Analytic Investigation on Drying Behavior of Water Film Spread on the Heated Metal Plate

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1. Introduction

The Integral Pressurized Water Reactor (iPWR) type Small Modular Reactor (SMR) offers the advantage of adopting a small and compact containment. This compact steel containment is usually maintained under vacuum during normal operation to minimize heat loss through radiative heat transfer from the outer wall of the reactor vessel to the environment[1]. The vacuum also provides an additional benefit by enabling convenient detection of Reactor Coolant System (RCS) leak rates[2].

The vacuum pump extracts gas from inside the containment. If the gas contains water vapor, it is condensed in the condenser, and the resulting condensate is collected in a condensate tank to measure the leak rate. The underlying assumption of this method is that any leaked coolant will immediately evaporate upon leakage.

This study investigates the drying behavior of leaked coolant using an analytical solution.

2. Methods and Results

2.1 Governing Equation Setup

The leaked coolant is expected to undergo flashing, as the RCS pressure is significantly higher than the containment pressure. Approximately 30–65% of the discharged liquid flashes into vapor [3]. The remaining liquid exchanges heat with the hot containment atmosphere and eventually settles at the bottom of the containment. Fig.1 shows a typical shape of containment bottom of iPWR type SMR.

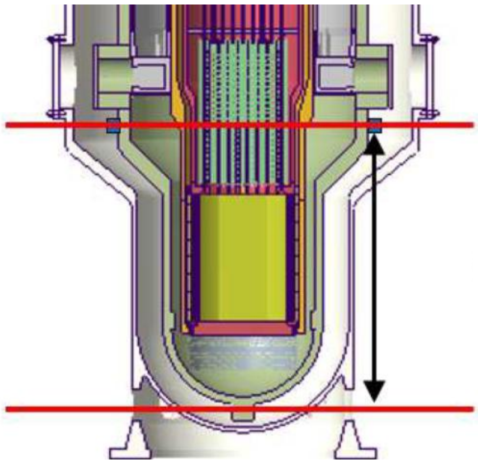


Fig. 1. Typical shape of iPWR type SMR containment bottom

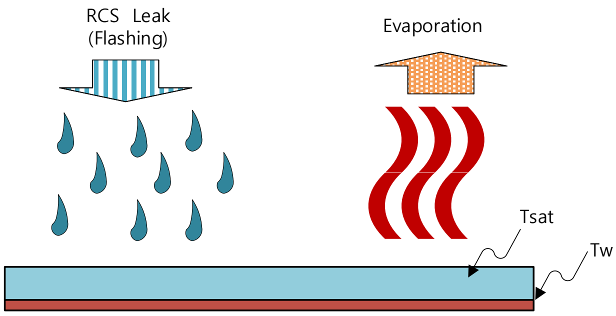


Fig. 2. Schematics of wall evaporation on the bottom of steel containment

To establish a simple governing equation, the configuration shown in Fig. 2 was assumed. A thin water film of thickness (δ) is formed across a fixed spread area (A_s). Although this assumption may not perfectly correspond to Fig. 2, it can be partially justified by considering the finite spread area and the geometry of the containment bottom or side wall. Moreover, it is considered that such a simplified model can provide sufficient insight into the drying behavior of the water film.

The mass balance on the surface of the plate can be conceptualized as follow:
(Water mass of the film) = (Inflow due to RCS leak) – (Evaporation rate driven by plate heat transfer)

$$\frac{d}{dt}(A_s\delta(t)\rho_L) = \dot{L} - \frac{A_s h(T_w - T_{sat})}{h_{fg}} \tag{1}$$

, where

A_s : Water film surface area, [m^2]
h : Heat transfer coefficient, [W/m^2-K]
$h = k/\delta$
h_{fg} : Latent heat, [J/kg]
k : Film thermal conductivity, [W/m-K]
$\dot{L}$: RCS leak rate into the film, [kg/s] (It represents the coolant that ultimately remains after undergoing flashing and heat transfer with the containment atmosphere)
T_{sat} : Film surface temperature (saturated), [K]
T_w : Wall temperature, [K]
t : Time, [sec]
δ : Thin film thickness (average) of water, [m]
ρ_L : Density of water film, [kg/m^3]

Initial condition can be defined as follow:

$$\delta(0) = \delta_0 \qquad \text{at } t = 0 \tag{2}$$

This initial condition implies that the initial thickness can be taken as an arbitrary constant, such as zero or a small finite value.

