Transactions of the Korean Nuclear Society Autumn Meeting
Changwon, Korea, October 30-31, 2025

Ultimate Ductility of Reinforced Concrete Water Storage Tank on Soil Site

Kyuho Hwang*, Tim J. Graf, Frederic F. Grant
Simpson Gumpertz & Heger
**Corresponding author: khwang@sgh.com*

***Keywords:** seismic analysis, soil-structure interaction, fixed base analysis, system ductility, trilinear force-displacement, bilinear force-displacement, inelastic energy absorption capability.

1. Introduction

Engineered structures are generally designed to develop ductility prior to loss of structural integrity. This kind of capability of a structure is not always invariant and may be dependent of site conditions. Hashimoto and et.al. [1] demonstrated that capability of a structure on soil site to absorb inelastic energy is less than on rock site. This study examines a reinforced concrete water storage tank on soil sites and rock sites, respectively, to see the difference in inelastic energy absorbing capability between the two tanks. Firstly, the tank on soil is seismically analyzed with the free field (FF) ground motion as input by considering soil-structure interaction (SSI) effect using SASSI2010. Secondly, a fixed base analysis of the same tank is performed with the same FF ground motion as input using SAP2000. A preliminary analysis indicates that the tank is governed by tangential shear failure occurring at the tank wall near the bottom. This study focuses on inelastic energy absorption capabilities of the concrete tank between the two analysis cases: the soil founded and fixed base. For purpose of the comparison, best-estimate (BE) analysis method is employed such that BE material properties and mean input ground motions are used in the response analyses.

2. Seismic Response Analyses

2.1. Input Motions

The FF ground motion applied to the SSI model is defined as 1E-4 mean uniform hazard response spectra (UHRS) as shown in Figure 1.

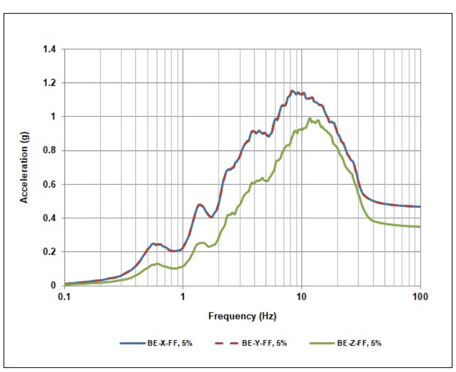

Fig. 1. 5% Damped UHRS (X, Y, and Z Directions)

The response spectra are anchored to peak ground accelerations (PGA) of 0.50g and 0.35g for the horizontal and vertical direction, respectively. Five sets of statistically independent time histories were generated to be compatible with the UHRS and were applied to the SSI model to obtain seismic demands (shears, moments, axial forces, and deflections) along the tank height and response spectra at the tank foundation. The UHRS is also applied to the tank model with the tank base fixed to generate seismic demands: shears, moments, axial forces, and deflections.

2.2. Analysis Models

The reinforced concrete tank is founded on soil at EL. 220 ft. It is 60 ft tall, 2 ft thick, and 44 ft in diameter, and is filled with water up to 46 ft from the top of the tank foundation. Median compressive strength of the concrete is 5,515 psi and median yield stress of the rebar is 71 ksi. The elastic moduli of concrete and rebar are 4,233 ksi and 29,000 ksi, respectively. Reinforcing steel ratios in the tank wall are 0.882% in the hoop direction and 0.693% in the meridional direction. The tank is modeled with lumped masses and stick elements. The soil media supporting the tank is 730 feet deep and is represented by 99 horizontal infinite soil layers above halfspace. The strained BE soil properties used for the SSI analyses are shown in Figure 2. Average shear wave velocity across the entire soil profile is approximately 1,800 fps.

