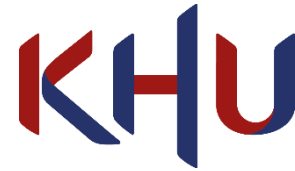


Criticality Experiment Benchmark Selection and New Critical Assembly Design Based on Similarity Analyses for LEU+ i-SMR System



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Contents

1. Introduction
2. Similarity Coefficient Generation
3. Similarity Test between Critical Experiments and LEU+ loaded i-SMR Core Systems
4. Design of Critical Assemblies similar to LEU+ loaded i-SMR Core Systems
5. Conclusion

1. Introduction

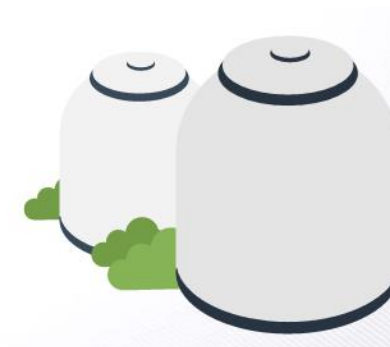
1. Selecting proper criticality experiments

- Various design methods and tools are employed in the development of new types of nuclear reactors and systems.
 - Must provide sufficient safety margins to maintain subcriticality under all conditions.
 - Calculated safety parameters (e.g., k_{eff}) must be systematically compared with experimental data to quantify uncertainties and biases in the design methods and tools.
- Selecting proper criticality experiments that accurately represent the target system $\Rightarrow$ crucial step
 - Mainly select benchmarks have been based on **qualitative similarities** – fuel type, fuel enrichment, pin pitch, etc..
 - It is not logical enough for regulators. $\Rightarrow$ lead to **quantitative** similarity analyses.



Nuclear Reactor A

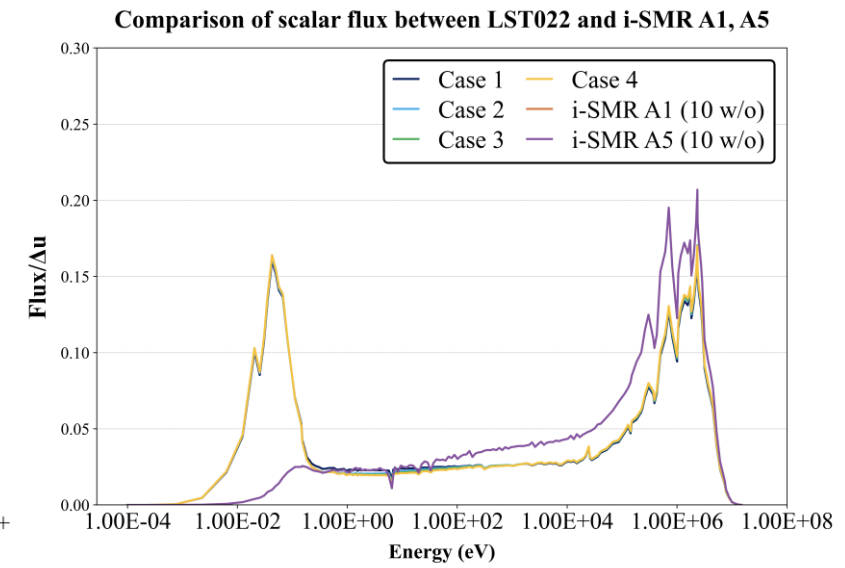
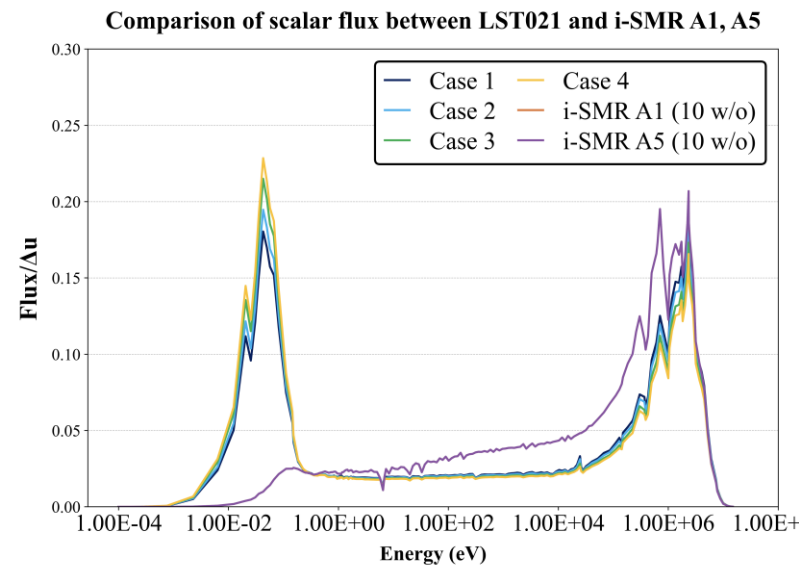
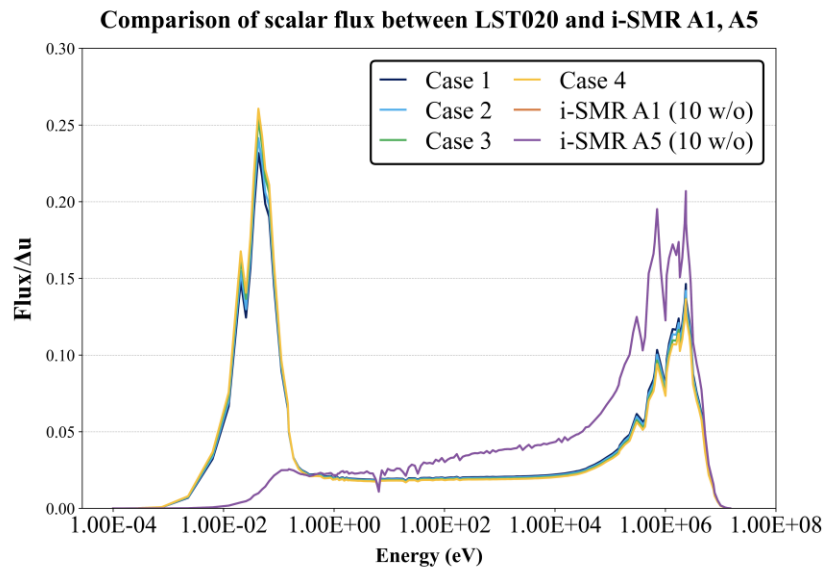
	Reactor A	Reactor B
Fuel	UO2	UO2
Enrichment	2.4 w/o	2.5 w/o
Pin pitch	0.625	0.710
Cladding	Al	Zr
Coolant	Light water	Light water



Nuclear Reactor B

2. LEU+ fuels loaded i-SMR System

- LEU+ fuels (5-10 wt.%) are proposed to improve economics and acceptability of SMRs.
- In the previous study $\Rightarrow$ the design code was validated by applying LEU+ fuels to the innovative SMR (i-SMR).
 - For the verification of the Monte Carlo (MC) solutions, analyzed criticality experiments (enrichment **10 wt.%**) : LST-020,021,022
 - There were discrepancies between their neutron energy spectra and those of the i-SMR Fuel Assembly system.



