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Constrained Dynamics Solver for Robotic Simulation via Augmented Lagrangian

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1. Introduction

Robotic simulation plays a critical role in validating unmanned operations. Robotic simulators typically model constrained rigid-body dynamics with contact conditions as either a cone complementarity problem (CCP) or a nonlinear complementarity problem (NCP) and solve them using various contact solvers.

MuJoCo [1] and Drake [2] adopt a CCP formulation with compliant contact models and smooth inequality constraints to obtain convex and differentiable formulations. This enables fast Newton-type solvers but compromises strict rigid contact consistency. Isaac Sim [3] uses an NCP formulation and relies on first-order solvers such as PGS/TGS for real-time performance, which often leads to limited convergence in ill-conditioned or tightly coupled multi-contact scenarios.

Recent solvers such as CANAL and SubADMM [4] address the NCP formulation with an augmented Lagrangian (AL) theory, achieving improved robustness and accuracy for rigid contact dynamics. However, these methods primarily focus on contact constraints and do not consistently incorporate other inequality constraints within the same complementarity framework.

In this work, we address this limitation by reformulating joint friction and joint limit as complementarity conditions and extending CANAL and SubADMM to handle these constraints within a unified complementarity framework.

2. Constrained Dynamics

In this section, we formulate a constrained dynamics problem for robotic simulation with three types of constraints: contact, joint friction, and joint limit. We denote the generalized velocity by v, and the constraint impulse by λ. Then, the dynamics equation is given by

$$Av - b - \sum_i J_i^T \lambda_i = 0 \quad (1)$$

where A denotes a joint-space inertia matrix, b denotes a bias force, and J_i is a constraint Jacobian for each constraint i. Combined with the appropriate condition between v and λ for each constraint type, the constrained dynamics solver solves the dynamics equation.

2.1. Augmented Lagrangian Solver

The AL method provides a robust framework for solving a general constrained optimization problem. Following [4], we define an AL function for the constrained dynamics as

$$\mathcal{L} = f(v) + \sum_i \left(u_i^T (J_i v - z_i) + \frac{\beta_i}{2} \|J_i v - z_i\|^2 \right) \quad (2)$$

where $f(v) = \frac{1}{2} v^T A v - b^T v$, β_i is a penalty parameter, z_i is a slack variable, and u_i denotes a dual variable associated with an auxiliary equality constraint $J_i v = z_i$. This auxiliary constraint effectively smoothens the original hard constraints. The AL-based constrained dynamics problem is then formulated as

$$\min_{v,z} \mathcal{L}(v, z; u) \quad \text{s.t.} \quad (z, \lambda) \in S_c \quad (3)$$

followed by the dual update

$$u_i \leftarrow u_i + \beta_i (J_i v - z_i) \quad (4)$$

where S_c denotes a feasible set defined by the contact, joint friction, and joint limit constraints, and the corresponding constraint impulse λ is directly computed from v and u.

2.2. Constraints

Here, we define the feasible set S_c for each constraint type and derive a closed-form expression for λ. For consistency, each constraint is formulated via a projection operator associated with its corresponding complementarity condition.

1) Contact: A contact condition for NCP is derived by the Signorini-Coulomb condition, which can be defined by the following cone complementarity condition:

$$\mathcal{C}_i \ni \lambda_i \perp J_i v + e_i + \left[0; 0; \mu_i \left\| J_{i,t} v \right\|\right] \in \mathcal{C}_i^* \quad (5)$$

where $\mathcal{C}_i$ is a friction cone with friction coefficient μ_i, e_i denotes a stabilizer term, and $J_{i,t}$ is a tangential component of J_i. The contact impulse λ_i is then computed using *strict* or *proximal* cone projection for CANAL and SubADMM, respectively. For more details, we refer to [4].

2) Joint friction: Motivated by the contact constraint, we formulate the dry friction model for each joint as the following complementarity condition:

$$\mathcal{C}_i \ni \begin{bmatrix}\lambda_i \\ 1\end{bmatrix} \perp \begin{bmatrix}J_i v \\ \mu_i \|J_{i,t} v\|\end{bmatrix} \in \mathcal{C}_i^* \tag{6}$$

where $\mathcal{C}_i$ is a two-dimensional cone with a half angle μ_i. This impulse λ_i is then computed in AL framework as

$$\lambda_i = \Pi_{\mu_i}^{\text{bound}}(-\beta_i J_i v - u_i) \tag{7}$$

where Π_{μ}^{bound} denotes a projection operator on a range $[-\mu_i, \mu_i]$.