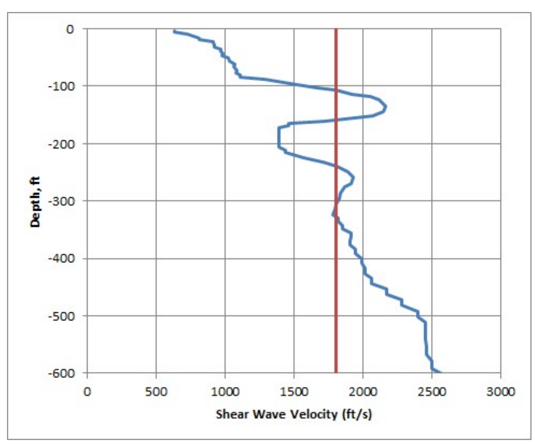

Fig. 2. Shear Wave Velocity of the BE Soil Deposit

Schematic drawings of the SSI model (left) and fixed base model (right) are presented in Figure 3. Each model has a foundation at Node 10. The SSI model includes a foundation mass at Node 10, which is in turn connected to the soil spring.

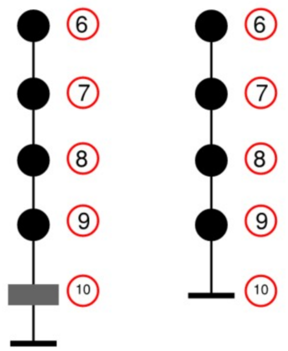

Fig. 3. SSI (Left) and Fixed Base (Right) Tank Models

2.3. Seismic Analysis Results

The SSI analysis was performed in the three orthogonal directions separately with the five sets of time histories as input for BE soil case. The response time histories at a node in one direction resulting from the three directional excitations were combined by the square root of sum of the squares. Nodal forces and displacements were extracted and average over the five analysis cases. Figure 4 compares the FF response spectrum to the response spectrum generated at the tank foundation in the horizontal direction. The frequency of the fixed base tank is 12.07 Hz while the SSI frequency is 3.17 Hz as shown in Figure 4.

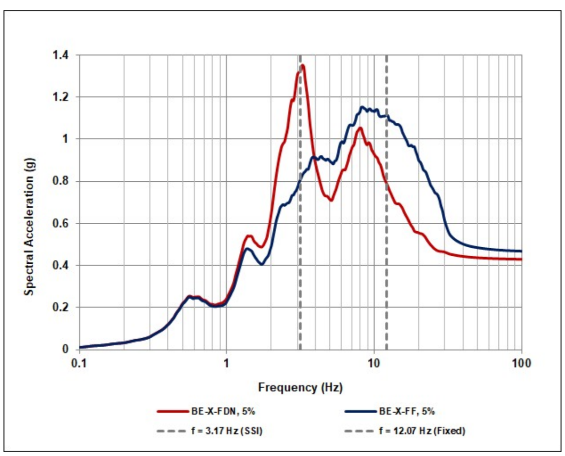

Fig. 4. Horizontal, Best Estimate, 5% Damped Response Spectra at the Tank Foundation and Free Field

A fixed base analysis is performed with the FF motion as an input (the blue curve in Figure 4). The tank is judged to respond at 7% damping.

The strength factors of the two analysis cases were calculated by the procedure delineated in EPRI 3002012994 [2] and ASCE 4-19 [3]. The analysis results from the seismic response analyses are summarized in Table 1 together with the strength factors. The deflection curves from the two analysis cases are also plotted in Figure 5, which demonstrates that there is substantial SSI effect in the deflection.

Table 1: Seismic Responses of the Tank

		SSI Analysis	Fixed Base Analysis	Unit
Seismic demands	Ps	600	473	kips
	Vs	3,755	4,183	kips
	Ms	130,600	158,040	ft-kips
Shear capacity	Cs	616.1	616.1	psi
Reduction in capacity	ΔCs	31.2	31.3	psi
Seismic shear demand	Ds	194.1	220.4	psi
Non-seismic demand	Dns	65.3	65.3	psi
Strength Factor	Fs	2.44	2.19	
Lateral nodal displacement	6	0.708	0.103	in
	7	0.569	0.076	in
	8	0.463	0.054	in
	9	0.354	0.026	in
	10	0.246	0.000	in
System frequency	f	3.17	12.07	Hz
System damping	ξ	7%	7%	

