

Enhancement of Global Modal Data Estimation Accuracy and Analysis of Error Sensitivity in Model Updating

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1. Introduction

Accurate prediction of the behavior of high-risk infrastructure such as nuclear power plants requires numerical models that reflect the structure's physical properties. Model updating calibrates these models using modal data (frequency, damping ratio, mode shape) estimated by applying a System Identification (SI) method to the measured responses. However, for large-scale structures like nuclear power plants, limitations on the number of sensors necessitate multi-setup testing, where responses are measured in sequence. The subsequent process of assembling the SI result from these setups can introduce significant errors, making the procedure highly sensitive to data quality.

Because displacement mode shapes are relatively insensitive to stiffness, these small measurement and assembly errors can be magnified during updating and produce large bias in stiffness estimates. While errors often arise from noise that can be canceled by aggregating multiple response sets, the assembly errors themselves must be addressed. We propose a multi-response integration framework that couples state-space system identification with convergence-based filtering, DBSCAN clustering, and weighted least-squares assembly to suppress this error amplification.

2. Methodology

2.1. Converged data integration and filtering

First, a System Identification (SI) method is applied to each response set to identify stable modes, which are defined by consistent frequency, damping, and mode shapes (MAC) across multiple system orders. The collected pool of converged modes is then subjected to a two-level statistical filter. To be retained for further analysis, a mode must first demonstrate consistency across the datasets (frequency variation <1%, damping variation <5%, and MAC >0.9) and then appear in a minimum percentage of the total datasets, a threshold governed by the *MinThds* Ratio. This rigorous process filters out inconsistent modes and significantly enhances the statistical reliability of the data.

2.2. Clustering process

The next step is to classify the filtered modal data by mode. To group physically identical modes and eliminate

any remaining outliers, the DBSCAN (Density-Based Spatial Clustering of Applications with Noise) algorithm is applied to the filtered data [1]. DBSCAN identifies dense regions of data as clusters, effectively separating them from sparse data points classified as noise. The process is controlled by two key parameters: the search radius (ϵ) and the minimum number of points required to form a cluster, which is defined by the *MinPts* Ratio. This ratio makes the parameter selection more intuitive by relating it to the total number of converged modes.

2.3. Optimal Clustering using Silhouette Analysis

The quality of the clustering results from various combinations of *MinThds* Ratio and *MinPts* Ratio is quantitatively assessed using the Silhouette value [2]. This metric measures how well-defined a cluster is by evaluating both its internal cohesion (how close points are to each other) and its separation from other clusters. A value close to 1 indicates a high-quality cluster. The optimal parameter set is selected by finding a balance: the one that achieves the highest possible Silhouette value while still identifying enough physically meaningful modes.

2.4. Derivation of Global Modal Data

Finally, the refined clusters are used to derive the global modal properties. First, common modes present across all measurement setups are identified. The global natural frequency and damping ratio for these modes are calculated using a variance-weighted average, giving more influence to more consistent (less varied) data. The global mode shape is then constructed from the local mode shapes using a least-squares assembly method [3]. This method minimizes the discrepancies at overlapping sensor locations, ensuring a physically consistent and accurate representation of the structure's global dynamic behavior.

3. Numerical Case study

3.1. Validation framework

The framework's effectiveness was validated using a 16-DOF shear-type numerical model under simulated Experimental Modal Analysis (EMA) conditions. The objective was to update an "initial" model with uniform stiffness to match a "true" model with randomly

generated story stiffnesses. A multi-setup scenario was simulated with three setups, each measuring 6 DOFs. To generate 60 total response sets, impulse loads were applied for 15 trials at each of four locations, selected to maximally excite the first four modes of the initial model. Crucially, white noise (up to ± 20 mm/s²) was added to all responses before applying the System Identification (SI) method, ensuring the final modal data used for updating realistically included measurement noise.

3.2. EMA based system identification

The Eigensystem Realization Algorithm (ERA) [4] was used to perform the SI on the 60 free vibration response sets. The proposed multi-response framework was then applied to this collection of datasets to filter, cluster, and assemble the global modal data. For comparison, a conventional method, which utilized only a single representative response set, was also analyzed. The global modal data estimated by each method are summarized in Table I. While both the proposed and conventional methods showed high agreement with the true system's modal data, there were key differences. The proposed method yielded superior accuracy for the mode shapes, which are critical for model updating. Conversely, the conventional method produced slightly more accurate estimates for the natural frequencies.

Table I: Comparison of Modal Properties Identified under EMA Conditions

Mode	Proposed method			Conventional method		
	F [Hz]	Error [%]	MAC	F [Hz]	Error [%]	MAC
1	0.591	0.69%	0.9999	0.588	0.08%	0.9997
2	1.795	0.01%	0.9999	1.794	0.04%	0.9995
3	3.205	0.07%	0.9998	3.211	0.13%	0.9999
4	4.520	0.03%	0.9993	4.517	0.04%	0.9942
5	5.324	0.16%	0.9980	5.331	0.04%	0.9921
6	-	-	-	6.386	0.49%	0.9786
Mean		0.19%	0.9994		0.13%	0.9940

3.3. Model updating results

The global modal data derived from both the proposed and conventional methods were used to update the stiffness parameters of the initial numerical model. As summarized in Table II, the results were definitive: the updating error from the conventional method was more than double that of the proposed method. This significant difference stems from the high sensitivity of the updating process, where the small improvements in mode shape accuracy achieved by our framework resulted in a substantial reduction in the final stiffness estimation error. This powerfully demonstrates that suppressing minor errors in the SI stage is critical for achieving reliable model updating results.

Table II: Comparison of Updating results

Mode	Proposed method Error [%]	Conventional method Error [%]
Mean	2.25	5.96

4. Conclusions

This study investigated the critical sensitivity of the model updating process to the quality of experimentally identified modal data. We demonstrated that minor errors in mode shape estimates, often introduced during the assembly of multi-setup test data, can be significantly amplified, leading to large biases in the updated stiffness parameters.

To address this error amplification, we proposed a multi-response integration framework that systematically enhances the accuracy of global modal data. By coupling a state-space SI method with convergence-based filtering, DBSCAN clustering, and weighted least-squares assembly, the framework effectively suppresses noise and inconsistencies inherent in multi-setup measurements.

The numerical validation powerfully illustrated the core finding of this research: a marginal improvement in the accuracy of the estimated mode shapes (an average MAC value increase from 0.9940 to 0.9994) led to a substantial improvement in the final updated model, reducing the mean stiffness error by more than half (from 5.96% to 2.25%). This result empirically confirms that the reliability of model updating is critically dependent on the precision of the initial SI stage. The proposed framework provides a robust method for achieving this necessary precision, highlighting its practical value for high-stakes applications like the structural health monitoring and performance assessment of nuclear power plants.

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List and number all bibliographical references in 9-point Times, single-spaced, at the end of your paper. When referenced in the text, enclose the citation number in square brackets, for example [1]. It is recommended that the number of references does not exceed five.

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