3. In this Study,

- How to select a criticality experiments $\Rightarrow$ suitable for **the i-SMR core system with LEU+ fuels (2 wt.% to 10 wt.%)**
 - Similarity Coefficient (LEU+ i-SMR core system $\Leftrightarrow$ ICSBEP benchmark problems)
 - Comparing the Neutron Energy Spectrum
 - Comparing EALF (Energy of Average Lethargy of Fission) as complementary tool.
- Design a critical assembly similar to LEU+ i-SMR core system
 - 8 wt.% and 10 wt.% enrichment critical assemblies
 - Perform similarity tests with i-SMR core system
- Similarity Coefficient $\Rightarrow$ deterministic based sensitivity and uncertainty (S/U) similarity analysis
 - McCARD MC code and SimTest Utility

2. Similarity Coefficient Generation

1. Similarity Coefficient Generation using S/U method

- Selection of the criticality experiment $\Rightarrow$ Provide computational justification to regulating body
 - Some researchers quantified the degree of similarity between criticality experiments and target system.
 - B. L. Broadhead et al. Sensitivity- and Uncertainty-Based Criticality Safety Validation Techniques, Nuclear Science and Engineering, 2004.
 - C. M. Perfetti and B. T. Rearden, Estimating Code Biases for Criticality Safety Applications with Few Relevant Benchmarks, Nuclear Science and Engineering, 2019.
- Similarity Coefficient : c_k
 - Similarity to the Pearson Coefficient
 - Quantify the degree of correlation of system I and II.
 - Range : $[-1, 1]$ / c_k approaches 1 = two systems are highly positively correlated.

$$c_k = \frac{cov[k_I, k_{II}]}{\sigma(k_I) \cdot \sigma(k_{II})}$$

$$cov[k_I, k_{II}] = \sum_{i,\alpha,g} \sum_{i',\alpha',g'} cov[x_{\alpha,g}^i, x_{\alpha',g'}^{i'}] \left(\frac{\partial k_I}{\partial x_{\alpha,g}^i} \right) \left(\frac{\partial k_{II}}{\partial x_{\alpha',g'}^{i'}} \right)$$

$x_{\alpha,g}^i$: α -type microscopic cross-section of isotope i for energy group g

2. Energy of Average Lethargy of Fission (EALF)

- Crucial concept in traditional criticality safety validation
 - One of the physical characteristics used to evaluate system similarity.
 - An average measure of the neutron energy spectra where fissions predominantly occur.
- Lethargy u of a neutron with energy E : $u = \ln(\frac{E_0}{E})$
 - E_0 : maximum neutron energy, 10 MeV

$$EALF = E_0 / e^{\bar{u}}$$

$$\bar{u} = \frac{\sum_m \sum_g (\bar{u} \times \sum_{fg}^m \phi_g^m)}{\sum_m \sum_g (\sum_{fg}^m \phi_g^m)}$$

m = number of a physical zone inside core

$\bar{u}_g$ = midpoint of the g th lethargy group, defined as lethargy of a neutron with energy $\bar{E}_g = \sqrt{E_g E_{g-1}}$

Σ_{fg} = group macroscopic fission cross section

ϕ_g = neutron flux within lethargy group g .

3. Similarity Test between Criticality Experiments and LEU+ loaded i-SMR Core System

1. McCARD/SimTest Code System

- McCARD : generate sensitivity coefficients using perturbation method.
 - Uncertainty of nuclear reaction cross-section
- SimTest : generate the S/U method based similarity coefficient.
 - Use sensitivity coefficients from McCARD.
 - Use the cross-section covariance data from evaluated nuclear data library.

