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Application of the History-Based Batch Method to the iDTMC Method for Reliable Real Variance Estimation

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1. Introduction

The Monte Carlo (MC) method is highly accurate for reactor analysis but requires high computational costs. To address this, various acceleration techniques have been developed to improve efficiency while maintaining accuracy. The Improved Deterministic Truncation of Monte Carlo (iDTMC) method integrates stochastic and deterministic techniques to accelerate the convergence of the fission source distribution (FSD) and truncates the solution. These techniques allow accurate subspace solutions with very few active cycles, significantly improving efficiency and reducing the computational cost. However, deterministic acceleration strengthens cycle-wise correlations, causing conventional variance estimators to underestimate uncertainty. To resolve this issue, this study applies the History-Based Batch method into the iDTMC framework.

2. Methods and Results

2.1 The iDTMC Method

The iDTMC method enhances the efficiency of MC simulations by integrating deterministic acceleration and truncation techniques [1]. It employs the p-CMFD method to accelerate the convergence of the FSD during inactive cycles and the p-FMFD method to generate pin-level reactor solutions during active cycles. The key distinction between p-CMFD and p-FMFD is that p-CMFD treats subassemblies as nodes, while p-FMFD treats individual pins within the subassemblies as nodes. The p-CMFD method solves the one-group neutron balance equation, Equation (1).

$$\sum_s \frac{A_s}{V_i}(J_{s1} - J_{s0}) + \Sigma_a^i \varphi_i = \frac{1}{k_{eff}} \nu \Sigma_f^i \varphi_i \qquad (1)$$

Here, A_s is the surface area, V_i is the volume of node i, s is the surface index, ϕ and J are the flux and current, respectively, Σ is the cross-section, and k_{eff} is the effective multiplication factor. The methodology consists of two stages. First, p-CMFD is applied during inactive cycles to accelerate FSD convergence. Once convergence is achieved, p-FMFD truncates the MC solution with accumulated parameters after skip cycles, operating independently to ensure numerical stability.

While iDTMC significantly improves computational efficiency, it also introduces stronger cycle-wise correlations due to the acceleration. As a result, variance estimation methods, which assume cycle-wise independence, tend to underestimate the real variance.

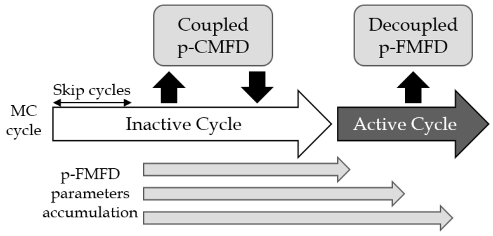

Figure 1. Schematic of the iDTMC method

2.2 History-Based Batch Method

Cycle-wise correlation in MC simulations arises from two primary sources: genealogical dependency and the normalization scheme for fission source neutron weights. To obtain independent estimates within a single MC run, Shim, Choi, and Kim developed the History-Based Batch Method [2]. This method partitions an MC simulation with N active cycles and M histories per cycle into N_B independent MC simulations, each with N active cycles and M/N_B histories per cycle. It achieves this by grouping M histories into N_B ancestor fission source neutrons groups at the start of the first active cycle and tracking each history through all N cycles. Here, N_B represents the number of History-Based Batches.

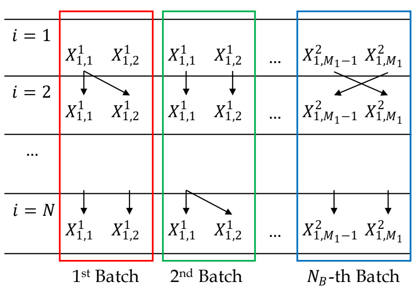

Figure 2. Schematic of the History-Based Batches

In standard MC simulations, the fission source weight is normalized at the ith cycle using Equation (2).

$$w_i = \frac{M}{M_i} \tag{2}$$

Here, M_i denotes the number of fission source neutrons generated in the $i-1$'th cycle. However, in the History-Based Batch Method, each batch is normalized separately to mitigate cycle-wise correlation caused by the normalization scheme. The weight normalization for the kth History-Based Batch is given by Equation (3).

$$w_i^k = \frac{M/N_B}{M_i^k} \tag{3}$$

Here, M_i^k represents the number of fission source neutrons in the$k$th History-Based Batch at ith cycle.

Using the History-Based Batch Method, the batch-average MC tally, Q^k, is computed as Equation (4).

$$Q^k = \frac{1}{N(M/N_B)} \sum_{i=1}^{N} \sum_{j \in k} f_i^k Q_{ij} \tag{4}$$

Here, Q_{ij} is the MC tally estimate of Q for jth history in ith cycle, and f_i^k is the weight correction for the kth History-Based Batch at ith cycle, defined as Equation (5).

$$f_i^k = \frac{w_i^k}{w_i} \tag{5}$$

Since the normalization process is applied separately to each History-Based Batch, the weights of different batches may vary—some becoming smaller and others larger. To compensate for this effect when tallying, we introduce the weight correction factor. From Q^k, the sample mean $\bar{Q}_{HB}$ and its variance $\sigma^2[\bar{Q}_{HB}]$ are estimated as Equation (6) and (7).

$$\bar{Q}_{HB} = \frac{1}{N_B} \sum_{k=1}^{N_B} Q^k \tag{6}$$

$$\sigma^2[\bar{Q}_{HB}] = \frac{1}{N_B(N_B-1)} \sum_{k=1}^{N_B} (Q^k - \bar{Q}_{HB})^2 \tag{7}$$

This batch-based grouping and normalization process effectively reduces cycle-wise correlation in MC tally calculations.

