

Modification and Validation of a Single-Phase Friction Factor Correlation for Helically Coiled Tubes

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1. Introduction

Small Modular Reactors (SMRs) have emerged as a promising solution in the nuclear industry due to their enhanced safety features, modularity, and compact design. To achieve compactness while enhancing safety, efficiency, and overall system performance, various innovative design features have been adopted. One of the key features is the use of helically coiled steam generators (HCSGs). HCSGs provide superior heat transfer performance while achieving a more compact configuration compared to conventional steam generators.

However, the helical geometry induces centrifugal forces that generate secondary flow, which significantly alter the flow structure. While this enhances heat transfer, it also leads to increased pressure drop due to intensified mixing and additional frictional losses. Accurate prediction of pressure drop is essential for reliable thermal-hydraulic design, pump sizing, structural integrity assessment, and overall system performance optimization in SMRs.

In this study, an existing single-phase friction factor correlation, Churchill correlation [1] was modified to improve its predictive capability across a broad range of Reynolds numbers and curvature ratios. The modified correlation was validated against the selected experimental datasets and compared with established models.

2. Single-Phase Frictional Pressure Drop Correlation

2.1 Churchill correlation for straight tubes

Churchill proposed a unified correlation for predicting the friction factor over the entire flow regime, including laminar, transitional, and turbulent flows. The correlation provides a single explicit expression, eliminating the need for iterative procedures required by the Colebrook equation. The correlation is applicable over a wide range of Reynolds number and relative roughness.

The Churchill correlation can be expressed in terms of the Fanning factor as follows:

$$f = 2 \left[\left(\frac{8}{Re} \right)^{12} + \frac{1}{(\alpha + b)^{1.5}} \right]^{1/12} \quad (1)$$

$$\alpha = \left[2.457 \ln \left(\frac{1}{\left(\frac{7}{Re} \right)^{0.9} + 0.27 \frac{\epsilon}{D}} \right) \right]^{16} \quad (2)$$

$$b = \left(\frac{37530}{Re} \right)^{16} \quad (3)$$

The first term in the Churchill correlation represents the laminar contribution, which approaches the analytical solution for developed flow in circular tubes ($f=16/Re$). The second term, governed by the auxiliary functions α and b , represents the turbulent contribution and reproduces the Colebrook type behavior by incorporating both Reynolds and relative roughness ϵ/D .

Although the Churchill correlation provides a robust framework for straight-tube flows, its direct application to helically coiled tubes is limited. In curved geometries, centrifugal forces generate Dean vortices, which significantly increase frictional losses compared to straight pipes. Consequently, straight-tube correlations tend to underestimate the friction factor in helically coiled tubes.

2.2 Modification of the Churchill Correlation [2]

To account for curvature effects in helically coiled tubes, the original correlation was modified in three aspects to account for curvature effects across different flow regimes.

First, the laminar term was corrected by multiplying it by the White correlation [3] to incorporate curvature-induced friction in the laminar regime. This modification allows the model to reflect the deviation from the straight-tube behavior due to secondary flow effects.

$$f_{white} = \left[1 - \left(1 - \left(\frac{11.6}{\text{Re} \sqrt{d/D}} \right)^{0.45} \right)^{1/0.45} \right]^{-1} \quad (4)$$

Second, the auxiliary function b , which governs the blending between laminar and turbulent contributions, was reformulated to explicitly control the transition Reynolds number.

Physically, the transition Reynolds number for straight tubes is approximately 2300. In the Churchill structure, the effective transition point is embedded in the balance between the laminar and turbulent terms. For $D/d=860$, the transition Reynolds number predicted corresponds to $\text{Re}=2301.4$, which is consistent with the straight-tube limit. Since transition in straight pipes occurs at approximately $\text{Re}=2300$, and the flow approaches straight-tube behavior as $D/d \rightarrow \infty$, the b term was modified to enforce a curvature-dependent critical Reynolds number:

$$b' = \left(\frac{37530 \cdot \frac{\text{Re}_{crit}}{2301.4}}{\text{Re}} \right)^{16} \quad (5)$$

$$\text{Re}_{crit} = 2.0 \cdot 10^4 \cdot \left(\frac{d}{D} \right)^{0.32} \quad (6)$$

where Re_{crit} represents the critical Reynolds number corresponding to the given curvature ratio. This modification enables the transition Reynolds number to be explicitly controlled while preserving the original blending structure of the Churchill correlation.

