Development of a SPH-DOM Solver for Radiative Heat Transfer with Complex Geometry

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1. Introduction

Molten salt reactor (MSR) is being developed as an advanced nuclear system, and its deployment requires credible safety analyses for specified accident scenarios. One postulated accident scenario considered for many MSR concepts is a rupture within the primary loop, which can result in molten fuel salt being spilled from the system [1]. In such an event, radiative heat transfer can play a major role in cooling due to the high temperature of the salt. In addition, the high freezing point of molten salt can lead to freezing during cooling, which may directly influence the salt relocation and spreading behavior. During solidification, fragmented crust may form and float, producing granular-flow-like behavior and highly irregular, complex interfaces. Modeling such geometries often requires substantial effort, and certain radiative heat transfer models may not be supported for such configurations in commercial mesh-based CFD tools.

Smoothed particle hydrodynamics (SPH), as a mesh-free method, represents the domain using discretized particles and tracks their motion, making it well suited for handling large deformations and free-surface flows. The discrete ordinates method (DOM) is a widely used deterministic method for solving radiative transfer with controllable accuracy. In this study, we develop an SPH-DOM solver to enable stable radiative transfer analysis in complex geometries. The solver is verified using established benchmark problems and demonstrated to operate robustly in such configuration, indicating its potential for future application to molten salt spreading simulations with radiative heat transfer.

2. Methodology

2.1. Radiative Transfer Model

Considering a gray, quasi-steady radiative field in a participating medium, the radiative transfer equation (RTE) for direction $\hat{\mathbf{s}}$ is written as

$$(1)\ \hat{\mathbf{s}}\cdot\nabla I(\mathbf{r},\hat{\mathbf{s}}) = \kappa I_b(\mathbf{r}) - \beta I(\mathbf{r},\hat{\mathbf{s}}) + \frac{\sigma_s}{4\pi}\int_{4\pi} I(\mathbf{r},\hat{\mathbf{s}}')\Phi(\hat{\mathbf{s}}',\hat{\mathbf{s}})d\Omega'$$

where I is the radiative intensity, and κ, β, σ_s are the absorption, extinction, scattering coefficients. Φ is phase function that scattered from $\hat{\mathbf{s}}'$ to $\hat{\mathbf{s}}$ [2]. For a diffuse gray opaque wall, the outgoing ($\hat{\mathbf{n}}\cdot\hat{\mathbf{s}} > 0$) intensity is

$$(2)\ I(\mathbf{r}_w,\hat{\mathbf{s}}) = \varepsilon_w I_b(\mathbf{r}_w) + \frac{\rho_w}{\pi}\int_{\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}'<0} I(\mathbf{r}_w,\hat{\mathbf{s}}')\,|\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}'|d\Omega'$$

The radiative heat flux is obtained by

$$(3)\ \mathbf{q}(\mathbf{r}) = \int_{4\pi} I(\mathbf{r},\hat{\mathbf{s}})\hat{\mathbf{s}}d\Omega$$

In DOM, the angular domain is discretized into ordinates $\{\hat{\mathbf{s}}_j\}_{j=1}^{n}$ with weights $\{\widehat{w}_j\}_{j=1}^{n}$, converting the RTE into directional transport equations and the heat flux becomes

$$(4)\ \hat{\mathbf{s}}_i\cdot\nabla I(\mathbf{r},\hat{\mathbf{s}}_i) = \kappa I_b(\boldsymbol{r}) - \beta I(\mathbf{r},\hat{\mathbf{s}}_i) + \frac{\sigma_s}{4\pi}\sum_{j=1}^{n} w_j I(\mathbf{r},\hat{\mathbf{s}}_j)\Phi(\hat{\mathbf{s}}_j,\hat{\mathbf{s}}_i)$$

$$(5)\ \mathbf{q}(\mathbf{r}) = \sum_{j=1}^{n} w_j I(\mathbf{r},\hat{\mathbf{s}}_j)\hat{\mathbf{s}}_j$$

Eq. (2) is similarly expressed as a weighted sum over incoming directions.

2.2. SPH approximation

In SPH, a scalar field $f(\mathbf{r})$ is approximated using a kernel function W.

$$(6)\ f(\mathbf{r}_p) = \sum_q \frac{m_q}{\rho_q} f(\mathbf{r}_q) W_{pq}(\mathbf{r}_{pq}, h)$$

where m, ρ are mass and density of particles, and the sum is over neighbors (q) within the support domain. Differential operators are evaluated by applying the operator to the kernel. A commonly used first-derivative form is

$$(7)\ \nabla f(\mathbf{r}_p) = \sum_q \frac{m_q}{\rho_q}\{f(\mathbf{r}_q) - f(\mathbf{r}_p)\}\nabla W_{pq}$$

To compensate for particle deficiency near boundaries /free surfaces and particle disorder, Shepard filter and kernel gradient correction can be applied.