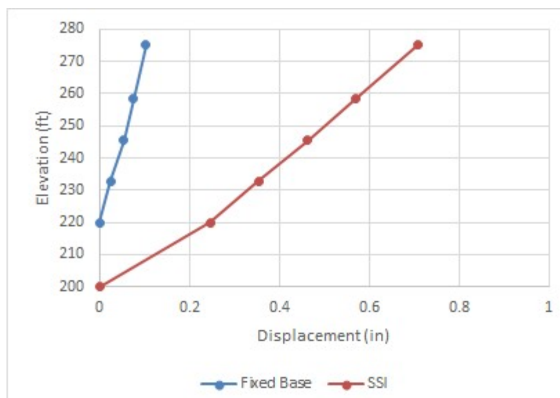

Fig. 5. Deflection from SSI and Fixed Base Analyses

3. Methodology

Under the lateral seismic loading, the largest seismic demands occur near the base of the tank. As the demands increase, cracks form in the concrete wall near the base. After concrete cracking the rebar starts to yield, and then the wall continues to deform without additional loading until the lateral displacement reaches its drift limit. Inelastic energy absorption capability of the concrete tank is investigated in this study by the Effective Frequency / Effective Damping (EF/ED) method delineated in EPRI 3002012994 [2].

A critical section of the tank is judged to be located at a height of approximately 12.8 ft from the top of foundation, which is the stick element between nodes 9 (critical section) and 10 (foundation). This wall segment is subjected to the seismic demands Ps, Vs, and Ms that are listed in Table 1. A trilinear force-displacement (F-D) curve is a better representation of the behavior of the concrete storage tank undergoing tangential shear failure. The EF/ED method requires estimation of the system ductility based on a bilinear force-displacement relationship at the critical section of the concrete tank. Hence, an equivalent bilinear F-D relationship is developed from which a median system ductility is derived. Equations in use for computing inelastic energy absorption factors for both SSI and fixed base analysis cases are taken from the ERPI fragility guide [2] and are listed in Table 2.

Table 2: Equations [2]

Equation	Description	Eqn.
$Vu = Fs \times Vs$	Shear capacity at Ultimate	1
$Vc = 0.8 \times \sqrt{fc} \times As$	Shear capacity at wall cracking	2
$As = 131.8\ ft^2$	Shear area of the tank	
$\Delta c = \Delta 9\ (Vc/Vs)$	Drift at node 9 at wall cracking	3
	$\Delta 9$ = Drift at node 9	
$\Delta y = \Delta c + (Vu - Vc) / k2$	Drift at node 9 at Yield	4
$k2 = s \times ke$	Stiffness of second slope in F-D curve	5
$ke = Vc / \Delta c$	Elastic stiffness in F-D curve	6
$\Delta u = 0.0075\ hs = 1.151$ in.	Drift at node 9 at Ultimate	7
$hs = 12.79$ ft	Height of critical wall segment	
$SF = \Delta y / \Delta 9$	Scale factor to be applied to elastic displacement for deflection at Yield	8
$\mu = \Sigma(m \times \delta u) / \Sigma(m \times \delta y)$	System ductility	9
$fs/f = 1/\sqrt{\mu}$	Secant frequency to elastic frequency	10
$fe/f = 1 - A \times (1 - fs/f)$	Effective frequency to elastic frequency	11
$A = 0.85$ or $Cf \times (1 - fs/f)$	Lesser of the two	12
$Cf = 2.3$		
$\xi h = 0.11 \times (1 - fs/f)$	Hysteretic damping ratio	13
$\xi e = \{(fs/f)/(fe/f)\}^2 \times (\xi + \xi h)$	Effective damping ratio	14
$F\mu = \{(fs/f)/(fe/f)\}^2 \times (Sa / Sae)$	Inelastic energy absorption factor	15

