

Feasibility Study of Applying the History-based Batch Method in Thermal-hydraulics Coupled Monte Carlo Analysis

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1. Introduction

The Monte Carlo (MC) method is widely recognized as one of the most accurate simulation techniques since it directly treats continuous-energy cross sections and complex geometries with minimal assumptions. Its stochastic nature provides both mean values and their associated uncertainties, which are essential for interpreting simulation results.

In steady-state MC eigenvalue calculations, the fission source distribution (FSD) is updated cycle by cycle and normalized to the number of fission neutrons [1]. Because of intercycle correlations in the FSD, tally estimates from different cycles are not independent, which leads to an underestimation of the sample standard deviation (SD) of the tally mean [2].

A variety of approaches have been explored to address this variance bias, i.e., the discrepancy between the true variance and the apparent variance of the tally mean. Among them, the history-based batch method (HBM) has shown particular promise [3]. The HBM scheme was recently implemented in the iMC code at KAIST (Korea Advanced Institute of Science and Technology), where it proved effective in estimating uncertainties of neutronics parameters such as multiplication factors and power distributions [4–6].

Building on this foundation, the present study extends the HBM framework to multi-physics coupling with emphasis on thermal-hydraulics (TH) feedback in MC simulations. In this context, both neutronics parameters and TH-related quantities, such as fuel temperature distributions, are evaluated in terms of mean values and statistically reliable uncertainties. This approach provides a more comprehensive assessment of reactor performance under coupled neutronics–TH conditions and improves the fidelity of uncertainty quantification in advanced reactor analysis.

2. History-based Batch Method

Consider a MC eigenvalue simulation with N active cycles, each consists of M neutron histories. Denote Q_{ij} as the tally Q estimate obtained from the j 'th neutron history in the i 'th cycle. Then, the mean value of Q and its related sample variance are computed as

$$\bar{Q} = \frac{1}{NM} \sum_i \sum_j Q_{ij}, \quad (1)$$

$$\sigma_s^2[\bar{Q}] = \frac{1}{NM(NM-1)} \sum_i \sum_j (Q_{ij} - \bar{Q})^2. \quad (2)$$

The real variance of the tally mean $\bar{Q}$ is written as

$$\begin{aligned} \sigma^2[\bar{Q}] &= E[\bar{Q}^2] - E[\bar{Q}]^2 \\ &= \frac{1}{NM} \sigma^2[Q_{ij}] \\ &\quad + \frac{1}{(NM)^2} \sum_{i,j} \sum_{i',j' \neq i,j} \text{cov}[Q_{ij}, Q_{i'j'}], \end{aligned} \quad (3)$$

where $\text{cov}[Q_{ij}, Q_{i'j'}]$ represents the covariance between Q_{ij} and $Q_{i'j'}$ and $E[\cdot]$ denotes the estimate of variable within the brackets.

The estimate of the sample variance is known as the apparent variance $\sigma_A^2[\bar{Q}] = E[\sigma_s^2[\bar{Q}]]$, which always underestimates the real variance. The discrepancy between the two can be mathematically expressed as

$$\begin{aligned} \sigma^2[\bar{Q}] - \sigma_A^2[\bar{Q}] &= \frac{1}{NM(NM-1)} \sum_{i,j} \sum_{i',j' \neq i,j} \text{cov}[Q_{ij}, Q_{i'j'}]. \end{aligned} \quad (4)$$

The history-based batch method resolves this issue by reinterpreting a MC eigenvalue simulation with N active cycles and M histories per cycle as N_B separate batch runs, each consisting of N active cycles and M/N_B histories per cycle. Care must be taken in this partitioning to ensure that the original tally mean $\bar{Q}$ is preserved. A detailed description of the HBM implementation in the iMC code can be found elsewhere [6]. Figure 1 illustrates the overall concept of the method where S_{ij} in the cartoon represents j 'th fission source in the i 'th cycle