Equation (1) can be simplified as follows:

$$A\delta\frac{d\delta}{dt} = B\delta - C \tag{3}$$

, where

$$A \equiv A_s\rho_L h_{fg}$$
$$B \equiv \dot{L}h_{fg}$$
$$C \equiv A_s k(T_w - T_{sat}) \tag{4}$$

A and B are positive constants, while the sign of C depends on the wall temperature and the film surface temperature(saturated).

2.2 Initial Exploration for the Solution

<u>Case 1: Steady solution</u>

At first, the steady-state solution can be considered.

Eq. (3) reduced to

$$0 = B\delta - C \quad \leftrightarrow \quad 0 = \frac{B}{h_{fg}} - \frac{C}{\delta h_{fg}} \tag{5}$$

And the solution is

$$\delta = \frac{A_s k(T_w - T_{sat})}{\dot{L}h_{fg}} = const. \tag{6}$$

This solution indicates that the inflow rate is equal to the drying rate. It can further be inferred that the steady-state film thickness is proportional to the wall temperature; in other words, a thicker film is sustained when the wall temperature is higher.

<u>Case 2: $C = 0$</u>

As mentioned in previous section, the variable C can have different sign depending on the driving temperature difference. For the case C=0, Eq. (3) reduced to:

$$A\delta\frac{d\delta}{dt} = B\delta \tag{6}$$

When the film thickness (δ) is nonzero, the above equation can be rearranged as

$$A\frac{d\delta}{dt} = B \tag{7}$$

The solution is obtained by applying the initial condition given in Eq. (2).

$$\delta(t) = \frac{B}{A}t + \delta_0 \tag{8}$$

That is,

$$\delta(t) = \frac{B}{A}t + \delta_0 = \frac{\dot{L}h_{fg}}{A_s\rho_L h_{fg}}t + \delta_0 = \frac{\dot{L}}{A_s\rho_L}t + \delta_0 \tag{9}$$

This case corresponds to zero wall heat transfer, and the film thickness increases over time from the initial thickness δ_0.

<u>Case 3: $C < 0$</u>

This case indicates that substantial condensation occurs on the film surface because the wall temperature is lower than the film temperature. Consequently, it can be physically inferred that the film becomes thicker fast, as condensate is added in addition to the inflow of the RCS leak, which by itself would only increase the film thickness linearly.

In this situation, a direct intuitive solution is not feasible; instead, a systematic solution process is required, as presented in the following subsection.

<u>Case 4: $C > 0$</u>

This case represents the typical situation, involving inflow from the RCS leak and outflow due to evaporation. Thus, the relative magnitude of each term becomes important.

If $B\delta > C$, the film thickness increases because the inflow exceeds the evaporation.

If $B\delta < C$, the film thickness decreases because the inflow is less than the evaporation. However, evaporation can cause the spread area of the film—which serves as the wall heat transfer area—to shrink, thereby reducing the evaporation rate.

<u>Case 5: $B = 0$</u>

This corresponds to the case with no additional inflow to the film due to RCS leak. Equation (3) reduces to:

$$A\delta\frac{d\delta}{dt} = -C \tag{10}$$

The solution with the given initial condition is:

$$\delta(t) = \sqrt{\delta_0^2 - \frac{2Ct}{A}} = \sqrt{\delta_0^2 - \frac{2k(T_w - T_{sat})}{\rho_L h_{fg}}t}$$

This solution shows that the film thickness decreases continuously until it vanishes.

2.3 Analytic Solution

Let's separate the variables in case of $B\delta - C \neq 0$.

$$A\delta\frac{d\delta}{B\delta - C} = dt \tag{11}$$

This equation can be more well suitable arranged.

$$\frac{A}{B}\left(1 + \frac{C}{B\delta - C}\right)d\delta = dt$$

(12)

By integrating both sides, the solution can be obtained.

$$\frac{A}{B}\left(\delta+\frac{C}{B}\ln(B\delta-C)\right)=t+c_1, \quad if\ B\delta-C>0$$

(13)

$$\frac{A}{B}\left(\delta-\frac{C}{B}\ln(C-B\delta)\right)=t+c_2, \quad if\ B\delta-C<0$$

(14)

$$\delta(t)=\delta_0, \quad if\ B\delta-C=0$$

(15)

The condition of Equation (15) is identical to the steady-state solution discussed in the previous subsection.

By applying the initial condition, the final solution can be obtained accordingly.

$$\frac{A}{B}(\delta-\delta_0)+\frac{AC}{B^2}\ln\left(\frac{B\delta-C}{B\delta_0-C}\right)=t, \quad if\ B\delta-C>0$$

(16)

$$\frac{A}{B}(\delta-\delta_0)+\frac{AC}{B^2}\ln\left(\frac{C-B\delta}{C-B\delta_0}\right)=t, \quad if\ B\delta-C<0$$

(17)

Same results are obtained! The solutions of Equations (16) and (17) are obtained in implicit form for the film thickness δ. It is difficult to express them explicitly in terms of δ.