4. Soil-Founded Case

4.1. 1g-Static Analysis

The lateral deflection of the tank resulted from the SSI analysis consists of pure translational components and components due to pure rotation of the foundation. The latter is considered not to contribute to the shearing distortion of the concrete tank wall. A 1g-static analysis is performed for the fixed base model in the horizontal direction to obtain purely translational displacement of the tank, i.e., no rotational components involved. Since the SSI frequency of the tank is 3.17 Hz and the corresponding spectral acceleration is 1.35g (Figure 4), the 1g displacements are scaled to the spectral acceleration level, 1.35g. Table 3 presents the 1.35g static deflection along the tank height together with the SSI displacement. It is worth comparing the SSI deflection to the 1.35g deflection shifted by the foundation displacement (0.246 inches) at Node 10, which is the dotted blue curve in Figure 6. This comparison concludes that a trilinear F-D curve of the critical wall segment should be developed by using the purely translational displacements.

Table 3: Translational Displacements from 1g Analysis

Node	EL.	1.35g Static w/o FDN displ	1.35g Static w/ FDN displ	SSI
	ft	inches	inches	inches
6	275.1	0.151	0.397	0.708
7	258.3	0.119	0.365	0.569
8	245.5	0.089	0.335	0.463
9	232.8	0.046	0.292	0.354
10	220	0.000	0.246	0.246
reference	200			0.000

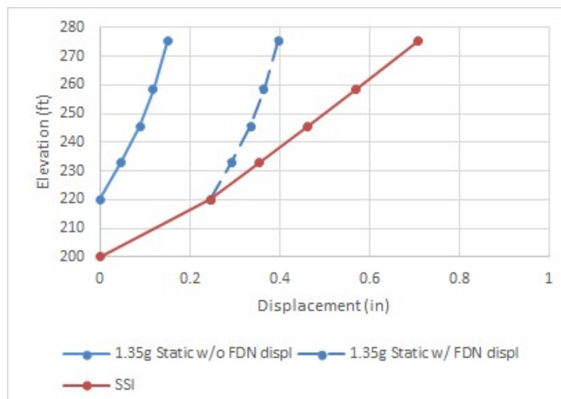

Fig. 6. 1.35g Static Displacement vs SSI Displacements

4.2 Force-Displacement Curve

System ductility of the tank is calculated by investigating the 12.8-ft critical wall segment spanning Node 9 to Node 10. A tri-linear F-D curve of the critical wall segment is defined by three control points: concrete cracking, yielding, and ultimate failure. Each of the three points is defined by nodal shear force and displacement, which are easily computed by plugging the data given in Table 1 into the equations in Table 2: concrete cracking by Eqn. 2 and 3, rebar yielding by Eqn. 1 and 4, and ultimate failure by Eqn. 1 and 7.

Next, a bi-linear F-D curve is computed by equating the area under the trilinear curve to the area under the bilinear curve. Table 4 summarizes the forces and displacements to form tri- and bi-linear curves, which are compared in Figure 7.

Table 4: Tri- and Bi-linear F-D Curve Data

	Tri-linear curve		Bi-linear curve	
	Displ, in	Shear, kip	Displ, in	Shear, kip
Origin	0	0	0	0
Cracking	0.013	1150	0	0
Yield	0.473	9182	0.105	9182
Ulimate	1.151	9182	0.990	9182