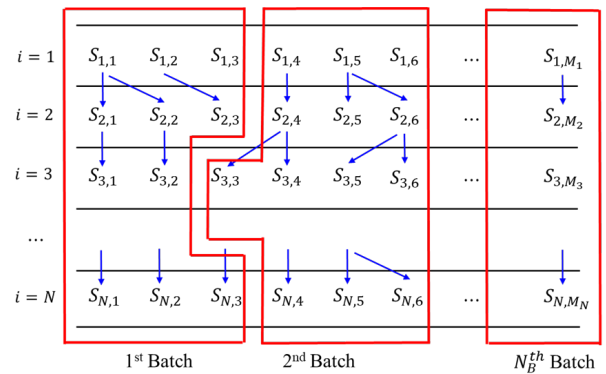


Fig 1. Graphical illustration of HBM.

The aforementioned description applies to MC runs without feedback. A similar strategy can be extended to simulations with feedback effects, such as thermal-hydraulics (TH) coupled MC analysis. In such cases, Picard iteration is typically performed between the neutronics and TH solvers. Since the primary interest lies in the converged solution, the HBM scheme is applied only in the final Picard iteration, where both neutronics and TH convergence are achieved.

By storing history-batch wise power distribution data, TH calculations can be performed for each batch, and the resulting batch-wise estimates of both neutronics and TH parameters are then used to evaluate uncertainties. This approach not only improves the estimation of neutronics uncertainties but also enables the quantification of TH-related variables, which is not feasible in conventional MC simulations without independent batch runs.

Figure 2 illustrates the implementation of the HBM scheme within the Picard iteration framework for TH-coupled MC analysis. Here, $Q_{MC}^{(k)}$ and $Q_{TH}^{(k)}$ denote the MC (neutronics) and TH-related variables at Picard iteration k . When the HBM scheme is applied, batch-wise quantities are evaluated, denoted as $Q_{MC,HB}$ and $Q_{TH,HB}$.

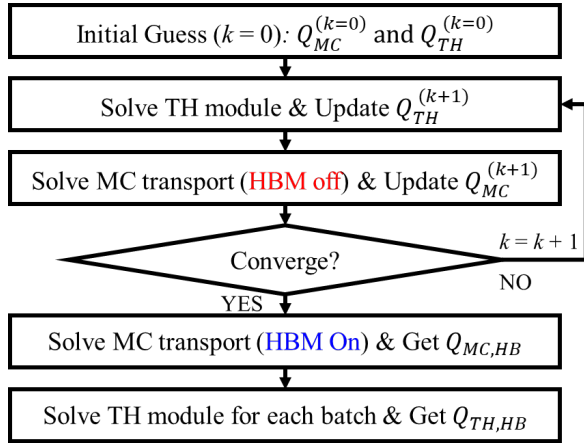


Fig 2. HBM scheme implemented in TH-coupled MC simulation.

3. Numerical Results

To examine the effectiveness of HBM in TH-coupled MC analysis, the iMC code was coupled with the subchannel code START [7] through MPI-based Picard iteration. For the preliminary study, a 17×17 -25 UOX fuel assembly was considered, which was previously used in validation studies for TH-coupled MC simulations [8]. The assembly specification is based on the OECD/NEA and U.S. NRC PWR MOX/UO₂ core transient benchmark [9], where the integral burnable absorber (IFBA) rods were replaced with unpoisoned fuel pins. The corresponding operating conditions are summarized in Table 1. Radially reflective boundary conditions were applied, while an axial vacuum boundary condition was imposed.

Table 1. UOX fuel assembly operating conditions.

Parameters	Value
Power	18.47 MWth
$\dot{m}$	82.12 kg/sec
P_{in}	15.45 MPa
T_{in}	560 K
Height	365.76 cm
Fuel pellet radius	0.3951 cm
Pin pitch	1.26 cm
Cladding thickness	0.573 mm