3) Joint limit: A joint limit constraint can also be represented by a two-dimensional complementarity condition as

$$\mathcal{C}_i^* \ni \begin{bmatrix}\lambda_i \\ \mu_i \|\lambda_i\|\end{bmatrix} \perp \begin{bmatrix}J_i v + e_i \\ 1\end{bmatrix} \in \mathcal{C}_i \tag{8}$$

where $\mathcal{C}_i$ is a two-dimensional cone with a half angle μ_i, and e_i denotes a bias term. This formulation effectively models a range $[-\mu_i - e_i, \mu_i - e_i]$ for the joint velocity $J_i v$. Then, the constraint impulse λ_i is computed in AL framework as follows:

$$\lambda_i = \Pi_{(\beta_i l_{b,i}, \beta_i u_{b,i})}^{\text{bound } v}(-\beta_i J_i v - u_i) \tag{9}$$

with the following projection operator designed for $J_i v \in [l_{b,i}, u_{b,i}]$:

$$\Pi_{(l,u)}^{\text{bound } v}(s) = \begin{cases} s + u & ,\text{if} - s > u \\ s + l & ,\text{if} - s < l \\ 0 & ,\text{otherwise} \end{cases}. \tag{10}$$

3. Experiments

Experiments are conducted to validate the proposed contact and joint constraints. First, we simulate a door handle mechanism with a revolute joint including joint friction and limit (Fig.1). A monotonically increasing and periodic joint torque is applied, and as shown in Fig.2 and 3, both solvers consistently enforce the joint constraints under varying parameters. Next, we simulate a contact scenario with Franka Emika PANDA 7-DoF manipulator and a box on its end-effector (Fig.1), which is randomly actuated. Fig.4 shows the convergence of both solvers. The proposed formulation maintains stable convergence, achieving residuals of 10^{-12} and 10^{-4} for CANAL and SubADMM, even in simulations including contact, joint friction, and joint-limit constraints.

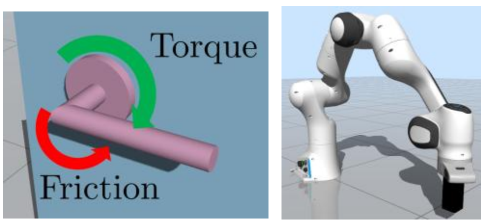

Fig. 1. A door handle (left) and a Franka (right).

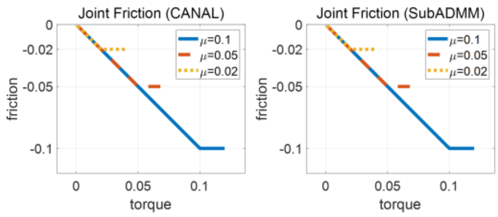

Fig. 2. Joint friction according to joint torque for each solver.

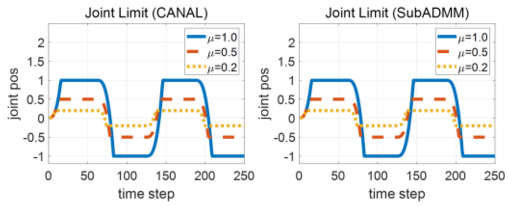

Fig. 3. Joint position with periodic torque for each solver.

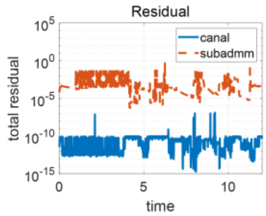

Fig. 4. Total residual for random action sequence of Franka.

4. Conclusion

We propose a unified framework of joint constraints in constrained dynamics solver based on the augmented Lagrangian. Each constraint is reformulated as a complementarity condition with its corresponding projection operator. When integrated with state-of-the-art contact solvers, our method achieves robust and stable convergence under diverse constrained scenarios.

Future works can extend this framework to incorporate additional equality constraints required in maximal coordinate and closed-chain systems.

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