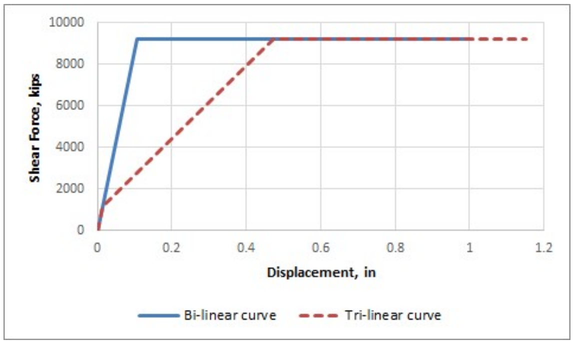

Fig. 7. Trilinear and Bilinear F-D Curves

As discussed above, the SSI elastic deflection contains lateral displacement due to rotation of the foundation, which does not contribute to shearing deformation of the concrete wall segment. Therefore, deflection at Yield (rebar yielding) is obtained by multiplying a scale factor (SF) to the wall deflection purely resulting from shearing distortion of the concrete wall. The scale factor is calculated by Eqn. 8 as shown below, where Δ9T is a pure translation at Node 9 excluding the effect of the foundation rotation.

- Δ9T = 0.046 in (w/o the foundation displacement)
- Δy = 0.105 in (bilinear F-D curve)
- SF = 1.105 / 0.046 = 2.27 (Eqn. 8)

As a result, a total tank deflection at Yield is a sum of the following displacement components.

- SF x deflection purely due to wall distortion
- Displacement purely due to foundation rotation
- Foundation displacement (0.246 in)

A tank deflection at Ultimate (ultimate failure) is the same as the yield deflection displaced by the drift limit (1.151 in per Table 2). Table 5 summarizes these values with the deflection at Yield and Ultimate, which are plotted in Figure 8.

The column named "Net Yield" in Table 5 is defined above as 2.27 times the deflection purely due to wall shearing, in which the foundation displacement (0.246 in) at Node 10 is excluded.

Table 5: Deflection Profiles at Yield and Ultimate

Node	El.	Net Yield	Rotation	Yield	Ultimate
	ft	inches	inches	inches	inches
6	275.1	0.342	0.311	0.900	1.785
7	258.3	0.271	0.204	0.720	1.606
8	245.5	0.202	0.128	0.576	1.462
9	232.8	0.105	0.062	0.413	1.298
10	220	0.000	0.000	0.246	0.246
reference	200			0.000	0.000

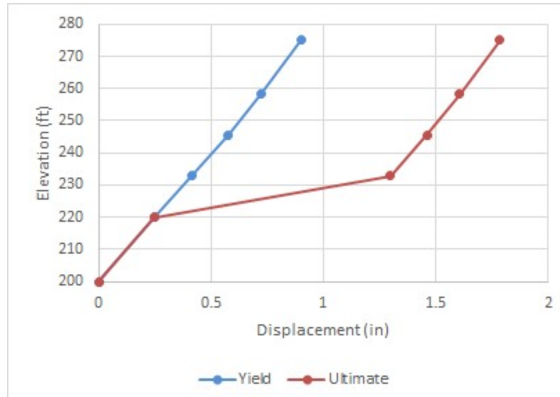

Fig. 8. Lateral Displacements at Yield and Ultimate

A system ductility is defined as a ratio of two amounts of energies consumed to reach yielding and ultimate failure. The system ductility of the tank is obtained by Eqn. 9 using the deflections at Yield and Ultimate and the nodal masses of the tank. Table 6 lists these quantities and presents a system ductility of 2.09 by Eqn. 9.

Table 6: System Ductility

Node	Mass, kip-s^2/ft	Yield, inches	Ultimate, inches	Mass x Y	Mass x U
6	32.89	0.899	1.785	354.8	704.5
7	17.17	0.72	1.606	148.3	330.9
13	69.34	0.649	1.535	540.0	1277.2
8	17.17	0.576	1.462	118.7	301.2
9	17.17	0.413	1.298	85.1	267.4
15	38.31	0.331	0.78	152.2	358.6
10	98.78	0.246	0.246	291.6	291.6
			Sum =	1690.7	3531.5
			μ =	2.09	

5. Fixed Base Case

Figure 9 compares the elastic deflection of the fixed base tank and the soil-founded tank, which illustrates that the fixed base deflection is not as significant as the SSI deflection. The procedure for system ductility of the fixed base tank is much simpler than the soil-founded tank case. The first step is to develop tri- and bi-linear F-D curves for the 12.8-ft wall segment. A tri-linear curve is defined as three control points of concrete cracking, yielding, and ultimate failure. Table

7 summarizes the parameters to form tri- and bi-linear curves, which are compared in Figure 10.