Finally, in the turbulent regime, an additional curvature correction based on the Ito correlation [4] was incorporated. In the Ito formulation, the term represents the increase in friction factor due to curvature effects in helically coiled tubes. In the present model, this term was added when ensuring that curvature-induced friction enhancement is applied only in the turbulent regime. Thus, the final form of the modified Churchill correlation incorporating curvature effects can be expressed as follows:

$$f = \begin{cases} 2 \left[\left(\frac{8 \cdot f_{white}}{\text{Re}} \right) + \frac{1}{(\sigma + b')^{1.5}} \right]^{1/12} & \text{Re} < \text{Re}_{crit} \\ 2 \left[\left(\frac{8 \cdot f_{white}}{\text{Re}} \right) + \frac{1}{(\sigma + b')^{1.5}} \right]^{1/12} + 0.00725 \left(\frac{d}{D} \right)^{0.5} & \text{Re} \geq \text{Re}_{crit} \end{cases} \quad (7)$$

3. Validation and Results

3.1 Methodology

The modified correlation was validated against published experimental data for single-phase flow in helically coiled tubes. The selected datasets cover a wide range of Reynolds number and D/d ratios representative of practical SMR operating conditions. The D/d ratio and Reynolds number for the datasets used in this study are summarized in Table 1.

Furthermore, four different models were compared in this study: the straight-tube Churchill correlation, the modified Churchill correlation proposed in the present work, and the recently published PNU model [5].

Table 1 Summary of experimental datasets used for single-phase friction factor validation

Author	Geometry		Reynolds Number
	D/d	Pitch (mm)	
Cioncolini et al. [6]	6.9-369.0	7-25	1000-60,000
Colombo et al. [7]	79.8	800	1800-30,000
Xiao et al. [8]	14.4-26.2	54-121	22,000-100,000
Global	6.9-369.0	7-800	1000-100,000

3.2 Results and Discussion

Figures 1–6 present validation results of the four friction factor correlations against the experimental datasets reported by Cioncolini (Figs. 1–5) and Colombo (Fig. 6).

The original Churchill correlation consistently underpredicts the friction factor over nearly the entire Reynolds number range, particularly in strongly curved geometries. This behavior is expected because the correlation was developed for straight tubes and does not account for curvature-induced secondary flows. In helically coiled tubes, centrifugal forces generate Dean vortices that enhance momentum transport and wall shear stress, resulting in higher friction factors than those predicted by straight-tube correlations.

The Ito correlation, which was developed for turbulent flow in curved tubes, provides good agreement with the experimental data in the fully turbulent regime. However, noticeable deviations appear in the laminar and transitional regions, reflecting the limited applicability of the correlation outside high-Reynolds-number conditions.

Both the modified Churchill correlation and the PNU model show strong agreement with the experimental data across all flow regimes. By incorporating curvature-dependent corrections and improved transition behavior, these models capture friction enhancement more consistently than the conventional straight-tube formulation.

A quantitative comparison of prediction accuracy is summarized in Figure 7 and Table 2. The PNU model exhibits the lowest Mean Absolute Percentage Error (MAPE) of 4.1%, indicating the best overall agreement with the combined experimental datasets. The Ito correlation yields a MAPE of approximately 7.5%, primarily due to its strong performance in the turbulent regime. The modified Churchill correlation results in a MAPE of 8.1%, demonstrating comparable overall accuracy.