2.3. SPH discretization of the DOM equations

In this study, we develop an SPH-DOM formulation in which each particle is treated as an intensity storage point for all ordinates. Each intensity-related term in Eq. (4) can be expressed by SPH approximations as

$$(8)\ \hat{\mathbf{s}}_i \cdot \sum_q \frac{m_q}{\rho_q}\left(I_{q,\hat{\mathbf{s}}_i} - I_{p,\hat{\mathbf{s}}_i}\right)\nabla W_{pq} = \kappa_p I_{b,p} - \beta_p \sum_q \frac{m_q}{\rho_q} I_{q,\hat{\mathbf{s}}_i} W_{pq} + \frac{\sigma_{s,p}}{4\pi}\sum_{j=1}^{n} w_j I_{p,\hat{\mathbf{s}}_j}\Phi(\hat{\mathbf{s}}_j, \hat{\mathbf{s}}_i)$$

Rearranging the Eq. (8) yields Eq. (9), the coefficients of intensities ($I_{p,\hat{\mathbf{s}}_i}, I_{q,\hat{\mathbf{s}}_i}$) can be assembled into a matrix.

$$(9)\ \sum_q \left(\frac{m_q}{\rho_q}\left(\hat{\mathbf{s}}_i \cdot \nabla W_{pq} + \beta_p W_{pq}\right) I_{q,\hat{\mathbf{s}}_i}\right) - \left(\sum_q \frac{m_q}{\rho_q}\hat{\mathbf{s}}_i \cdot \nabla W_{pq}\right) I_{p,\hat{\mathbf{s}}_i} = \kappa_p I_{b,p} + \frac{\sigma_{s,p}}{4\pi}\sum_{j=1}^{n} w_j I_{p,\hat{\mathbf{s}}_j}\Phi(\hat{\mathbf{s}}_j, \hat{\mathbf{s}}_i)$$

2.4. Iterative Method

From Eq. (9), for each discrete ordinate $\hat{\mathbf{s}}_i$, we obtain a sparse linear system $\mathbf{A}_i \mathbf{I}_i = \mathbf{S}_i$, where $\mathbf{I}_i$ is the vector of particle intensities in direction i. The matrix $\mathbf{A}_i$ is constructed from Eq. (9) and the boundary condition, while $\mathbf{S}_i$ **c**ontains emission and the in-scattering source.

Each directional system is solved using Bi-CGSTAB, a Krylov iterative method suitable for large, sparse, and non-symmetric linear systems, providing robust and stable convergence. To handle angular coupling introduced by the in-scattering term, an outer iteration is employed. At outer iteration $k+1$, $\mathbf{S}_i$ is assembled by treating intensities in other directions as constants taken from the previous outer iterate k. With updated intensity fields $\mathbf{I}_i^{k+1}$ for all ordinates, the boundary condition (wall outgoing intensity) is updated, and this procedure is repeated until the intensity field converges.

Once converged, the radiative heat flux is evaluated by Eq. (5), and the resulting energy change can be computed from $-\nabla \cdot \mathbf{q}$.

3. Results & Discussion

This section presents three representative results to demonstrate the accuracy and robustness of the proposed SPH–DOM solver: (i) a 2D semicircular enclosure with an internal circular wall, (ii) a 3D hexahedral enclosure, and (iii) a 2D dam-break geometry. The first two cases are widely used benchmark problems in the radiative transfer research, allowing direct verification of the present implementation. The dam-break case is included to demonstrate applicability to complex, fragmented free surfaces.

3.1. 2D semicircular enclosure with circular wall

Fig. 1 illustrates a 2D semicircular enclosure with an internal circular wall. The region between the semicircular wall and the inner circle is filled with a participating medium, and all boundaries are cold, black surfaces. This case is a standard benchmark to assess the accuracy of radiative transport in curved geometries. Fig. 2 compares the dimensionless radiative heat flux along the bottom wall for absorption coefficients $\kappa = 10, 1, 0.1$ against the exact solution and reference data from prior studies [3]. The present SPH–DOM results show close agreement with the exact solution and previous studies across all three optical conditions, confirming that the proposed formulation reliably captures the wall heat-flux distribution in this configuration.

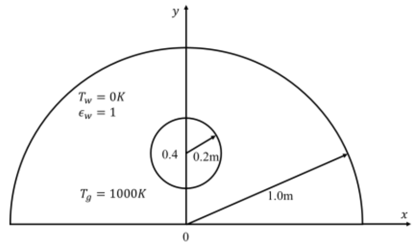

Fig. 1. 2D semicircular enclosure with circular wall

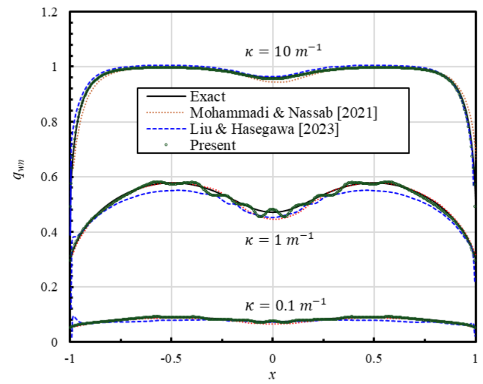

Fig. 2. Dimensionless radiative heat flux at the bottom wall

3.2. 3D hexahedral enclosure

Fig. 3 shows the 3D hexahedral enclosure used as a benchmark problem [4]. The enclosure is filled with a participating medium at 100 K, and all walls are treated as cold, black surfaces. This case is commonly used to validate radiative transfer models in three-dimensional geometries. Fig. 4 compares the dimensionless radiative heat flux for absorption coefficients $\kappa = 10, 1, 0.1$ against the exact solution, including a resolution study ($\Delta x = 0.05, 0.02, 0.01$). The present SPH–DOM results closely follow the exact profiles for all optical conditions, and improved agreement is observed as the particle resolution is refined, demonstrating stable and convergent behavior of the proposed method in 3D.