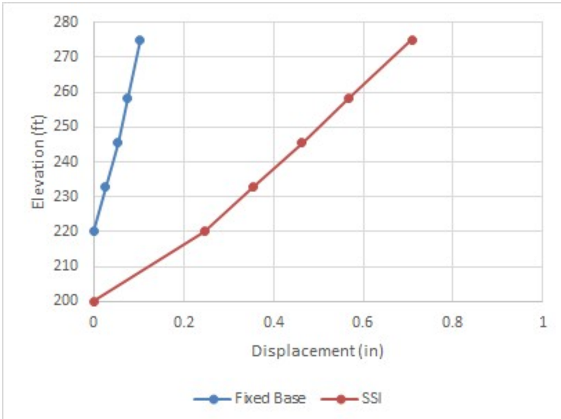

Fig. 9. Elastic Displacements of the Tank from SSI and Fixed Base Analyses

Table 7: Tri- and Bi-linear F-D Curve Data

	Tri-linear curve		Bi-linear curve	
	Displ, in	Shear, kip	Displ, in	Shear, kip
Origin	0	0	0	0
Cracking	0.0064	1128	0	0
Yield	0.2345	9154	0.0521	9154
Ulimate	1.1511	9154	1.071	9154

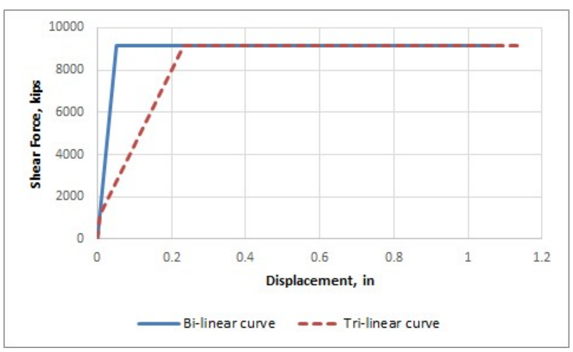

Fig. 10. Trilinear and Bilinear F-D Curves

On the contrary to the soil-founded case, deflection of the fixed base tank at Yield is simply obtained by multiplying a scale factor to the elastic deflection since this deflection merely includes the displacement component contributing to shearing distortion of the concrete wall. The scale factor is calculated by Eqn. 8.

- $\Delta 9T$ = 0.026 in (Table 1)
- Δy = 0.052 in (bilinear curve)
- SF = 0.052 / 0.026 = 2.03 (Eqn. 8)

Deflection at Ultimate is the same as the yield deflection that is laterally displaced by the drift limit (1.151 in). Table 8 summarizes these values with the deflection at Yield and Ultimate, which are also plotted in Figure 11.

Table 8: Deflection Profiles at Yield and Ultimate

Node	EL.	Elastic	Yield	Ultimate
	ft	inches	inches	inches
6	275.1	0.103	0.209	1.228
7	258.3	0.076	0.154	1.173
8	245.5	0.054	0.110	1.129
9	232.8	0.026	0.052	1.071
10	220	0.000	0.000	0.000

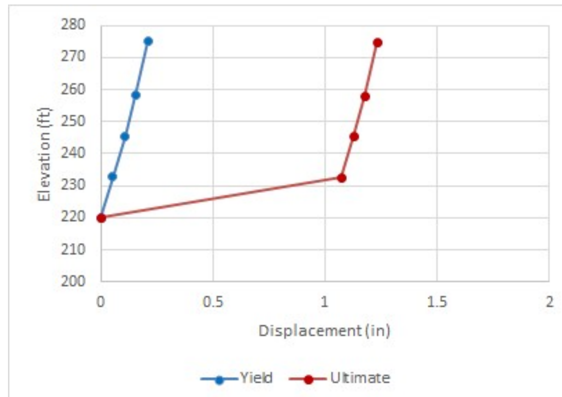

Fig. 11. Lateral Displacements at Yield and Ultimate

Lastly, the system ductility of the tank is obtained by Eqn. 9 using the yield and ultimate deflections and nodal masses of the tank. Table 9 lists these quantities and presents a system ductility of 8.85 by Eqn. 9.