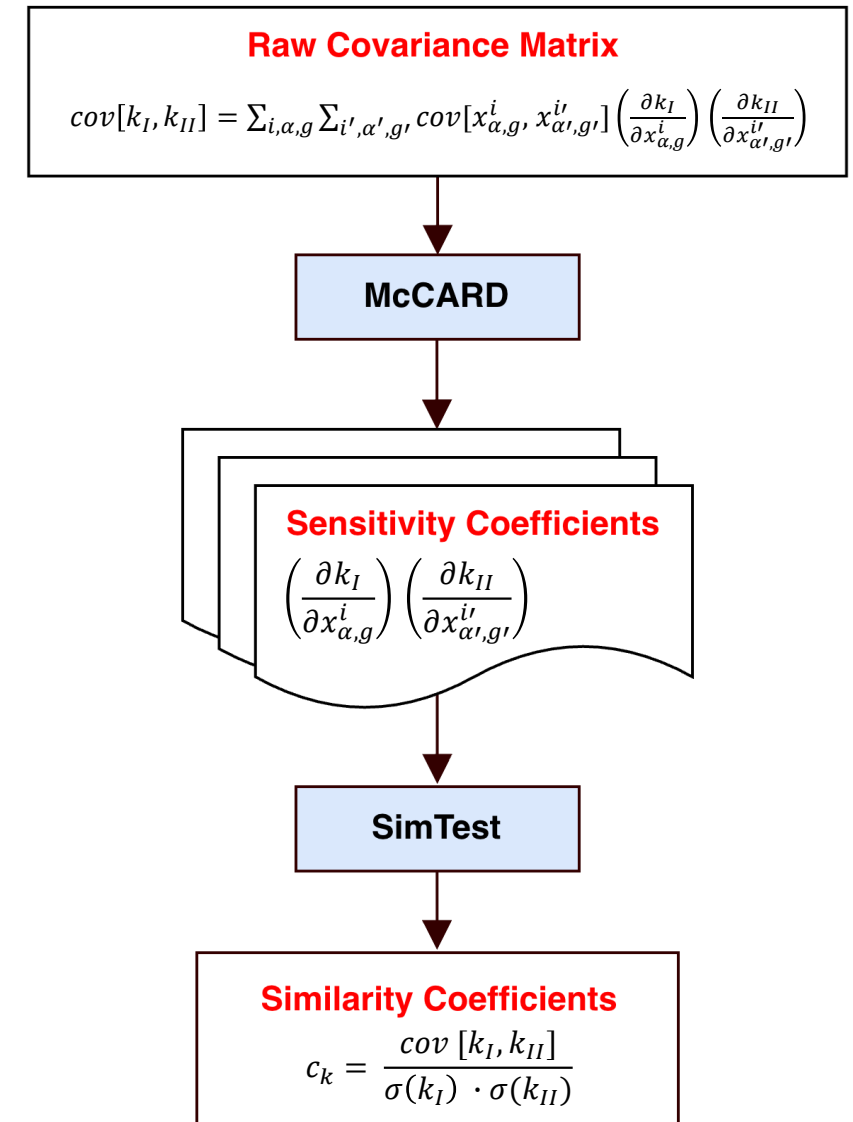


Fig 1. Flow chart of McCARD/SimTest Code System

2. Similarity Tests

- Test Methods
 - 1) Similarity Coefficients
 - 2) Neutron Energy Spectrum
 - 3) EALF (Energy of Average Lethargy of Fission)

- Criticality Experiments – ICSBEP benchmark book, 15 problems (HMF, LMT, LCT, LST categories)
Target System – LEU+ loaded i-SMR Core (2, 4, 6, 8, 10 wt.%)
 - ENDF/B-VII.1 covariance data matrix with the LANL 30-group structure (^{235}U , ^{238}U)
 - HMF(HEU-MET-FAST), LMT(LEU-MET-THERM), LCT(LEU-COMP-THERM), LST(LEU-SOL-THERM)

- McCARD eigenvalue calculation – 200 inactive cycles, 800 active cycles, 80,000 histories per cycle

2. Similarity Tests – Criticality Experiments Description

Table I. Description of the selected critical experiment benchmarks and LEU+ loaded i-SMR core system

Short Name	Ref No.	Spectrum	²³⁵ U Enrichments (wt.%)	Pin Pitch (cm)	Pellet Radius (cm)
i-SMR Core 5 Cases	[2]	Thermal	2.0 / 4.0 / 6.0 / 8.0 / 10.0	1.26	0.4096
Flattop25	[3]	Fast	93.2	-	-
GODIVA		Fast	94.0	-	-
HMF002c2		Fast	97.6	-	-
HMF032c1		Fast	94.0	-	-
IPENMB01		Thermal	4.35	1.50	0.4245
LCT001c1		Thermal	2.35	2.032	0.635
LCT003c1		Thermal	2.35	1.684	0.5588
LMT007c2		Thermal	4.95	1.53	0.38645
LST002		Thermal	4.90	-	-
LCT022c1		Thermal	10.0	0.70	0.208
LST020c1		Thermal	10.0	-	-
LST021c1		Thermal	10.0	-	-
LST022c4		Thermal	10.0	-	-
LCT085c1		Thermal	6.50	1.27	0.06
LCT085c13		Thermal	6.50	1.27	0.06

2. Similarity Test Results – Similarity Coefficients

- HEU benchmarks $\Rightarrow$ 0.232 to 0.498 / LEU benchmarks $\Rightarrow$ 0.738 to 0.992
 - U.S. NRC : critical safety analyses should be conducted using the critical experiments with c_k value in excess of 0.90.
 - Broadhead et al : a target application should have more than 20 experiments with c_k value greater than 0.80.

	2 wt. %	4 wt. %	6 wt. %	8 wt. %	10 wt. %	flattop25	GODIVA	HMF002c2	HMF032c1	IPENMB01	LCT001c1	LCT003c1	LMT007c2	LST002	LCT022c1	LST020c1	LST021c1	LST022c4	LCT085c1	LCT085c13
i-SMR 2wt. %	1.000	$\approx$ 1.000	0.998	0.993	0.985	0.259	0.232	0.256	0.249	0.934	0.992	0.972	0.780	0.949	0.839	0.922	0.922	0.927	0.807	0.823
i-SMR 4wt. %	$\approx$ 1.000	1.000	$\approx$ 1.000	$\approx$ 1.000	$\approx$ 1.000	0.302	0.314	0.292	0.317	0.941	0.977	0.965	0.803	0.932	0.888	0.910	0.910	0.917	0.844	0.855
i-SMR 6wt. %	0.998	$\approx$ 1.000	1.000	$\approx$ 1.000	$\approx$ 1.000	0.351	0.382	0.335	0.380	0.916	0.939	0.933	0.790	0.893	0.897	0.873	0.873	0.881	0.844	0.850
i-SMR 8wt. %	0.993	$\approx$ 1.000	$\approx$ 1.000	1.000	$\approx$ 1.000	0.395	0.443	0.375	0.437	0.882	0.894	0.894	0.768	0.844	0.891	0.826	0.827	0.835	0.831	0.834
i-SMR 10wt. %	0.985	$\approx$ 1.000	$\approx$ 1.000	$\approx$ 1.000	1.000	0.438	0.498	0.414	0.491	0.840	0.843	0.848	0.738	0.791	0.876	0.775	0.776	0.785	0.810	0.808
flattop25	0.259	0.302	0.351	0.395	0.438	1.000	0.758	0.991	0.930	-0.176	-0.223	-0.232	-0.293	-0.099	-0.069	-0.079	-0.080	-0.080	-0.173	-0.192
GODIVA	0.232	0.314	0.382	0.443	0.498	0.758	1.000	0.657	0.895	0.703	0.697	0.656	0.623	0.752	0.680	0.764	0.764	0.761	0.637	0.650
HMF002c2	0.256	0.292	0.335	0.375	0.414	0.991	0.657	1.000	0.887	-0.191	-0.210	-0.226	-0.335	-0.096	-0.117	-0.085	-0.087	-0.087	-0.219	-0.234
HMF032c1	0.249	0.317	0.380	0.437	0.491	0.930	0.895	0.887	1.000	-0.074	-0.220	-0.200	-0.077	-0.075	0.143	-0.021	-0.022	-0.020	0.045	0.016
IPENMB01	0.934	0.941	0.916	0.882	0.840	-0.176	0.703	-0.191	-0.074	1.000	0.947	0.959	0.986	0.914	$\approx$ 1.000	0.923	0.925	0.931	$\approx$ 1.000	$\approx$ 1.000
LCT001c1	0.992	0.977	0.939	0.894	0.843	-0.223	0.697	-0.210	-0.220	0.947	1.000	$\approx$ 1.000	0.997	0.952	$\approx$ 1.000	0.951	0.952	0.959	$\approx$ 1.000	$\approx$ 1.000
LCT003c1	0.972	0.965	0.933	0.894	0.848	-0.232	0.656	-0.226	-0.200	0.959	$\approx$ 1.000	1.000	$\approx$ 1.000	0.894	$\approx$ 1.000	0.892	0.893	0.902	$\approx$ 1.000	$\approx$ 1.000
LMT007c2	0.780	0.803	0.790	0.768	0.738	-0.293	0.623	-0.335	-0.077	0.986	0.997	$\approx$ 1.000	1.000	0.718	$\approx$ 1.000	0.748	0.750	0.759	$\approx$ 1.000	$\approx$ 1.000
LST002	0.949	0.932	0.893	0.844	0.791	-0.099	0.752	-0.096	-0.075	0.914	0.952	0.894	0.718	1.000	0.910	0.997	0.997	0.999	0.877	0.896
LCT022c1	0.839	0.888	0.897	0.891	0.876	-0.069	0.680	-0.117	0.143	$\approx$ 1.000	$\approx$ 1.000	$\approx$ 1.000	$\approx$ 1.000	0.910	1.000	0.720	0.722	0.732	0.969	0.957
LST020c1	0.922	0.910	0.873	0.826	0.775	-0.079	0.764	-0.085	-0.021	0.923	0.951	0.892	0.748	0.997	0.720	1.000	$\approx$ 1.000	$\approx$ 1.000	0.858	0.878
LST021c1	0.922	0.910	0.873	0.827	0.776	-0.080	0.764	-0.087	-0.022	0.925	0.952	0.893	0.750	0.997	0.722	$\approx$ 1.000	1.000	$\approx$ 1.000	0.860	0.880
LST022c4	0.927	0.917	0.881	0.835	0.785	-0.080	0.761	-0.087	-0.020	0.931	0.959	0.902	0.759	0.999	0.732	$\approx$ 1.000	$\approx$ 1.000	1.000	0.875	0.894
LCT085c1	0.807	0.844	0.844	0.831	0.810	-0.173	0.637	-0.219	0.045	$\approx$ 1.000	$\approx$ 1.000	$\approx$ 1.000	$\approx$ 1.000	0.877	0.969	0.858	0.860	0.875	1.000	0.986
LCT085c13	0.823	0.855	0.850	0.834	0.808	-0.192	0.650	-0.234	0.016	$\approx$ 1.000	$\approx$ 1.000	$\approx$ 1.000	$\approx$ 1.000	0.896	0.957	0.878	0.880	0.894	0.986	1.000

