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Correlation Analysis between CuMUF Accumulation and Stratum-Level Throughput in Sequential MBPs: A Case Study of a Hypothetical Bulk Handling Facility

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1. Introduction

In a bulk handling facility, Material Unaccounted For (MUF) is a key metric in nuclear material accountancy, defined as the difference between the book inventory and the physical inventory [1, 2]. It is calculated for a single Material Balance Period (MBP) using Equation (1):

(1) $MUF = PB + X - Y - PE$

Where PB is the inventory of Physical Beginning, X represents total receipts, Y represents total shipments, and PE is the inventory of Physical Ending. The term (PB + X - Y) constitutes the book inventory at the end of an MBP, making MUF the discrepancy between what the records claim should be present and what physically exists.

While a single MUF value is a critical indicator, safeguards analysis often extends across multiple, sequential MBPs. The Cumulative MUF (CuMUF), which is the algebraic sum of MUFs over these consecutive periods, is an important diagnostic tool. A persistent, unidirectional trend in the CuMUF can indicate underlying systematic biases in measurement procedures, even if each individual MUF remains within its statistical uncertainty.

In facilities with long-term, consistent operations, the ending inventory of one period (PE) becomes the beginning inventory (PB) of the next. This leads to a long-term cancellation effect of the inventory components on the CuMUF trend. As a result, the relative contributions of receipts (X) and shipments (Y) become the dominant factors driving any persistent trend. It is therefore a rational approach to investigate the root cause of a CuMUF trend by analyzing the correlation between the throughput of specific receipt and shipment strata and the period-wise MUF values [3]. Such a diagnostic approach is essential for enhancing the efficiency of verification activities for regulatory bodies, and it is becoming increasingly important for states to establish independent analysis capabilities to strengthen their own safeguards technical systems [4].

Therefore, this paper focuses on applying correlation analysis as a tool to identify the root cause of persistent CuMUF trends in a hypothetical facility.

2. Method

2.1. Hypothetical Facility (HBF-01) and Data

A 5-MBP dataset was constructed for HBF-01 (Hypothetical Bulk-handling Facility-01), a hypothetical facility designed by generalizing the characteristics of a real facility (see Table 1). The dataset was engineered so that MUF_i exhibits a strong positive correlation with UO_2 receipts and foreign shipments, and a negative correlation with UF_6 receipts. The uncertainty for each MBP, $\sigma(MUF_i)$, was set to a level where individual MUFs are not statistically significant, allowing for an investigation of trends that are not yet statistical alarms. For measurement uncertainty parameters where historical data is insufficient, international target values (ITV) can be utilized as a reference [5].

Table I: Hypothetical isotope MUF, its standard deviation, and cumulative MUF (in kg ^{235}U) for the five-MBP evaluation period at HBF-01.

Year	MBP	MUF_i	$\sigma(MUF_i)$	$CuMUF_i$
2020	1	21.5	14.8	21.5
2021	2	41.2	23.1	62.7
2022	3	18.1	12.5	80.8
2023	4	20.8	15.2	101.6
2024	5	33.7	21.9	135.3

2.2. Material Balance Components

Unless otherwise specified, MUF in this paper refers to isotope MUF (^{235}U), which is the primary focus for safeguards concerning fissile material [1]. The analysis of element MUF is considered out of scope, although parallel analysis is recommended for actual facility applications.

For a detailed analysis, the total receipts (X) and shipments (Y) are disaggregated into their primary strata. The total receipt (X_t) is the sum of receipts from UF_6 cylinders (X_{UF6}) and UO_2 powder (X_{UO2}), as shown in Equation (2).

(2) $X_t = X_{UF6} + X_{UO2}$

The total shipment (Y_t) consists of fuel assemblies shipped to foreign (Y_{SF}) and domestic (Y_{SD}) clients, and measured discards to retained waste (Y_{TW}), as shown in Equation (3).

$$(3) \quad Y_t = Y_{SF} + Y_{SD} + Y_{TW}$$

The hypothetical throughput data for these strata are presented in Table 2. Under the assumption of PB-PE cancellation in long-term accumulation, the CuMUF trend is closely linked to systematic biases within these specific strata.

Table II: Hypothetical throughput data (in kg U) for the major receipt and shipment strata at HBF-01 over the five-MBP evaluation period.

MBP	X_{UF_6}	X_{UO_2}	Y_{SF}	Y_{SD}	Y_{TW}
1	3,005	203	1,367	1,228	2
2	1,920	354	2,350	2,030	2
3	2,786	182	1,710	1,433	1
4	2,686	182	1,953	1,428	1
5	1,789	299	2,142	1,774	3

2.3. Correlation Analysis Design

To investigate the relationship between material flow and MUF, the Pearson correlation coefficient (r) can be calculated. This coefficient quantifies the linear relationship between two variables—in this case, a stratum's throughput (T) and the corresponding MUF (M) over n MBPs. The calculation is performed using Equation (4).

$$(4) \quad r_{T,M} = \frac{\sum_{i=1}^{n}(T_i-\bar{T})(M_i-\bar{M})}{\sqrt{\sum_{i=1}^{n}(T_i-\bar{T})^2}\sqrt{\sum_{i=1}^{n}(M_i-\bar{M})^2}}$$

Here, T_i and M_i are the individual throughput and MUF values for the i-th MBP, and $\bar{T}$ and $\bar{M}$ are their respective mean values. A threshold of $|r| \geq 0.65$ is used as a criterion for identifying candidate strata whose measurement procedures may warrant a more detailed review. This approach is a practical application of error propagation principles to diagnose a complex system [6].