Table 9: System Ductility

Node	Mass, $kip\text{-}s^2/ft$	Yield, inches	Ultimate, inches	Mass x Y	Mass x U
6	32.89	0.2089	1.228	82.4	484.7
7	17.17	0.1538	1.173	31.7	241.7
13	69.34	0.1322	1.151	110.0	957.7
8	17.17	0.1099	1.129	22.6	232.6
9	17.17	0.052	1.071	10.7	220.7
15	38.31	0.0264	0.544	12.1	250.1
			Sum =	269.6	2387.5
			μ =	8.85	

6. Results and Conclusions

With the system ductility, inelastic energy absorption factors of the tank can be computed by Eqn. 10 to 15 (Table 2). The resulting inelastic energy absorption factors, $F\mu$, of the two analyses (soil-founded and fixed base) are summarized in Table 10.

The fixed base tank frequency is 12.07 Hz, which is close to the spectral peak of the ground response spectrum and therefore, it was expected that its strength factor would be smaller than the soil-founded tank. However, the tank frequency is shifted to 5.3 Hz where the spectral acceleration demand goes down (Figure 1). As a result, its strength factor became comparable to the strength factor of the soil-founded tank.

Table 10: Inelastic Energy Absorption Factors

Equation	Soil-Founded	Fixed Base	Eqn.
$\mu = \Sigma(m \times \delta u) / \Sigma(m \times \delta y)$	2.09	8.85	9
$fs/f = 1/\sqrt{\mu}$	0.69	0.34	10
A = 0.85 or Cf x (1 -fs/f)	0.71	0.85	12
fe/f = 1 - A x (1 - fs/f)	0.78	0.44	11
Frequency shift from f to fe	3.2 to 2.5 Hz	12.1 to 5.3 Hz	
$\xi h = 0.11 \times (1 - fs/f)$	3.39%	7.30%	13
$\xi e = \{(fs/f)/(fe/f)\}^2 \times (\xi + \xi h)$	8.14%	8.51%	14
Sa at f, ξ	0.72g	1g	
Sae at fe, ξe	0.56g	0.76g	
$F\mu = \{(fe/f)/(fs/f)\}^2 \times (Sa / Sae)$	1.64	2.23	15
Strength factor, Fs	2.45	2.19	
Capacity factor = Fs x Fμ	4.02	4.88	

It should be noted that the system ductility of the fixed base tank ($\mu = 8.85$) is much larger than the soil-founded tank ($\mu = 2.09$), but the resulting inelastic energy absorption factors Fμ are 2.23 for the fixed base tank and 1.64 for the soil-founded tank such that their inelastic energy absorption factors are not as different as the system ductility.

This study also demonstrates that the inelastic energy absorption capability of soil-founded tank is less than the fixed base tank, which agrees to Hashimoto's conclusion in Reference [1].

REFERENCES

[1] Hashimoto, P.S., J.M. Jimenez-Chong, D.K. Nakaki, and R.P. Kennedy, "Nonlinear Response of a Structure with Significant Soil-Structure Interaction Effects for Application to Seismic Fragility Evaluation", Transactions, SMiRT-25, Charlotte, NC, 4-9 Aug. 2019
[2] Electric Power Research Institute, Seismic Fragility and Seismic Margin Guidance for Seismic Probabilistic Risk Assessments, EPRI 3002012994, Palo Alto, CA, 2018.
[3] American Society of Civil Engineers, Seismic Analysis of Safety-Related Nuclear Structures, ASCE/SEI 4-16, American Society of Civil Engineers, Reston, VA, 2017