Meta-learning Enhanced PINN for Steady-state Incompressible Laminar Flow Analysis

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1. Introduction

Accurate simulation of incompressible laminar flows is imperative in fluid dynamics modeling. However, traditional numerical methods frequently encounter challenges concerning computational intensity and geometric complexity [1-2]. Physics-Informed Neural Network (PINN) presents a compelling alternative by integrating the Navier-Stokes equations into neural network frameworks, effectively serving as PDE solvers. The efficacy of PINN is contingent on the initial parameter settings, which considerably influence convergence speed and solution accuracy [3-4].

This paper explores the application of meta-learning techniques to refine these initial parameters, with the aim of improving both the generalizability and efficiency of PINN. By learning meta-parameters that represent a global optimization solution, we extend the applicability of PINN across varied fluid dynamics scenarios. This study demonstrates how meta-learning can significantly broaden the solution space, offering an effective strategy for faster adaptation to the complexities of modeling incompressible laminar flows.

2. Methods and Results

2.1 PINN for Steady-state Incompressible Flow

In this study, the focus is on two-dimensional, steady-state, incompressible laminar flow, which can be described by the Navier–stokes equations. The conservation of mass is expressed by the following equation:

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

where $\mathbf{u} = (u_x, u_y)$ represents the velocity field. The momentum conservation equation, neglecting external body forces, is stated in Equation (2):

$$(\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \nu \nabla^2 \mathbf{u} \tag{2}$$

where p denotes the pressure field and ν is the kinematic viscosity. The central concept of this study is to embed this physical knowledge within the PINN. Specifically, the PINN consists of a multi-layer perceptron (MLP) that takes spatial coordinates $\mathbf{c} = [x, y]$ as inputs and outputs the velocity and pressure fields, i.e., $(u_x(\mathbf{c}), u_y(\mathbf{c}), p(\mathbf{c}))$. The training of the PINN is guided by a total loss function, L_{total}, which enforces both the governing equations and the boundary conditions. This total loss is defined by Equation (3):

$$L_{total} = L_{res} + 2L_{bc} \tag{3}$$

where L_{res} captures the residual errors for Equations (1) and (2), and L_{bc} penalizes deviations from the required Dirichlet or Neumann conditions at the domain boundaries. The factor of 2 is used here to weight the boundary condition loss appropriately.

To sample the physical domain more thoroughly, Latin Hypercube Sampling (LHS) is employed, which disperses collocation points (i.e., the points at which Eq. (1) and (2) are enforced) throughout the fluid region in a statistically uniform manner, thereby enhancing the generalization capability of the network.

In summary, the PINN for incompressible laminar flow is designed to satisfy Eq. (1) and (2) by minimizing the total loss function in Eq. (3). This incorporation of core physical constraints continuity, momentum conservation, and boundary conditions directly within the training process.

2.2 Meta-learning Strategy

To enhance the adaptability of the PINN across varying obstacle geometries, we employ a meta-learning approach that aims to find a set of robust initial parameters, denoted by θ_{meta}. In the meta-training phase, we only use a single circular obstacle while systematically altering its size or position in the domain across multiple training instances. By consecutively training on these different circle-based configurations, the network learns fundamental flow features and converges to θ_{meta} , which encodes generalized knowledge of incompressible laminar flow around circular shapes.

Once θ_{meta} is established, it serves as the initial parameter set for fine-tuning the PINN on more complex domains. Even when faced with entirely different shapes such as triangles or rectangles the initialization derived from circles allows the model to converge more rapidly and achieve higher accuracy than a randomly initialized network. In principle, this approach capitalizes on shared flow characteristics learned from circular obstacles and extends them to other geometrical contexts with minimal additional training.

2.3 Experimental Details

A series of experiments was designed to illustrate the benefits of the proposed meta-learning strategy. The computational domain, measuring 1.1 m in length and 0.41 m in height, is employed to fine-tune the PINN, as illustrated in Fig. 1. The fluid has a dynamic viscosity of 2×10^{-2} kg/(m·s) and a density of 1 kg/m^3. The velocity profile is defined as $\boldsymbol{u}(0, y) = 4U_{max}(H - y)y/H^2$ with $U_{max} = 1.0$ m/s, ensuring the flow remains laminar by maintaining a small Reynolds number.

During the meta-training phase, the same domain is utilized, but only a single circular obstacle, whose position or size is varied, is used to generate multiple training configurations. The meta-training phase involves a total of 5,000 cycles, with five circle configurations selected at random from a pre-generated set in each cycle. The PINN is trained sequentially for two iterations per configuration.