Fig 2. Similarity Coefficients for 20x20 benchmark matrix

2. Similarity Test Results – Neutron Energy Spectrum

- LCT benchmarks(LCT022c1, LCT085c13) and LEU+ i-SMR core system
 - LCT022c1 (10 wt.%) $\Leftrightarrow$ i-SMR core (10 wt.%) : $c_k = 0.876$
 - LCT085c13 (6.5 wt.%) $\Leftrightarrow$ i-SMR core (6 wt.%) : $c_k = 0.850$
- ✓ Quite high similarity, similar spectrum

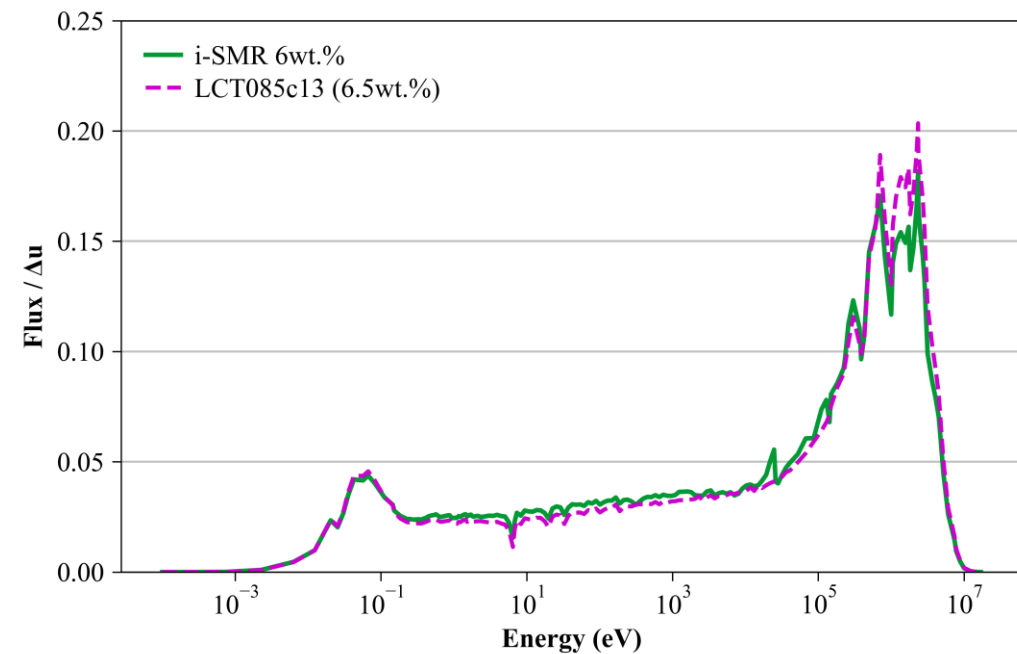
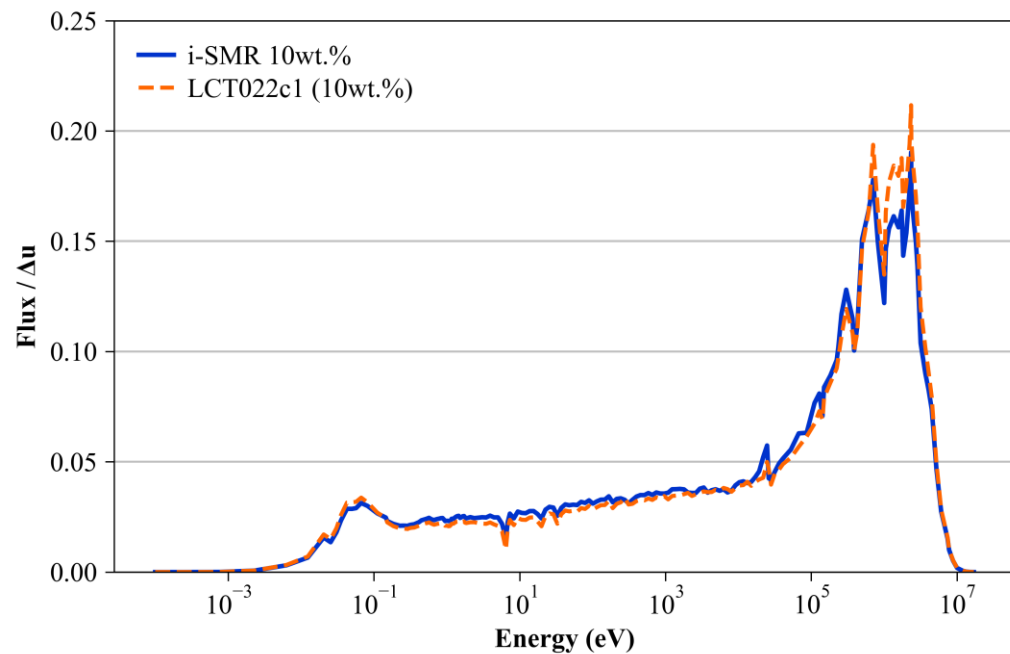