Figure 3 shows the evolution of normalized L2-norm values for power, fuel temperature, and coolant temperature distributions over Picard iteration. Each fuel pin was divided into four equal-volume regions to represent the temperature distribution. The MC calculation used 25 inactive cycles, 200 active cycles, and 500,000 histories per cycle. The normalized L2-norm for a 3-D field sequence $\{X_n(i, j, k)\}$ defined on grid indices $i = 1, \dots, N_x$, $j = 1, \dots, N_y$, $k = 1, \dots, N_z$ is given by

$$E_n^{rel}(X) = \frac{E_n^{abs}(X)}{(\sum_{i,j,k} [X_n(i, j, k)]^2)^{\frac{1}{2}}}, \quad (5)$$

$$E_n^{abs}(X) = \left(\sum_{i,j,k} [\Delta X_n(i, j, k)]^2 \right)^{\frac{1}{2}}, \quad (6)$$

where subscript n is the Picard iteration index. The node-wise discrepancy $\Delta X_n(i, j, k)$ is defined as

$$\Delta X_n(i, j, k) = X_n(i, j, k) - X_{n-1}(i, j, k). \quad (7)$$

From 30 independent batch runs, the reference (real) variance for quantity of interest was obtained. The number of Picard iteration was fixed at five, which was found sufficient to achieve convergence. The resulting node-wise uncertainties were used to compute an L2-norm-type metric, termed the uncertainty level, by applying Eqs. (5) and (6) with $\Delta X_n(i, j, k)$ replaced by $\sigma_{n=5}^{ref}(i, j, k)$. The corresponding results for power, fuel temperature, and coolant temperature are shown in Fig. 3, where the L2-norm fluctuations remain within the 2σ range, confirming convergence within five iterations.

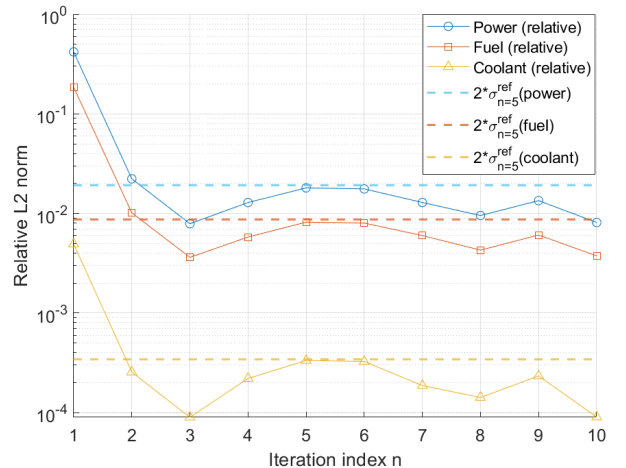


Fig 3. Evolution of normalized L2-norm values

Figures 4 and 5 present the reference normalized power and fuel temperature distributions, together with their associated uncertainties (1σ), obtained from 30 independent batch runs with the number of Picard iterations fixed at five. As shown, a non-negligible level of uncertainty is observed in the fuel temperature distribution. The displayed uncertainty distributions correspond to the uncertainty levels defined in Fig. 3.

The HBM calculation used a batch size of 10,000 (50 history batches), considered adequate for reliable results based on previous studies [6]. Figure 6 compares the power distribution uncertainty from a conventional MC run with the HBM result in Fig. 7, showing improved variance estimation for neutronics parameters. For the multiplication factor, the difference was marginal as shown in Table 2. Figure 8 shows the fuel temperature uncertainty from the HBM run, which, although relatively small, demonstrates the advantage of HBM since conventional MC cannot provide uncertainty estimates for TH-related quantities.