2.3 iDTMC with History-base Batch Sampling

The iDTMC method exhibits strong cycle-wise correlations, making accurate variance estimation challenging. To address this issue, various techniques, such as the Correlated Sampling Method [3] and the Autoregressive (1) Model [4], have been employed. These methods generate p-FMFD parameters from a probability density function obtained through correlated sampling or the autoregressive model. The sampled parameters include one-group total, absorption, and nu-fission cross-sections, as well as flux and interface current. Using these sampled parameters, multiple p-FMFD problems are formulated. Each perturbed p-FMFD problem is then solved either directly or using first-order perturbation theory, and the resulting standard deviation is used to estimate the real variance.

In this study, we adopt the History-Based Batch method for sampling p-FMFD parameters. Implementing this method in iDTMC involves some modifications. To maintain consistency with conventional MC simulations, we preserve the standard weight normalization process. However, to ensure the conservation of histories within each History-Based Batch, a weight correction factor is applied when banking fission source neutrons, as described by Equation (8).

$$n = \left[w_i \left(\frac{\nu\sigma_f}{\sigma_t}\right) \left(\frac{1}{k_{eff}}\right) f_i^k \right] \tag{8}$$

Here, n represents the number of fission source neutrons.

For the iDTMC calculation itself, we use non-corrected tallied values. However, when accumulating p-FMFD parameters, we store both non-corrected and corrected values. During the sampling process, since all the considered parameters follow a normal distribution, the non-corrected accumulated parameters' mean is used as the sample mean, while the corrected parameters' variance is used as the sample variance.

Latin Hypercube Sampling (LHS) is employed to generate independent uniform random numbers (URNs), which are then transformed to follow a normal distribution. The generated values are adjusted based on the computed mean and variance, producing perturbed p-FMFD problems. These problems can be solved either directly or using first-order perturbation theory. The variance obtained from solving these perturbed problems is then used to estimate the real variance.

A schematic representation of the iDTMC method with History-Based Batch sampling is provided in Figure 2. This new sampling method enables a more accurate estimation of the real variance by explicitly accounting for the variance of the sampled parameters.

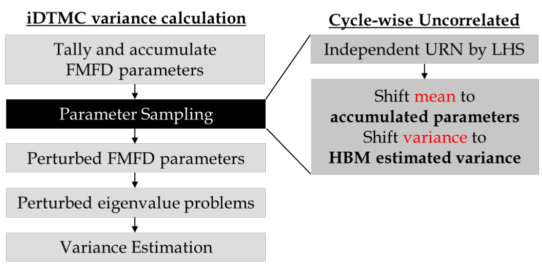

Figure 3. Schematic of the iDTMC method with History-Based Batch sampling

3. Numerical Results

To estimate the real variance, we solve the small modular reactor (SMR) problem previously studied in [4]. The geometry of the problem is shown in Figure 4. A total of 1.5 × 10^6 histories were simulated using 30 inactive cycles and 10 active cycles. Additionally, 15 cycles were skipped before accumulating p-FMFD parameters, and p-CMFD acceleration was applied in the inactive cycles after skipping the first cycle. All calculations were performed in parallel using 200 Intel® Xeon® Gold 6148 cores.

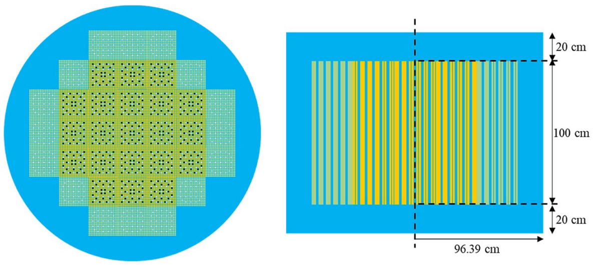

Figure 4. Cross-sectional (left) and side (right) view of the SMR model

To investigate the effect of different history-based batch sizes, we set the number of history-based batches to 30, 40, 50, and 60. The minimum batch size of 30 was chosen to ensure that the number of samples was sufficient for a normal distribution approximation. The maximum batch size of 60 was selected based on the mesh resolution of the p-FMFD problems. Since pins are used as fine meshes, the number of histories per batch is given by M/N_B. If too many batches are used, the number of histories per batch becomes too low, leading to reduced or even zero tallied neutron counts for each parameter in the fine meshes.