Fig 3. Neutron energy spectra of LCT benchmarks and i-SMR core system

2. Similarity Test Results – Neutron Energy Spectrum

- LST, LMT benchmarks(LST020c1, LMT007c2) and LEU+ i-SMR core system
 - LST020c1 (10 wt.%) $\Leftrightarrow$ i-SMR core (10 wt.%) : $c_k = 0.775$
 - LMT007c2 (4.95 wt.%) $\Leftrightarrow$ i-SMR core (4 wt.%) : $c_k = 0.803$
 - LST020c1 (10 wt.%) $\Leftrightarrow$ i-SMR core (2 wt.%) : $c_k = 0.922$
 - ✓ Different physical properties, High similarity, Different spectrum shape $\Rightarrow ??$
 - ✓ Dominated by common nuclear data uncertainties

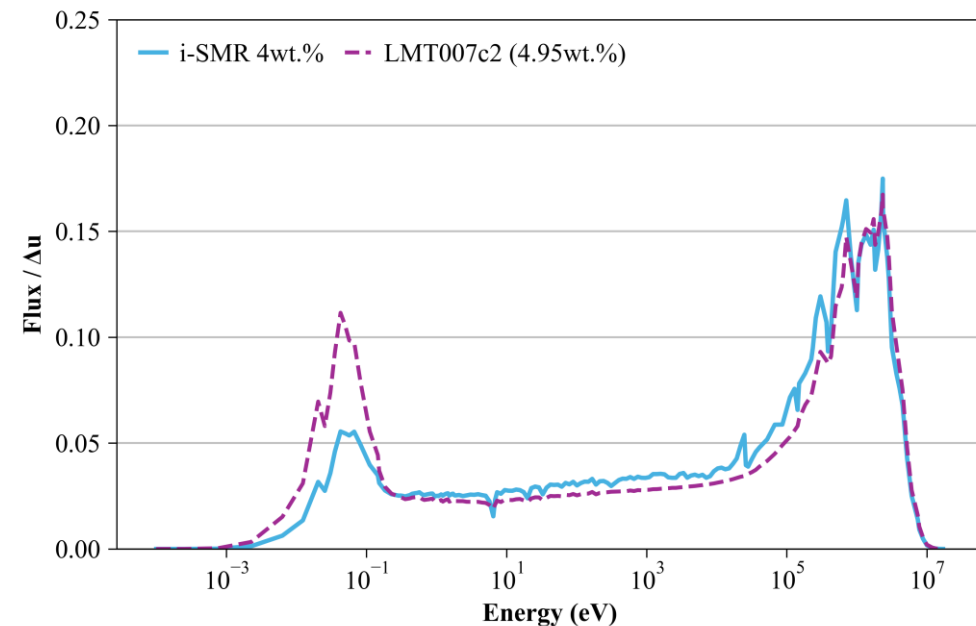
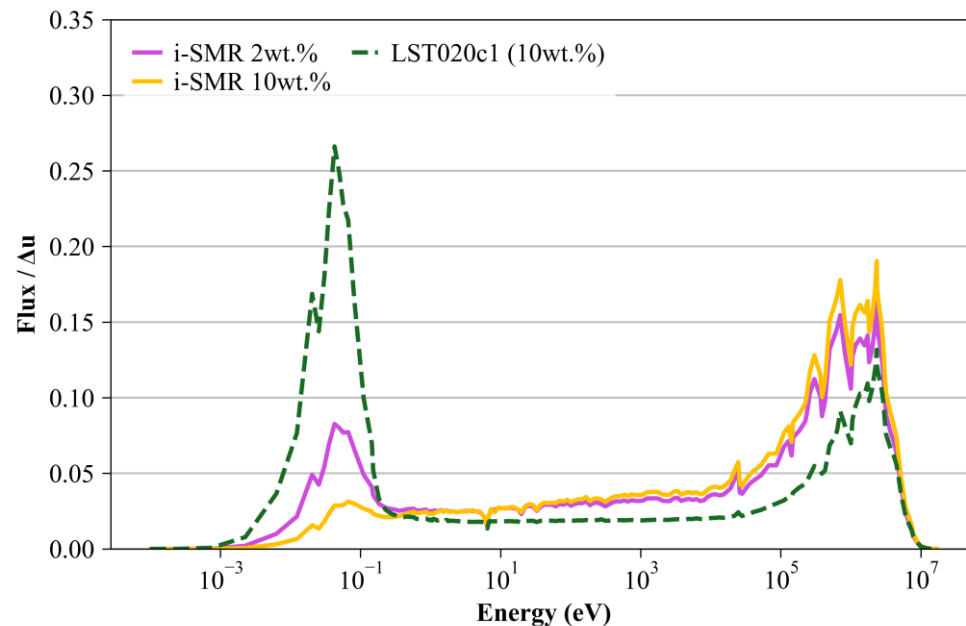


Fig 4. Neutron energy spectra of LST, LMT benchmarks and i-SMR core system

2. Similarity Test Results – EALF (Energy of Average Lethargy of Fission)

- LST020c1 (10 wt.%), LMT007c2 (4.95 wt.%), LCT085c13 (6.5 wt.%), LCT002c1 (10 wt.)
- i-SMR core : as enrichment increases, fission occurs more in the relatively high energy region