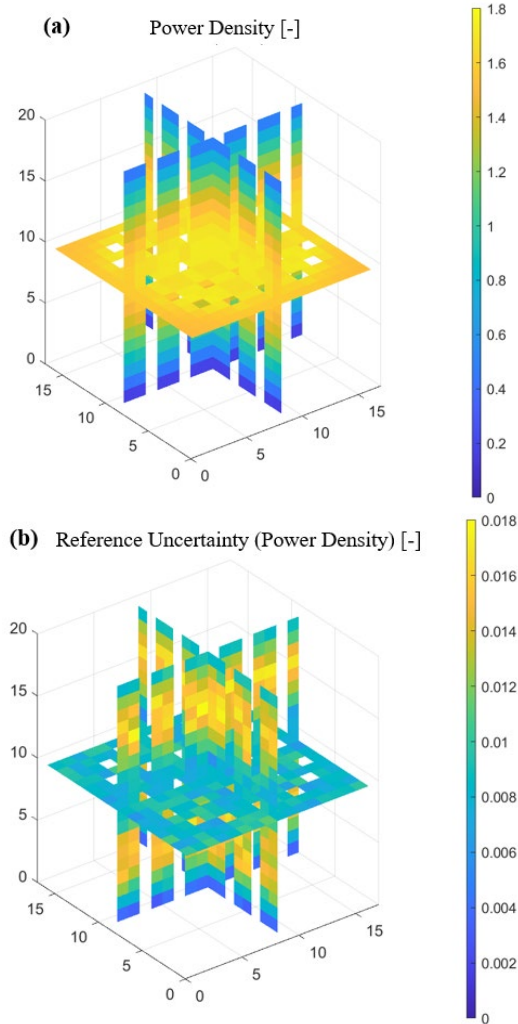


Fig 4. Reference normalized power distribution: (a) mean value and (b) associated uncertainty (1σ).

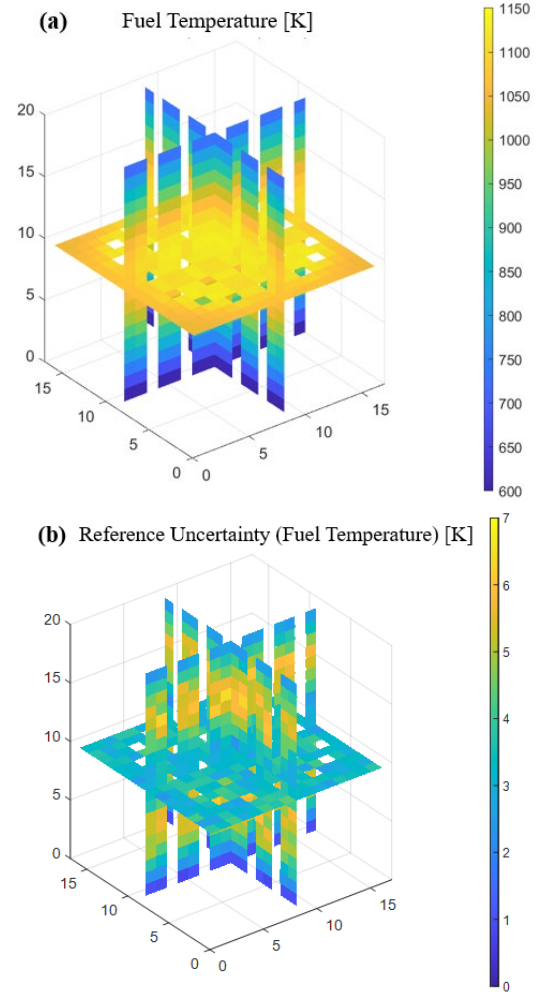


Fig 5. Reference fuel temperature distribution: (a) mean value and (b) associated uncertainty (1σ).

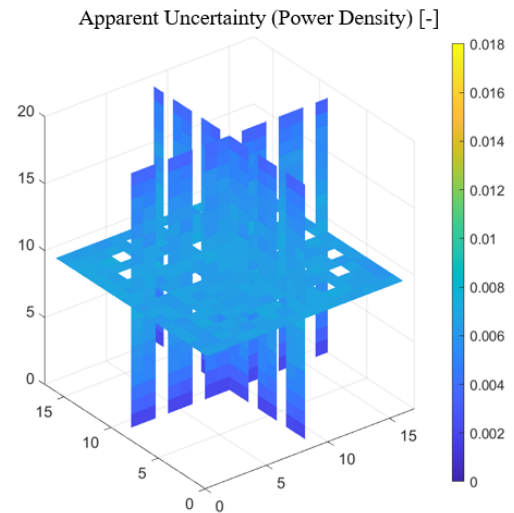


Fig 6. Power distribution uncertainty (1σ) from the conventional MC run.