The network architecture consists of the MLP with one input layer, eight hidden layers of 40 neurons each, and one output layer producing $u_x(\boldsymbol{c})$, $u_y(\boldsymbol{c})$, and $p(\boldsymbol{c})$. The training process utilizes the Adam optimizer with a learning rate of 5×10^{-4} and approximately 20,000 collocation points, which are uniformly distributed via LHS.

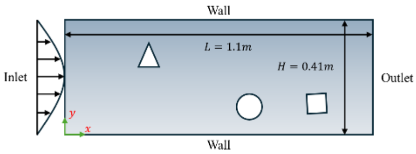

Fig. 1. Schematic of the computation model.

2.4 Experimental Results

In this study, a comparison was made between two primary models: one model is a PINN that was trained from the beginning, while the other is a PINN that was initialized with meta-learned parameters. The training loss trends reveal that the meta PINN converges at a faster rate and achieves a lower overall loss, as illustrated in Fig. 2. Of particular note is the purple dotted line at approximately 50,000 iterations, which marks the point at which the meta PINN achieves a loss value lower than the minimum value recorded by the scratch PINN, and the green dotted line at around 100,000 iterations, which indicates the lowest loss attained by the scratch PINN.

It is noteworthy that, despite initializing the PINN with meta parameters derived from a domain containing solely a circular obstacle, the model, as illustrated in Fig. 3 and Table 1, attains a similar solution with fewer iterations. By leveraging the flow characteristics acquired from the circular domain, the PINN adapts more efficiently to complex geometries, achieving rapid convergence and a lower final loss. This meta-parameter approach facilitates the incorporation of essential flow characteristics and boundary features, thereby promoting a more robust and accurate solution for complex geometries.

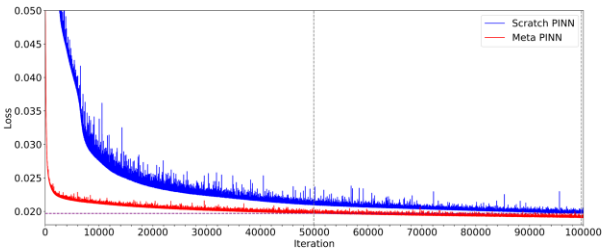

Fig. 2. Comparison of training loss trends for the scratch PINN and the meta PINN.

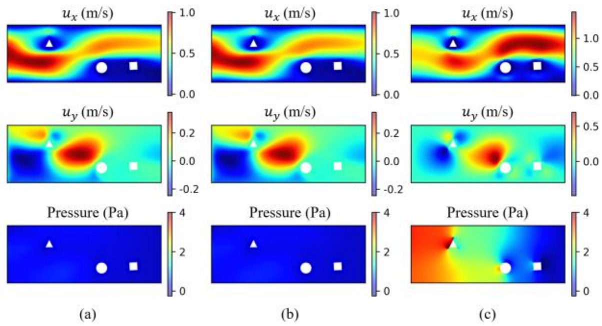

Fig. 3. Analysis of velocity and pressure fields: (a) scratch PINN after 100,000 iterations, (b) meta PINN after 50,000 iterations initialized from meta parameters, and (c) reference solution from OpenFOAM.

Table I: Evaluation of mean absolute error in velocity and pressure fields across different models.

Model	u_x (m/s)	u_y (m/s)	Pressure (Pa)
Scratch PINN	0.2211	0.0741	1.4979
Meta PINN	0.2214	0.0738	1.5005

3. Conclusions

This study proposes a meta-learning-enhanced PINN framework, designed to enhance convergence speed while maintaining accuracy in solving steady-state incompressible laminar flow problems. The framework is trained initially on simpler circular domains, demonstrating effective knowledge transfer and faster adaptation to a more complex geometry incorporating features such as triangles and rectangles. This underscores the efficacy of employing a robust initialization strategy in PDE solvers based on PINN, mitigating the likelihood of converging to local minima and enhancing solution accuracy.

The optimized meta-parameters provide deeper insights into flow phenomena, such as boundary-layer formation and wake behavior, by preserving critical domain-specific features. This approach enhances PINN stability and computational efficiency, showing promise for diverse fluid-flow scenarios. Leveraging this adaptability, subsequent studies will concentrate on systematically testing its performance across a broad spectrum of geometries to rigorously validate its robustness and consistently high average performance, thereby confirming its generalizability. Furthermore, we will focus on the implementation of this framework in transient flows and even more intricate domains, with a focus on parameter optimization and domain decomposition methods. This will serve to broaden the capabilities of PINN in the realm of computational fluid dynamics.

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