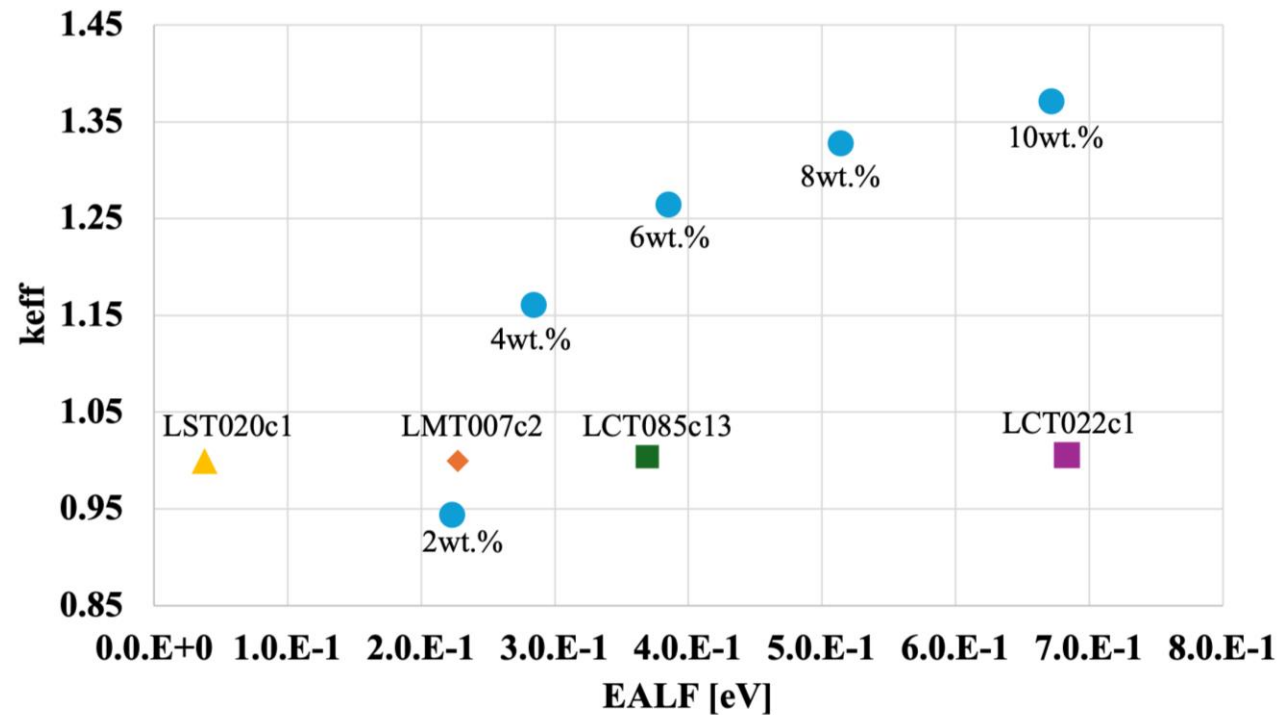


Fig 5. EALF of benchmarks and i-SMR core system

4. Design of Critical Assemblies similar to LEU+ loaded i-SMR Core System

1. Draft Design of Critical Assemblies

- Construct critical assemblies using the i-SMR **A1** fuel assembly
 - Fuel ^{235}U enrichment - 8 wt.%, 10 wt.%
 - Composed of fuel, water, air, SS304
 - McCARD eigenvalue calculation – 200 inactive cycles, 800 active cycles, 80,000 histories per cycle , ENDF/B-VII library
 - Multiplication factor : approached to the criticality when stochastic uncertainty is less than 10pcm.

Table II. Specifications of LEU+ i-SMR critical assemblies

Parameter	Value (8 wt.%)	Value (10 wt.%)	Unit
# of Assemblies	5	5	#
Reflector Outer Radius	75.708	75.708	cm
SS304 Thickness	10	10	cm
Total Radius	85.708	85.708	cm
Fuel Height	40	40	cm
Air Height	24.395	28.575	cm
Moderator Height	30.605	26.425	cm
Bottom Reflector Height	5	5	cm
Total Height	60	60	cm

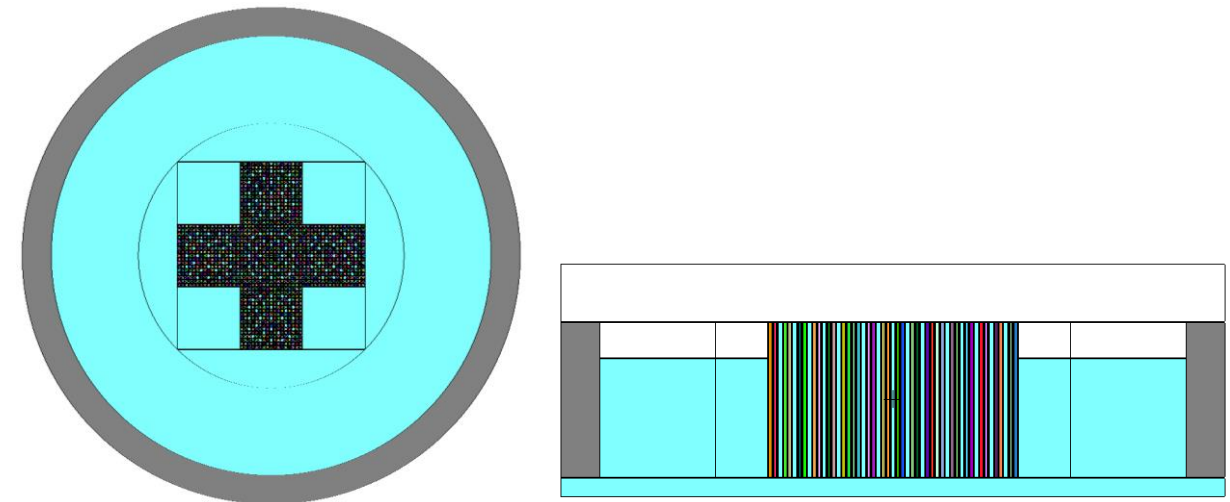


Fig 6. Cross-section and Elevation view of Critical Assembly

2. Similarity Test Results

■ Similarity Coefficient

- CA 8 wt.% $\Leftrightarrow$ i-SMR core 8 wt.% : $c_k = 0.84570$
- CA 10 wt.% $\Leftrightarrow$ i-SMR core 10 wt.% : $c_k = 0.87425$

