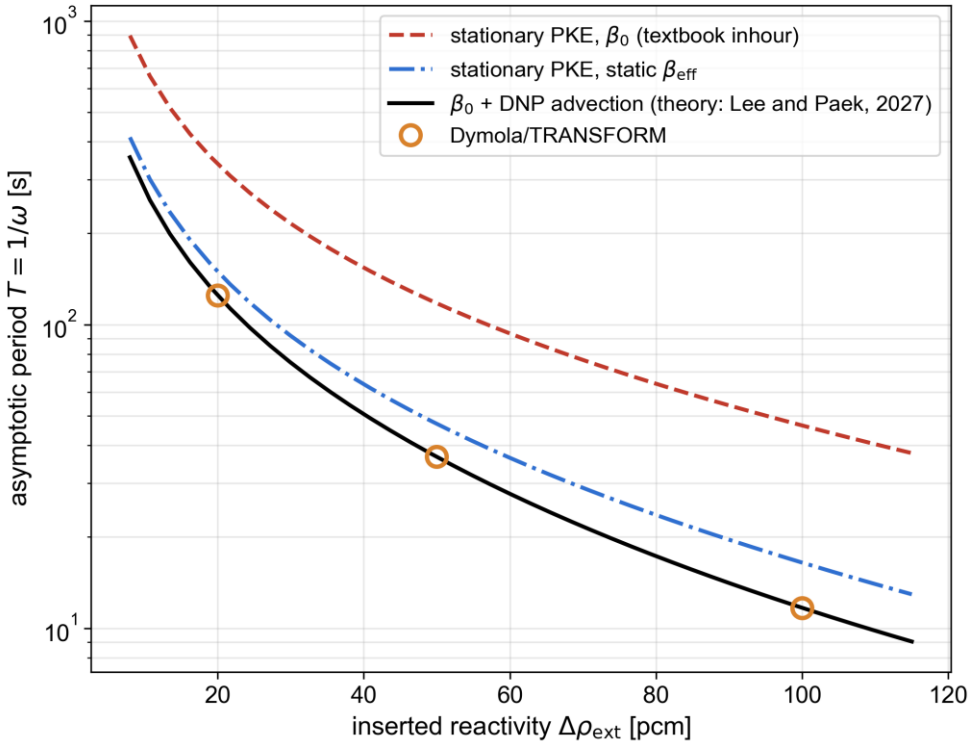
$$\rho^{\infty}_{\text{drift}}(\omega) = -\sum_i \beta_i \frac{\lambda_i}{\lambda_i + \omega}\left[1 - F(\lambda_i + \omega)\right] \qquad F(\mu) = 1 - \frac{(1 - e^{-\mu\tau_C})(1 - e^{-\mu\tau_L})}{\mu\tau_C(1 - e^{-\mu(\tau_C + \tau_L)})}$$

Benchmark 3: Predicted Period for a Reactivity Insertion

▪ Period Prediction: Three Kinetics Models

- Static models are **non-conservative** for a reactivity insertion
- β_0 with explicit drift gives the **shortest period**

$$\frac{dN}{dt} = \frac{\rho - \beta}{\Lambda} N(t) + \sum_i \lambda_i C_i$$



Period for a +50 pcm insertion

Kinetics model	T [s]
β_0 + DNP advection (TRANSFORM)	**36.7**
Stationary PKE, static β_{eff}	47.1 (+28%)
Stationary PKE, β_0	117.6 (3.2×)

Curves: inhour solution of each model, on the same 10-cell mesh

$\beta = \beta_0$, $C = C^{core}$ with advection → 36.7 s
$\beta = \beta_{eff}$, C without advection → 47.1 s
$\beta = \beta_0$, C without advection → 117.6 s

Conclusions and Outlook

Conclusions

- **Derived** the shadow-precursor observer ρ_{drift}: drift reactivity in general transients, beyond steady state and constant power
- **Implemented** as six one-way ODEs in Dymola/TRANSFORM; $\rho_{tot} = \rho_{code} + \rho_{drift}$ shows what $-\rho_{loss}$ and ρ_{code} cannot
- **Verified** on three benchmarks against closed-form and textbook inhour references, better than 2×10^{-9}
- Drift reactivity depends **on the power history even at fixed flow**; static models predict **non-conservative** (longer) periods

Outlook

- Six one-way ODEs: **applicable to any system code** with DNP advection
- MSR transient analysis and control-system design; **experimental check** via inverse kinetics in rod-fixed pump transients
- Next: pump coastdown transient analysis with the observer

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