3. Results

3.1. MBP-wise MUF and CuMUF Trend

Figure 1 illustrates the trend of MUF_i and $CuMUF_i$ for the 5-MBP period. The $CuMUF_i$ gradually increases, and while a clear positive trend is visible, each individual MUF_i remains within its $\pm 3\sigma$ uncertainty band and is not statistically significant on its own. This scenario was designed to represent a possible challenge in safeguards: investigating the cause of a growing anomaly even in the absence of a statistical alarm, as each MUF_i value remains within its uncertainty band.

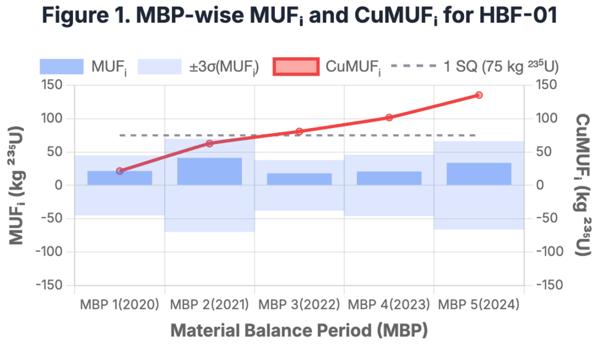

Fig. 1. Isotope MUF (MUF_i) with its $\pm 3\sigma$ uncertainty and the cumulative MUF ($CuMUF_i$) for five sequential material balance periods at the hypothetical facility HBF-01.

3.2. Throughput–MUF Correlation

The correlation analysis, summarized in Figure 2 and Table 3, reveals strong relationships between MUF and specific strata. The throughput of UO_2 receipts (X_{UO_2}) shows the highest positive correlation ($r = 0.974$), followed by foreign shipments (Y_{SF}) with $r = 0.861$. In contrast, UF_6 receipts (X_{UF_6}) show a strong negative correlation ($r = -0.762$). These results suggest that the measurement processes associated with these three strata are the most likely contributors to the observed MUF variability and its cumulative trend. The direct linear relationships are visualized in the scatter plots in Figure 3.

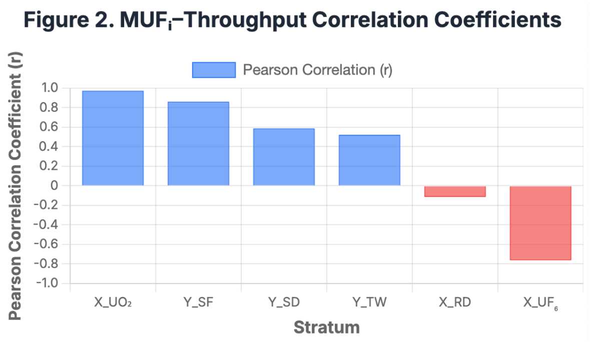

Fig. 2. Pearson correlation coefficients (r) between isotope MUF and the throughputs of major material balance strata over the five-MBP evaluation period.

Table III: Calculated Pearson correlation coefficients (r) between the isotope MUF and the throughput of each material stratum.

Stratum	Correlation(r)
X_{UF_6}	-0.762

X_{UO_2}	0.974
Y_{SF}	0.861
Y_{SD}	0.587
Y_{TW}	0.523

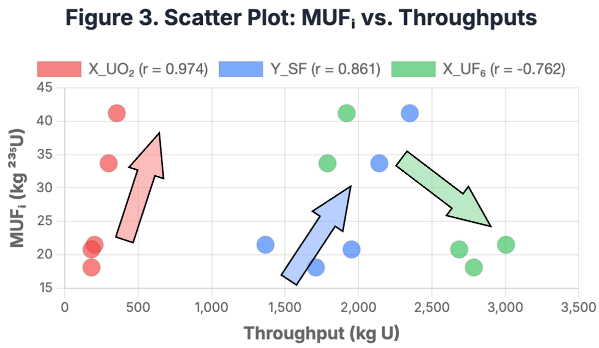

Fig. 3. Scatter plots illustrating the linear relationships between isotope MUF (MUF$_i$) and the throughputs of the three most correlated strata: X_{UO_2}, Y_{SF}, and X_{UF_6}.

3.3. Procedural Implications

The procedural implications of this analysis are significant. The strong positive correlation for the X_{UO_2} stratum suggests a potential positive bias in the measurement of weight, U-concentration, or isotope ratio, or could stem from the use of conservative nominal values. This finding calls for a thorough review of the sampling and analysis procedures and a reinforcement of QA/QC. Similarly, for the Y_{SF} stratum, a review is needed to check for any differences in the isotope ratio measurement procedures between assemblies for foreign shipment versus domestic use. Both of these implications underscore the validity of this analytical approach for pinpointing specific areas for procedural improvement.

For the key strata identified in this way, the inspection plan can be optimized by applying more precise verification methods or by increasing the number of verification samples. For example, by using software that can evaluate detection probabilities for different diversion scenarios [4], it is possible to establish an efficient sampling plan that reduces the overall inspection burden while maintaining the verification goals for the strata that showed high correlation.

4. Conclusion

This study shows that, in a facility with a persistent CuMUF trend, treating the cancellation of inventory effects (PB and PE) as a working hypothesis is a practical approach to focus an investigation on receipts (X) and shipments (Y) as the primary drivers of MUF variation. We illustrate that a targeted review of measurement procedures for strata identified via correlation analysis is a direct and efficient path to identifying the root cause of the issue. The framework of this study serves to generalize this diagnostic procedure, contributing to the enhancement of national safeguards inspection capabilities [7]. For future work, a comparative study of σ(MUF) frameworks can be proposed, along with research into how these diagnostic findings can be better integrated into optimized sampling strategies for on-site inspections.

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