3. Sample Calculation and Discussion

3.1 Reference Value of each Variable for Sample Calculation

Reference values are selected as Fig. 3, and all these values are for typical normal operation with maximum allowable RCS leak (0.1gpm = 6.31ml/s) for containment atmosphere control.

Variable definition	
Heat transfer Area of water film	$As:=1\ m^2$
RCS leak rate (volume)	$Lq:=0.1\ gpm$
Density of RCS coolant	$rho\_RCS:=748.7\ \frac{kg}{m^3}$
RCS leak rate (mass)	$Lm:=Lq\cdot rho\_RCS=0.005\ \frac{kg}{s}$
Latent heat at 0.1bar	$hfg01:=(2584.78-191.8324)\cdot\frac{kJ}{kg}=(2.393\cdot10^6)\ \frac{J}{kg}$
Density of water at 0.1bar sat.	$rhof01:=989.87\ \frac{kg}{m^3}$
Thermal conductivity at 0.1bar sat. water	$kw01:=0.6357\ \frac{W}{m\cdot K}$
Sat. temperature at 0.1bar	$Ts01:=45.833\ °C$
Wall temperature	$Tw:=150\ °C$
Variables in solution	
A	$AA:=As\cdot rhof01\cdot hfg01=(2.369\cdot10^9)\ N$
B	$BB:=Lm\cdot hfg01=(1.13\cdot10^4)\ W$
C	$CC:=As\cdot kw01\cdot(Tw-Ts01)=66.219\ \frac{kg\cdot m^3}{s^3}$

Fig. 3. Reference values in the governing equation

3.2 Critical Initial Film Thickness

The zero of argument value in log can be critical value, as the log calculation can be deferent.

$$\delta_{cr}=\frac{C}{B}=5.858mm$$

(18)

This means the film of initial thickness over this value cannot be dried out.

As can be seen in Eq. (4) the critical thickness increases as the film spread area increases.

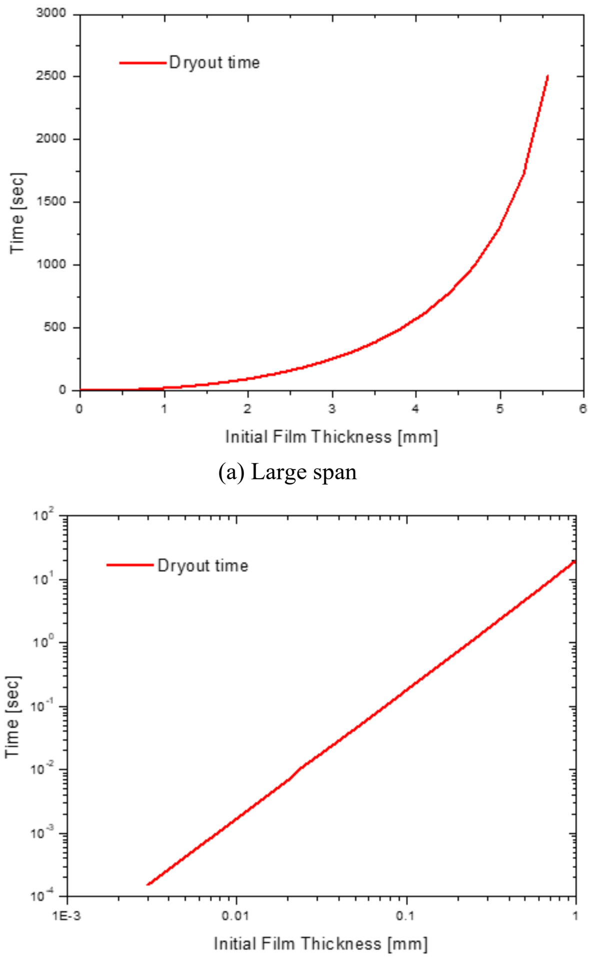
3.3 Dryout Time

Dryout time can be easily calculated by letting $\delta=0$ in Eqs. (16) or (17).

$$t_{Dryout}=\frac{A}{B}\delta_0+\frac{AC}{B^2}\ln\left(\frac{-C}{B\delta_0-C}\right)$$

(19)

Calculation results can be depicted as Fig. 4.



(b) Small span

Fig. 4. Dryout time according to the initial film thickness

3.4 Dryout Behavior

The solution given in Eq. (16) is a kind of implicit form for the film thickness. In order to get the thickness behavior according to time, actual value can be obtained using numerical method such as Newton Laphson method.