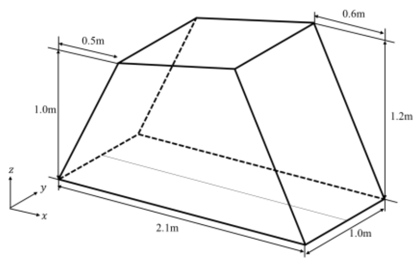

Fig. 3. 3D hexahedral enclosure

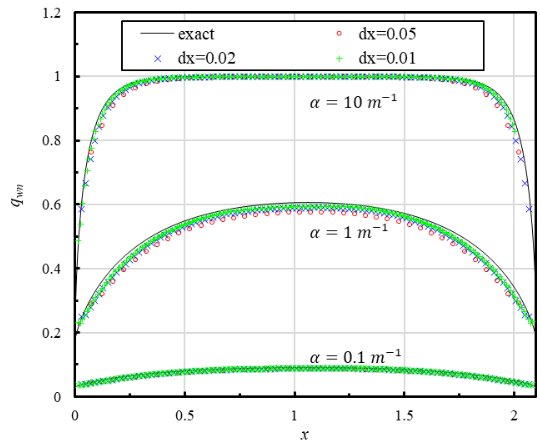

Fig. 4. Dimensionless radiative heat flux at x=0.5, z=0

3.3. 2D dam-break geometry

Fig. 5 shows a snapshot geometry from a 2D dam-break free-surface evolution and the corresponding intensity fields for two representative directions computed by the proposed SPH–DOM solver. In this demonstration, the fluid and walls are set to 1000 K, while the surrounding gas is set to 300 K. The absorption coefficient is fixed at $\kappa = 0.1$. Since no analytical solution is available for this complex configuration, verification is performed through a particle-resolution convergence test. As shown in Fig. 6, the radiative heat flux on the top wall is evaluated while progressively increasing the number of particles, and the results show clear convergence with resolution refinement. These results indicate that the developed solver remains stable and yields consistent radiative heat-flux predictions even for irregular free surfaces and fragmented particle geometries.

4. Conclusion

This work developed an SPH–DOM solver for radiative heat transfer analysis in complex geometries. The gray, quasi-steady radiative transfer equation was discretized using SPH kernel approximations, yielding sparse directional systems solved by Bi-CGSTAB with an outer iteration.

The solver was verified using two established benchmark problems, showing good agreement with exact solutions and consistent convergence behavior. In addition, a 2D dam-break free-surface geometry was analyzed to assess robustness under complex configurations, where particle-resolution tests confirmed stable and convergent heat-flux predictions. These results support the use of the proposed SPH–DOM framework as a practical basis for future coupled simulations involving radiative cooling in solidifying molten-salt spreading, including MSR-related spill scenarios.

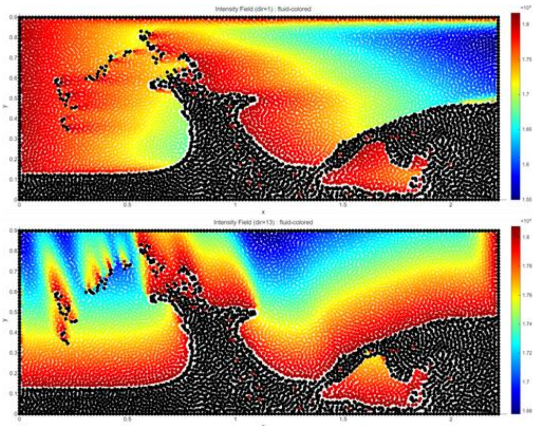

Fig. 5. Intensity field by ordinates for 2D dam-break geometry

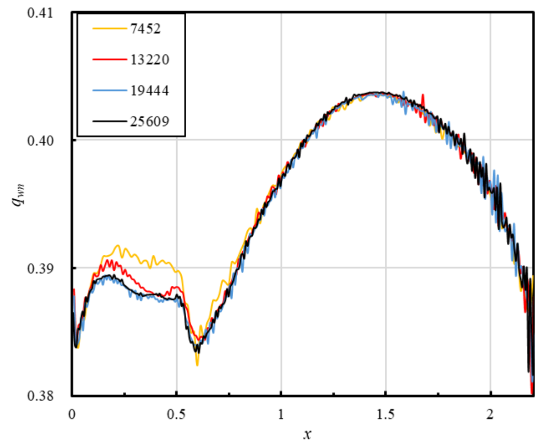

Fig. 6. Radiative heat flux at the top wall

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