An important distinction emerges in cases with relatively low curvature ($D/d > 150$), as shown in Figures 4 and 5. In these conditions, the modified Churchill correlation provides better agreement with the experimental data than the PNU model. This behavior suggests that the present formulation maintains stronger asymptotic consistency, smoothly recovering straight-tube characteristics as curvature effects diminish.

Furthermore, since practical SMR steam generator designs often operate in moderate-to-low curvature configurations, this characteristic enhances the applicability of the proposed correlation to realistic system conditions.

Overall, the results confirm that incorporating curvature effects into the Churchill correlation significantly enhances predictive capability. While the PNU model provides the best overall statistical performance, the modified Churchill correlation offers competitive accuracy with improved consistency in the low-curvature limit, making it suitable for system-level thermal-hydraulic analyses spanning a wide range of geometric conditions.

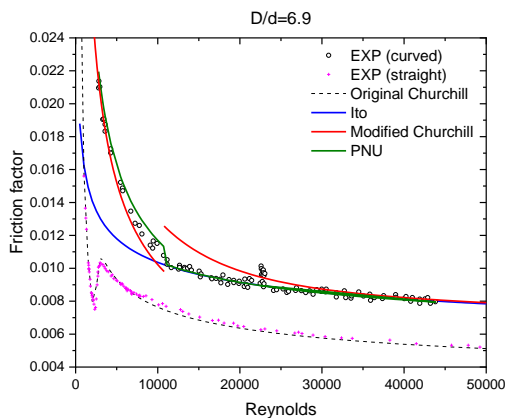


Figure 1 Validation results for the Cioncolini experiment ($D/d=6.9$)

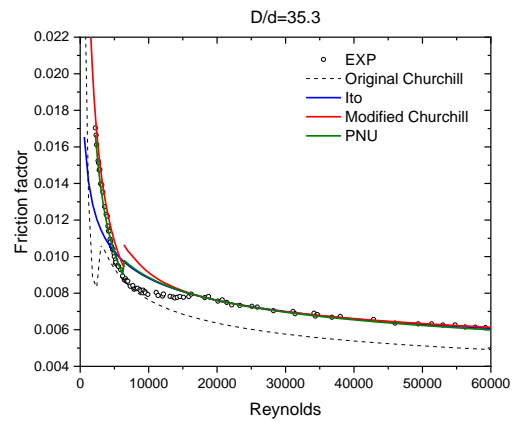


Figure 2 Validation results for the Cioncolini experiment ($D/d=35.3$)

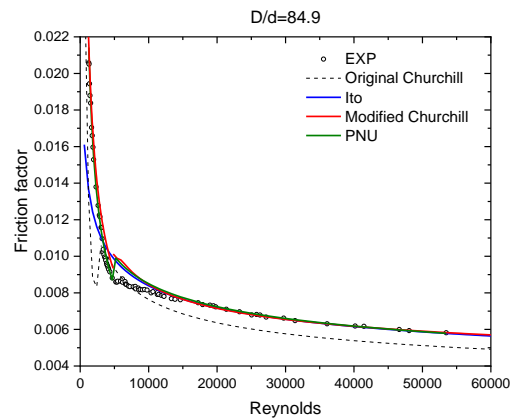


Figure 3 Validation results for the Cioncolini experiment ($D/d=84.9$)

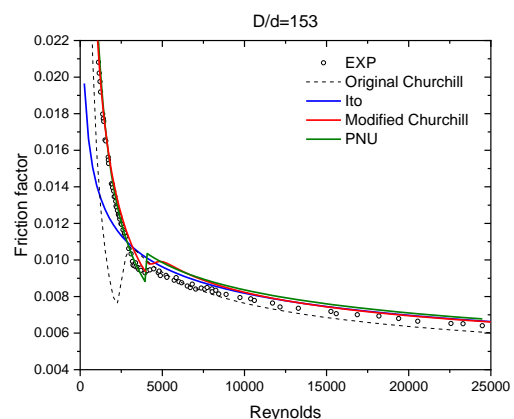


Figure 4 Validation results for the Cioncolini experiment ($D/d=153$)

