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Preliminary Study of Radiation Dose Assessment for Gamma-ray Measurements in Liquid Radioactive Waste Management

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1. Introduction

Accurate radiation dose assessment is essential for the safe design and operation of facilities that use radioactive sources (e.g., nuclear fuel, accelerators, radionuclides) and for protecting workers. Historically, ICRP-74 has been widely used to convert radiation flux to dose, but changes in radiation weighting factors and the Kinetic Energy Released in Material (kerma) approximation for gamma rays can produce discrepancies and overestimates.

Two common spectrum-dose approaches are the G(E) function method and spectral unfolding. The G(E) method converts counts in energy segments to dose using channel-specific factors and is attractive for embedded systems because it is simple and stable against ill-posed inversion. Unfolding is computationally heavier but can identify nuclides while producing dose estimates. Various algorithms and ensemble approaches have been used to obtain G(E), including least squares and first-order, gradient optimizers such as ADAM; ensemble combinations improve generalization.

However, least square assumes homoscedastic, independent channel errors-assumptions violated in practice because noise is energy-dependent (heteroscedastic). In particular, LaBr3(Ce) detectors show pronounced nonlinearity below ~400 keV, causing larger uncertainties at low energies. To address these issues, we propose a Bayesian Weighted Least Squares (B-WLS) approach, that explicitly models energy-dependent uncertainties and assigns greater statistical weight to low-energy measurements. This study compared unfolding combined with ICRP-74 conversion and the improved G(E) method using B-WLS, aiming to provide more robust, detector-aware dose estimates for complex radiation fields.

2. Methods and Results

The study employs both simulated and experimental gamma-ray spectra data to systematically compare dose estimation approaches, specifically evaluating the unfolding method with the GRAVEL algorithm and the B-WLS enhanced G(E) function method. Simulations utilize fluence-to-dose conversion coefficients recommended by ICRP-74 and ICRP-116, enabling analysis of updated radiological protection guidelines on dose estimation. Experimental validation involves spectral measurements from scintillation detectors at various source.

2.1 Gravel algorithm

The GRAVEL algorithm operates by iteratively refining an initial estimation of the true spectrum until a satisfactory concordance is achieved between the predicted and empirically measured spectra. The fundamental iterative formula for GRAVEL is expressed as:

$$\Phi_j^{K+1} = \Phi_j^K \cdot \exp\left(\frac{\sum_i W_{ij}^K \ln\left(\frac{N_i}{\sum_{j'} R_{ij} \Phi_{j'}^K} \right)}{\sum_i W_{ij}^K} \right). \quad (1)$$

In the formulation, Φ_j^K signifies the fluence within the j-th energy interval following the K-th iteration. N_i represents the measured count in the i-th channel of the pulse height spectrum. R_{ij} represents the response matrix that the i-th pulse height interval to the j-th energy interval.

An essential component of the GRAVEL algorithm is the weight factor W_{ij}^K, which is explicitly defined as:

$$W_{ij}^K = \frac{R_{ij} \Phi_j^K}{\sum_{j'} R_{ij} \Phi_{j'}^K} \cdot \frac{N_i^2}{\sigma_i^2}. \quad (2)$$

In this formulation, σ_i corresponds to the estimated measurement uncertainty for bin i, conventionally approximated by the square root of N_i. This weighting mechanism ensures that channels exhibiting a high number of events exert a more significant influence on the folding process, while channels with fewer counts are less substantially modified, frequently preserving their shape from the initial estimate.

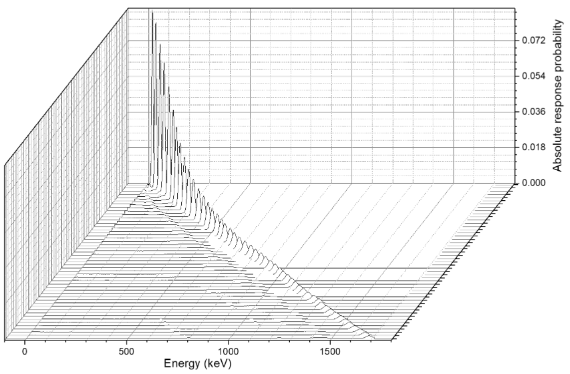

Figure 1. To obtain the energy-dependent spectra and corresponding dose required for the stripping method and the G(E) function. MCNP simulations were performed as follows: monoenergetic gamma sources spanning the spanning the energy range of interest were modeled, with the resulting detector spectra by channel and the ambient dose for each energy.

2.2 Bayesian-Weighted Least Square

Integrating B-WLS regression into G(E) function analysis offers substantial advantages in terms of robust parameter estimation and comprehensive uncertainty quantification. While LSM effectively addresses correlated and heteroscedastic error terms, B-GLS further enhance this by incorporating Bayesian inferential principles, allowing for the representation of uncertainty in parameter space through probability distributions and the utilization of prior information.

The fundamental linear regression model for estimating the G(E) function is as follows:

$$y = X\beta + \epsilon \tag{3}$$

y represents the dependent variable vector, X is the matrix of explanatory variables, β is the vector of parameters to be estimated for the G(E) function, and ϵ is the error vector.

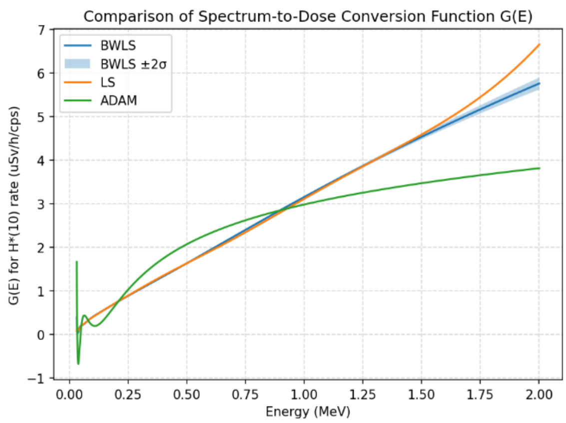

Figure 2. Comparison of Spectrum-to-Dose conversion function, G(E), derived using B-WLS, LS, and ADAM methods.

The results indicate that the Bayesian method demonstrated superior accuracy compared to other evaluated techniques. Specifically, the BWLS approach yielded dose estimations of 9.066, 13.55, and 50.8 uSv/h for Ba-133, Cs-137, and Co-60 respectively, closely aligning with the theoretical dose values (6.53, 10.81, and 51.21 uSv/h). In contrast, the stripping method consistently exhibited significant overestimations, reflecting larger deviations from theoretical expectations. Furthermore, the BWLS estimates exhibited smaller discrepancies than those obtained through Adaptive Moment Estimation (ADMA) and Least Square (LS) methods, emphasizing its robustness and suitability for accurate dose assessment in practical applications involving gamma-ray spectroscopy of liquid radioactive waste.

3. Conclusions

In summary, B-WLS demonstrates both superior accuracy and promising operational applicability. With targeted field validation and detector-specific calibration, B-WLS can realistically be adopted as more reliable spectrum-to-dose conversion method for practical radiation monitoring tasks.

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