

Development of a Fracture Energy Regularized Continuum Damage Model for Tensile Cracking in Concrete with Crack Width Evaluation

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1. Introduction

Reinforced concrete is the primary structural material for nuclear containment buildings because it provides both load-carrying capacity and radiation shielding. However, concrete is much weaker in tension than in compression, and tensile cracking is a critical failure mode that can threaten structural integrity and leak-tightness. In containment buildings, tensile cracking may provide leakage paths and thus has direct relevance to safety assessment. In practical finite element analyses for containment structures, the Concrete Damaged Plasticity (CDP) model is widely used, but it often requires a relatively large number of input parameters and careful calibration, and its post-peak response can be sensitive to mesh characteristics when strain localization develops. In this study, we develop an Abaqus/Standard user material subroutine (UMAT) based on the Mazars isotropic damage model [1] to provide a computationally efficient alternative for containment cracking assessment, while improving (i) the fracture-energy regularization by revising the characteristic length definition to better reflect the actual element geometry, and (ii) the interpretability of continuum damage analysis by computing crack width within the UMAT framework.

2. Mazars damage model

The Mazars damage model is formulated using a scalar damage variable $d \in [0,1]$ that reduces the elastic stiffness without introducing permanent strains. The damaged constitutive relation is expressed as $\{\sigma\} = [E_d]\{\varepsilon\}$ with $[E_d] = (1-d)[E_0]$, where $[E_0]$ is the undamaged isotropic elastic stiffness. Damage initiation and evolution for both tensile and compressive behavior are governed by an equivalent strain measure $\tilde{\varepsilon}$ based on the positive principal strains ε_i^+ as

$$\tilde{\varepsilon} = \sqrt{(\varepsilon_1^+)^2 + (\varepsilon_2^+)^2 + (\varepsilon_3^+)^2}. \quad (1)$$

Then, a loading function is defined $f(\tilde{\varepsilon}, \varepsilon_{d0}) = \tilde{\varepsilon} - \varepsilon_{d0}(d)$, where ε_{d0} is initially the elastic limit strain and, after damage initiation, stores the maximum attained equivalent strain in the loading history. When $f < 0$, damage does not evolve; otherwise, damage is accumulated and the damage variable d is calculated as a

linear combination of the tensile and compressive damage components d_T, d_C as

$$d = \alpha_T d_T + \alpha_C d_C \quad (2)$$

where α_T and α_C are weighting factors. Since the Mazars model assumes that damage is driven only by positive strains, the weighting factors are given as

$$\alpha_T = \frac{\sum_{i=1}^3 \langle \varepsilon_{T_i} \rangle^+}{\varepsilon_V^+}; \alpha_C = \frac{\sum_{i=1}^3 \langle \varepsilon_{C_i} \rangle^+}{\varepsilon_V^+} \quad (3)$$

where

$$\varepsilon_V^+ = \sum_{i=1}^3 \langle \varepsilon_{T_i} \rangle^+ + \sum_{i=1}^3 \langle \varepsilon_{C_i} \rangle^+, \quad (4)$$

$$\varepsilon_{T_i} = \frac{1+\nu}{E} \langle \hat{\sigma}_i \rangle^+ - \frac{\nu}{E} \sum_{j=1}^3 \langle \hat{\sigma}_j \rangle^+, \quad (5)$$

$$\varepsilon_{C_i} = \frac{1+\nu}{E} \langle \hat{\sigma}_i \rangle^- - \frac{\nu}{E} \sum_{j=1}^3 \langle \hat{\sigma}_j \rangle^-, \quad (6)$$

Here $\langle a \rangle^+ = \frac{1}{2}(a + |a|)$ and $\langle a \rangle^- = \frac{1}{2}(a - |a|)$, and $\hat{\sigma}$ denotes the equivalent stress measure.

A major issue in strain-softening damage models is mesh dependency, because strain localization tends to concentrate into a narrow band whose width is governed by the element size. To mitigate this issue, fracture energy regularization is introduced in the present Mazars formulation by enforcing that the area under the post-peak stress-strain response corresponds to the volumetric fracture energy, i.e., $g_f = G_f/L_c$, where L_c is the element characteristic length. In this study, a bilinear softening model is adopted for the tensile response (d_T), while the MC2010 stress-strain relation is employed to describe the compressive behavior (d_C).

3. Crack width evaluation

In the present work, the crack width is evaluated by reconstructing an equivalent displacement discontinuity from the strain field in a manner consistent with the crack-band regularization discussed in Section 2. Since a continuum damage formulation provides stiffness degradation but not an explicit displacement discontinuity, an equivalent crack opening width is reconstructed from local kinematics by assuming that

cracking localizes into a band of finite width h . The crack opening is then estimated as [2]

$$w_n \approx h \varepsilon_{cr,n}. \quad (7)$$

The crack-related normal strain $\varepsilon_{cr,n}$ is defined as the positive part of the difference between the total normal strain and the elastic normal strain:

$$\varepsilon_{cr,n} = \langle \varepsilon_n - \varepsilon_n^e \rangle^+, \quad (8)$$

The total normal strain is obtained by projecting the strain tensor onto the crack normal direction \mathbf{n} :

$$\varepsilon_n = \mathbf{n}^T \boldsymbol{\varepsilon} \mathbf{n}. \quad (9)$$

The elastic strain is computed from the current stress and the undamaged stiffness:

$$\boldsymbol{\varepsilon}^e = [E_0] \boldsymbol{\sigma}, \quad \varepsilon_n^e = \mathbf{n}^T \boldsymbol{\varepsilon}^e \mathbf{n}. \quad (10)$$

In this work, the principal strains are evaluated from $\boldsymbol{\varepsilon}$, and \mathbf{n} is taken as the eigenvector associated with the maximum positive principal strain. Additionally, the band width h is selected to be consistent with the length scale used for fracture energy regularization, i.e., $h = L_c$.

4. Conclusions

In this study, the fracture energy regularized-continuum damage model is developed to evaluate tensile cracking behavior of concrete in containment structures based on the Mazars isotropic damage framework. The proposed formulation retains the equivalent-strain-driven damage concept and the tension–compression damage decomposition, while introducing practical enhancements for containment applications. First, fracture-energy regularization is employed to reduce mesh dependency in the strain-softening regime by relating the dissipated energy to the element characteristic length. Second, a crack-width evaluation capability is incorporated by reconstructing an equivalent displacement discontinuity from the local strain and stress fields using a crack-band interpretation, so that the crack opening can be reported together with the damage variable as a leakage-relevant indicator. The proposed Abaqus/Standard UMAT will be verified through numerical examples, including uniaxial tension/compression tests for energy consistency, mesh sensitivity studies with different element types, and benchmark problems, to demonstrate robustness and applicability to the structural integrity assessment of containment concrete structures.

REFERENCES

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