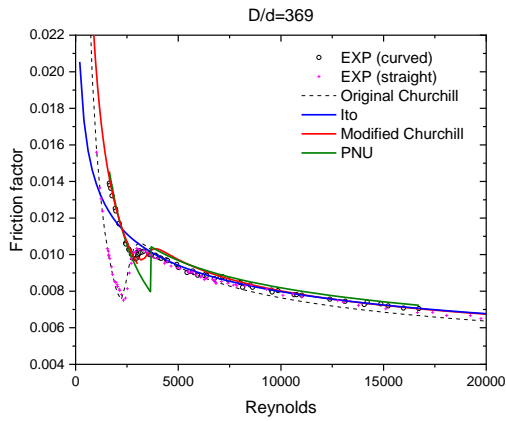


Figure 5 Validation results for the Cioncolini experiment ($D/d=369$)

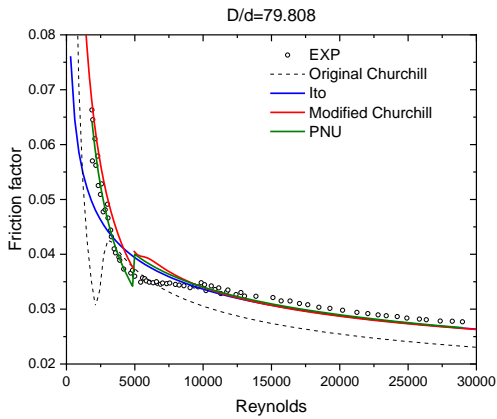


Figure 6 Validation results for the Colombo experiment ($D/d=79.8$)

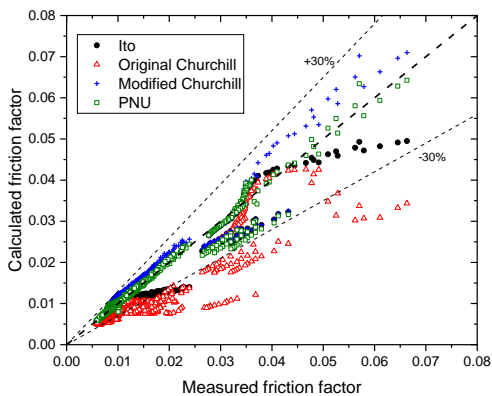


Figure 7 Calculated vs. measured friction factors for the correlations validated against the experimental datasets

Table 2 MAPE of the evaluated friction factor correlations

Correlation	MAPE
Churchill	18.1
Ito	7.5
Modified Churchill	8.1
PNU	4.1

4. Conclusions

In this study, the Churchill friction factor correlation was modified to improve its applicability to single-phase flow in helically coiled tubes. The modified correlation was validated against published experimental datasets covering a wide range of Reynolds numbers and curvature ratios. Comparative analysis was performed against the original Churchill correlation, the Ito correlation, and the recently proposed PNU model.

Quantitative evaluation indicates that the PNU model yields the lowest overall MAPE of 4.1%. The modified Churchill correlation results in a slightly higher overall MAPE of approximately 8%, indicating somewhat reduced global accuracy compared to the PNU model. However, its predictive performance improves as the D/d ratio increases. This trend suggests that the present formulation more effectively recovers straight-tube behavior as curvature effects diminish.

Overall, the proposed modification extends the applicability of the Churchill correlation to helically coiled tubes by incorporating curvature-dependent effects while preserving its unified and computationally efficient structure. Although the PNU model exhibits the lowest overall prediction error, the modified Churchill correlation demonstrates improved agreement in low-curvature conditions and better consistency with the straight-tube limit. These characteristics make the proposed correlation suitable for system-level thermal-hydraulic analyses of helically coiled steam generators in SMR applications.

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