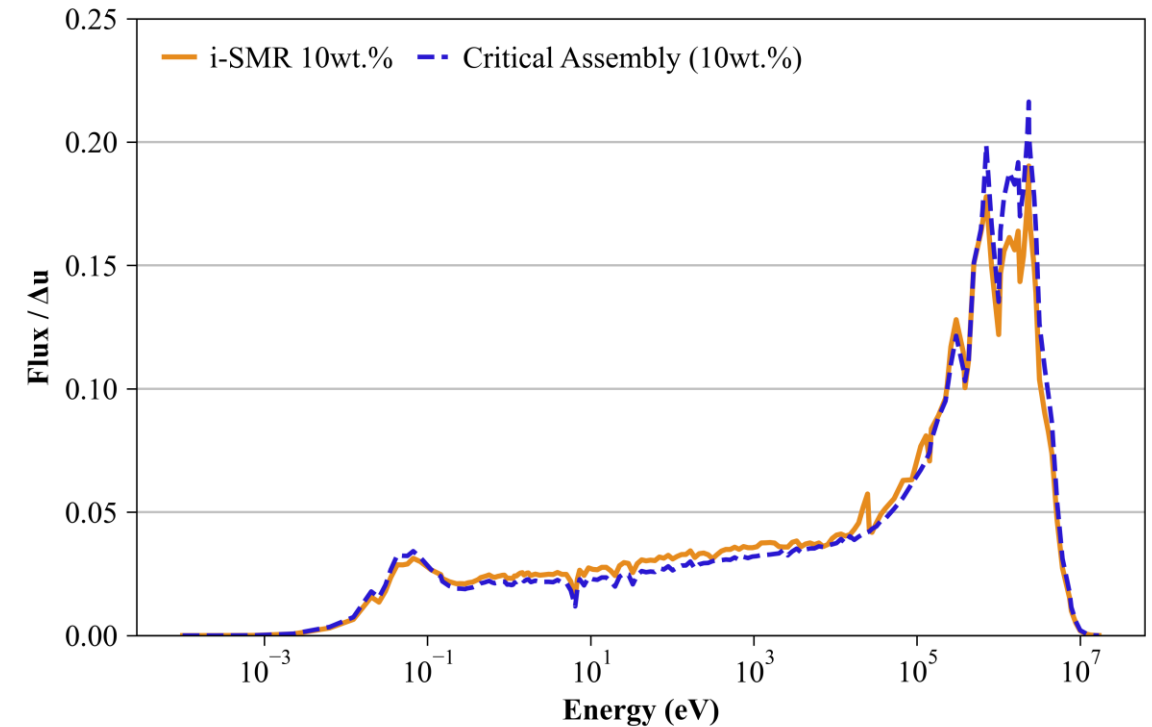
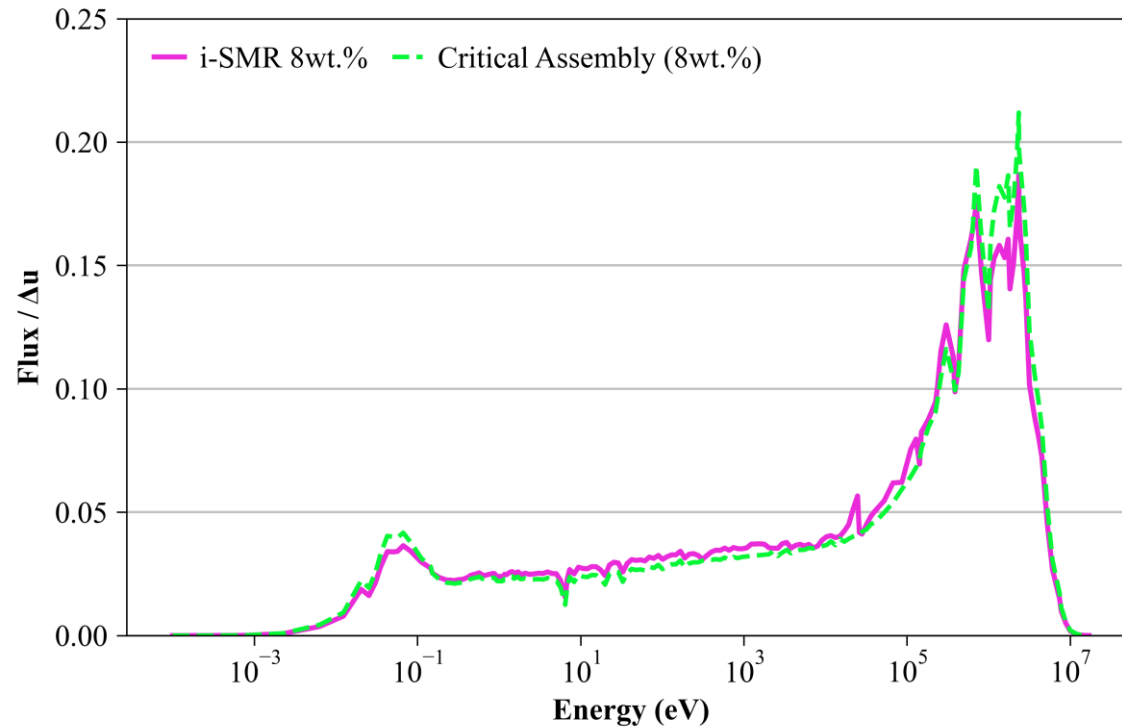


Fig 7. Neutron energy spectra of critical assembly and i-SMR core system

2. Similarity Test Results

■ Similarity Coefficient

- CA 8 wt.% $\Leftrightarrow$ i-SMR core 8 wt.% : $c_k = 0.84570$
- CA 10 wt.% $\Leftrightarrow$ i-SMR core 10 wt.% : $c_k = 0.87425$