Let's define a function for Newton-Laphson method to solve Eq. (16) for a given time *t*.

$$f(t,\delta)=\frac{A}{B}(\delta-\delta_0)+\frac{AC}{B^2}\ln\left(\frac{B\delta-C}{B\delta_0-C}\right)-t \quad (20)$$

And then, the derivative of Eqs. (16) with respect to δ is:

$$\frac{\partial f(t,\delta)}{\partial\delta}=\frac{A}{B}+\frac{AC}{B^2}\frac{B}{B\delta-C} \quad (21)$$

Film thickness can be obtained for a given time t. Let's consider the initial condition (0.0 sec, 1.0mm) for a sample calculation. The dryout time corresponding to this initial thickness of 1.0 mm is calculated to be 20.223 seconds using Eq. (16). The resulting thickness profile is illustrated in Fig. 5. As time progresses, the film becomes thinner, which accelerates the drying process due to the enhanced heat transfer coefficient associated with reduced thickness. For the other initial thickness similar trends are calculated.

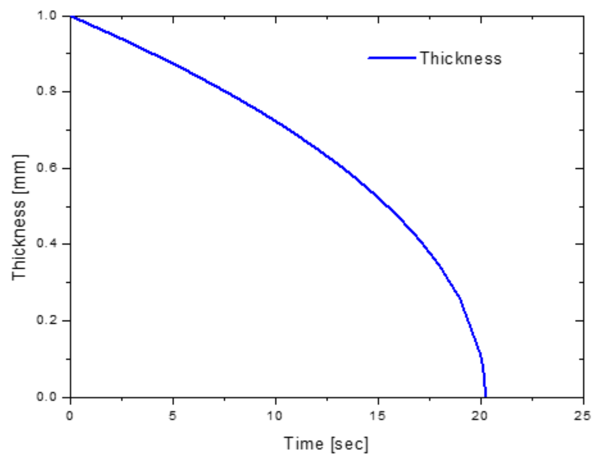


Fig. 5. Film thickness behavior (Initial thickness 1.0mm)

3.5 General Review of Thin Film Behavior

Several statements regarding film thickness at the engineering scale can be found:

(1) The maximum plausible thickness of the stable film is associated with the capillary length. For water at 20 °C and 1 atm, the capillary length is known to be less than 3.0 mm[5].

(2) The critical thickness is approximately 100 μm, at which point the influence of Van der Waals forces decreases sharply. Beyond this thickness, the regime transitions to one governed by macroscopic fluid mechanics, such as gravity and viscosity [6].

(3) The thickness of a visible thin liquid film on a stainless steel surface is typically defined as being in the range of 10 to 100μm for phase change [7].

Thus, very rapid dryout is expected in the thin film case.

4. Conclusions

This paper examines the dryout behavior of thin films using an analytical model and its solution. The governing equation was formulated and solved analytically. A detailed review provided insights into the steady-state solution, the no heat transfer case, and the no-inflow case. The analytical solution was obtained in implicit form. Based on this solution and typical operating variables, the critical initial thickness was calculated, representing the balance between inflow and evaporation. The corresponding dryout time for a given initial thickness was also provided, and the dryout behavior for specific initial thicknesses was calculated. Finally, the film thickness under typical conditions was discussed.

Although the analysis has certain limitations in modeling, it offers valuable insights into the dryout behavior of thin films.

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REFERENCES

[1] Homepage of Innovative SMR Development Agency, February 24, 2026.

[2] S. J. Hong et al., Review of the Performance Features of Containment Evacuation System in Integral Pressurized Water Reactor, Transactions of the Korean Nuclear Society Spring Meeting, Jeju, Korea, May 22-23, 2025

[3] S. J. Hong et al., Discussion on the Flashing Phenomena of High Pressure Break Flow, Transactions of the Korean Nuclear Society Autumn Meeting, Changwon, Korea, October 24-25, 2024.

[4] H.M. Seo, Evaluation of Evaporation Rate for RCS Leakage into Containment Vessel of Integral-Type SMR, Transactions of Korea Society of Fluid Machinery, Pyeongchang Alpsensia Resort, 2025. 6. 25~ 6. 28.

[5] Wikipedia, Capillary length, February 2026.

[6] Gennes, Wetting: statics and dynamics, Reviews of Modern Physics, Vol. 57, No. 3, Part I, July 1985.

[7] Carey, Liquid-Vapor Phase-Change Phenomena, CRC Press, Taylor & Francis Group, 3rd Ed., 2020