Table 2. Estimated k_{eff} and uncertainty

Type	Mean	Uncertainty [pcm]
Reference	1.27128	11.2
Conventional	1.27128	9.3
HBM	1.27124	10.7

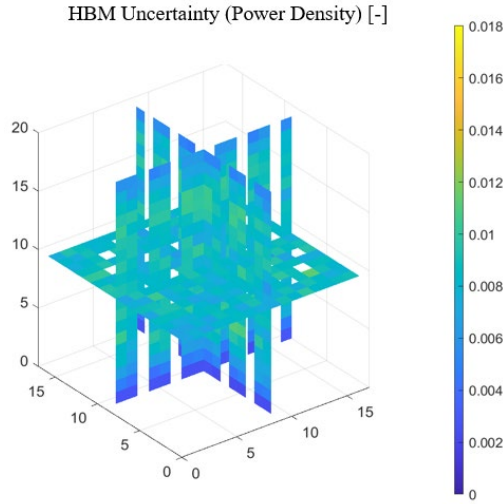


Fig 7. Power distribution uncertainty (1σ) from the HBM run (batch size = 10,000).

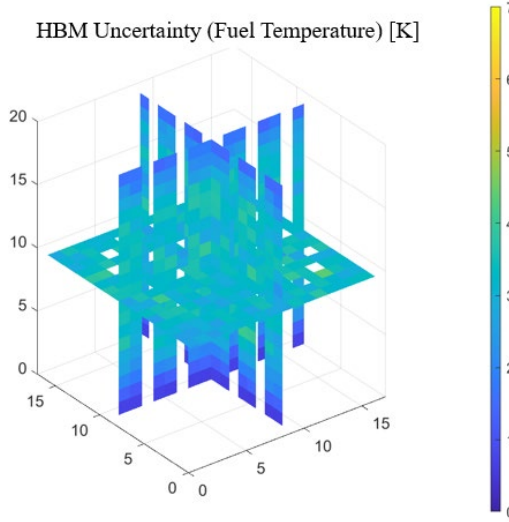


Fig 8. Fuel temperature distribution uncertainty (1σ) from the HBM run (batch size = 10,000).

Table 3. Comparison of uncertainty levels.

Variable (X)	$\sigma_{n=5}^{ref}(X)$ [Ref]	$\sigma_{n=5}^{MC}(X)$ [Apparent]	$\sigma_{n=5}^{HBM}(X)$ [HBM]
Power Density	9.664E-03	4.508E-03	6.453e-03
Fuel Temperature	4.366E-03	-	2.964e-03
Coolant Temperature	1.707E-04	-	8.266e-05

For a more quantitative comparison highlighting the significance of HBM in TH-coupled MC simulations, uncertainty levels were evaluated for both apparent and HBM-based estimations, corresponding to the dashed lines in Fig. 3 calculated using Eqs. (5)–(6). Table 3 summarizes these results. No values are reported for the conventional MC run (apparent uncertainty) regarding TH-related variables, as a single MC run cannot provide such estimates. In contrast, the HBM approach yields improved uncertainty levels for the power distribution and, importantly, provides meaningful estimates for fuel

and coolant temperature uncertainties. The number of Picard iterations used for convergence was fixed at five, as noted earlier.

4. Conclusions

In this work, the history-based batch method (HBM) was extended to thermal-hydraulics (TH) coupled Monte Carlo (MC) analysis and implemented in the iMC code with the subchannel code START. By applying the HBM scheme to the final Picard iteration, batch-wise uncertainties could be evaluated not only for neutronics parameters, such as multiplication factors and power distributions, but also for TH-related quantities, including fuel and coolant temperatures.

The numerical results based on a 17×17 –25 UOX assembly demonstrated that HBM provides improved variance estimation for neutronics parameters compared to conventional MC runs. More importantly, the method enables uncertainty quantification of TH-related variables, which is not possible with conventional single-run MC simulations. With a reasonable batch size, the tally mean is preserved and the estimated uncertainties closely match the reference values obtained from independent batch runs.

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