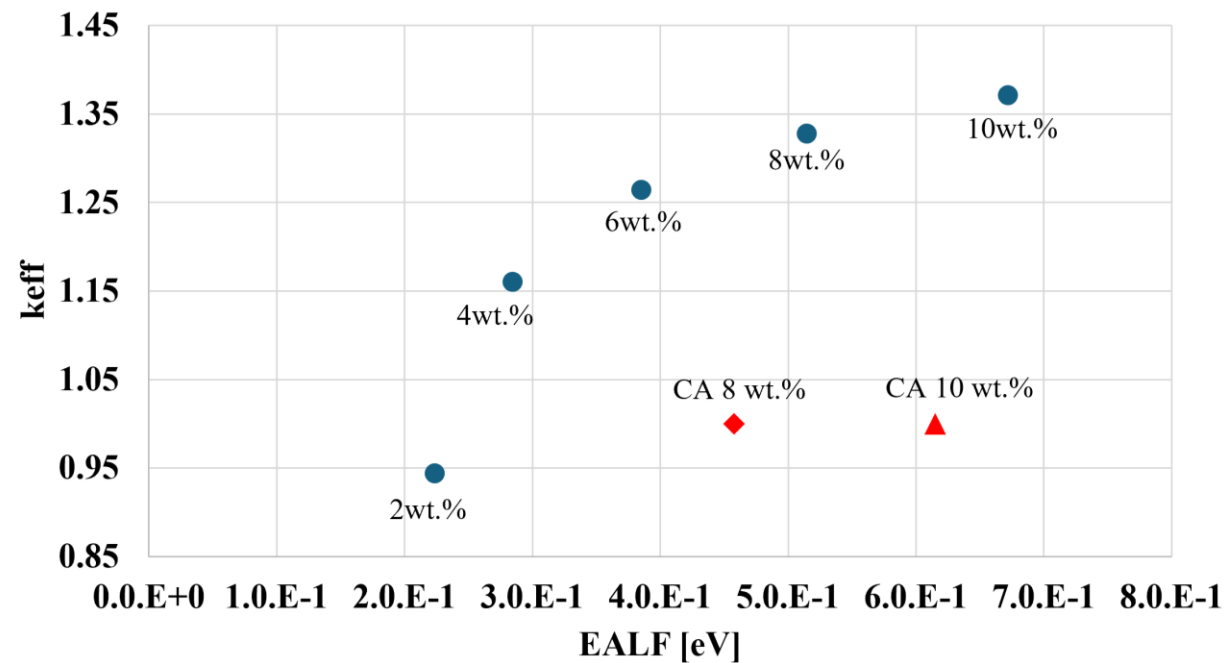


Fig 8. EALF of critical assembly and i-SMR core system

3. Discussions

- Code Validation
 - Performed by comparing calculated and experimental critical values.
 - Requires proper benchmark experiments representing the target system.
- Limitation for LEU+ systems (6-10 wt%) :
 - The number of benchmark experiments in the ICSBEP database is very limited.
- Reference criteria for similarity coefficient (0.8-0.9) :
 - Proposed by NRC and several studies based on LEU benchmark datasets.
 - These values may not be directly applicable to LEU+ systems.
- This study extends the use of similarity analysis from validation to the design of a new LEU+ critical assembly representing the LEU+ loaded i-SMR core.

5. Conclusion

1. Criticality Experiment Selection based on Similarity analyses

- Similarity Analyses between i-SMR core system and criticality experiments
 - McCARD/SimTest code system
 - LEU benchmarks (c_k : 0.738-0.992) showed higher similarity than HEU benchmarks (c_k : 0.232-0.498).
 - Compared the neutron energy spectrum and EALF additionally.
 - ✓ LST, LMT benchmarks showed differences compared to the i-SMR core systems.
⇒ LCT benchmarks were the most similar criticality experiments of these.
 - Despite the different physical properties, there was a case that the similarity was high. (LST020c1 10 wt.% ⇔ i-SMR 2 wt.% : 0.922)
 - ✓ **Limitation of relying solely on the similarity coefficient for the multiplication factor**
 - ✓ **Physical indices (spectrum, EALF) must complement similarity coefficients.**

2. Design Application of Similarity analyses

- Designed new critical assemblies with high similarity to i-SMR core system. (8 wt.%, 10 wt.%)
- Verified similarity through similarity coefficient, neutron energy spectrum, EALF.
 - c_k : 0.84570, 0.87425
 - Show potential to be utilized in the design of new critical assemblies for next-generation reactors.

3. In the future,

- In the process of selecting a similar critical experiment for the target system, we intend to simulate a method that can consider the neutron energy spectrum and EALF together, including the similarity coefficient for the multiplication factor.

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Thank You For Listening



1. Used α -types Nuclear Data in Nuclear Reaction Cross-section Covariance Matrix

MT	Reaction Type	Description
2	(z,z0)	Elastic scattering cross section for incident particles
4	(z,n)	Inelastic scattering cross section
102	(z, γ)	Radiative capture
16	(z,2n)	Production of two neutrons and a residual
17	(z,3n)	Production of three neutrons and a residual
18	(z,fission)	Particle-induced fission
452	$\bar{\nu}$	Average total number of neutrons released per fission event

2. Descriptions of Selected ICSBEP Benchmark Problems

Short Name	Benchmark ID	Description
FLATTOP25	HEU-MET-FAST-028	^{235}U (93.24 wt.%) Sphere reflected by normal uranium
GODIVA	HEU-MET-FAST-001	Bare, highly enriched uranium (94 wt.%) sphere
HMF002c2	HEU-MET-FAST-002 case2	Topsy 8-Inch-Tuballoy-Reflected Orally Assemblies (97.67 wt.%)
HMF032c1	HEU-MET-FAST-032 case1	^{235}U (94 wt.%) Spheres surrounded by natural-uranium reflectors
IPENMB01	LEU-COMP-THERM-077 case1	Water-moderated squared-pitched lattices UO ₂ (4.3486 wt.%)
LCT001c1	LEU-COMP-THERM-001 case1	Water-moderated UO ₂ (2.35 wt.%) Fuel Rods in 2.032 cm square-pitched arrays
LCT003c1	LEU-COMP-THERM-003 case1	Water-moderated UO ₂ (2.35 wt.%) Fuel Rods in 1.684 cm square-pitched arrays
LMT007c2	LEU-MET-THERM-007 case2	Water-Moderated and Water-Reflected 0.30 in. Diameter U (4.95 wt.%) metal rods in square-pitched arrays
LST002	LEU-SOL-THERM-002	174-liter spheres of low enriched (4.9 wt.%) uranium oxyfluorine solutions
LCT022c1	LEU-COMP-THERM-022 case1	Uniform water-moderated hexagonally pitched lattices of rods with UO ₂ (10 wt.%) fuels
LST020c1	LEU-SOL-THERM-020 case1	Water-reflected uranyl nitrate solution in 80cm cylindrical water tank (10.0 wt.%)
LST021c1	LEU-SOL-THERM-021 case1	Unreflected uranyl nitrate solution in 80cm cylindrical water tank (10.0 wt.%)
LST022c4	LEU-SOL-THERM-022 case4	Borated concrete-reflected uranyl nitrate solution in 28cm thick slabs (10.0 wt.%)
LCT085c1	LEU-COMP-THERM-085 case1	Regular hexagonal lattices of low-enriched U (6.5 wt.%) fuel rods in light water
LCT085c13	LEU-COMP-THERM-085 case13	Regular hexagonal lattices of low-enriched U (6.5 wt.%) fuel rods in light water

3. i-SMR Critical Assembly Design – Fuel Assembly A1