The difference between the averaged eigenvalues from iDTMC with History-Based Batch sampling with 30 independent batch calculations and the reference eigenvalue obtained from 45 independent batch calculations are summarized in Table 1. The reference eigenvalue is 1.1258 ± 15.29 pcm.

Number of history-based batches	Difference [pcm]
30	7.44
40	10.8
50	10.3
60	12.1

Table 1. Calculated eigenvalue from iDTMC with History-Based Batch sampling and its difference

The calculated eigenvalues remain within 1σ of the real variance, indicating that the method produces reasonable results. This confirms that iDTMC with History-Based Batch sampling does not introduce significant difference in the eigenvalue calculation.

The averaged estimated variance of eigenvalue at 10th active cycle obtained using History-Based Batch sampling with 30 independent batch calculations is presented in Table 2 and Figure 5. The variance was estimated using first-order perturbation theory. HB at Figure 5 represent the number of history-based batches

Number of history-based batches	Estimated k_{eff} standard deviation [pcm]	Relative error [%]
30	12.62±0.62	17.5
40	12.57±0.78	17.8
50	12.63±1.1	17.4
60	12.57±0.97	17.8

Table 2. Estimated $\boldsymbol{k_{eff}}$ variance and its relative error from iDTMC with History-Based Batch sampling

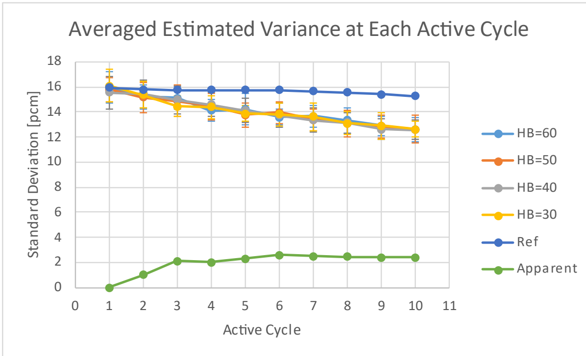

Figure 5. Estimated $\boldsymbol{k_{eff}}$ variance at each active cycle using History-Based Batch sampling

Table 2 reports the variance estimated at the 10th active cycle, while Figure 5 illustrates the variance estimation throughout all active cycles. The variance estimation at the 10th active cycle shows a 3 pcm difference regardless of batch sizes. Given that the apparent variance computed from a single batch exhibits a 2 pcm, applying History-Based Batch sampling significantly improves variance estimation accuracy. A similar trend is observed when solving all the perturbed p-FMFD problems directly.

By solving all the perturbed p-FMFD problems, we can also estimate the variance of the power distribution. The overall standard deviation of the power distribution is evaluated using the average standard deviation, as defined in Equation (9).

$$\bar{\sigma}_p = \frac{1}{N_{pins}} \sum_{i=1}^{N_{pins}} \sigma_i \tag{9}$$

Here, N_{pins} represents the number of fuel pin, i is the fuel pin index, and σ_i is the relative standard deviation of the i th fuel pin. The real variance of the power distribution, obtained from 45 independent batch calculations, is 3.913 pcm. Table 3 presents the estimated variance of the power distribution and its relative error, while Figure 6 illustrates the estimated and real variance of the power distribution at the midplane.

Number of history-based batches	Estimated $\bar{\sigma}_p$ [pcm]	Relative error [%]
30	4.083	4.4
40	4.087	4.5
50	4.087	4.4
60	4.098	4.7

Table 3. Estimated $\bar{\boldsymbol{\sigma}}_{\boldsymbol{p}}$ and its relative error from iDTMC with History-Based Batch sampling

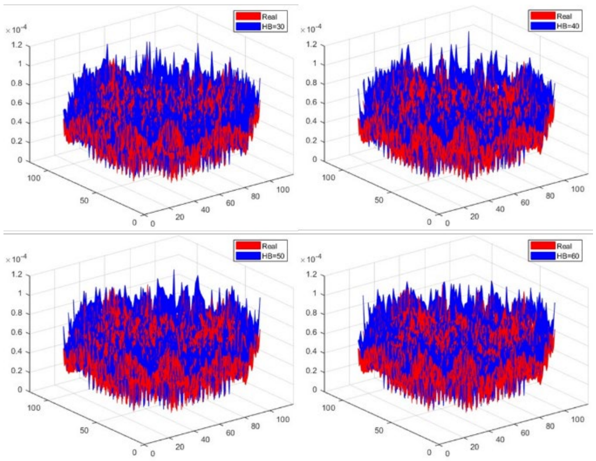

Figure 6. Estimated $\bar{\boldsymbol{\sigma}}_{\boldsymbol{p}}$ and the real variance at the midplane

As shown in Table 3, the estimated power distribution variance remains within 5% of the real variance, regardless of the number of history-based batches. Additionally, Figure 6 confirms that the estimated and real variance closely overlap, demonstrating the accuracy of the variance estimation method. When averaging the relative error of variance for each pin, the average error remained within 10% across all batch sizes, further validating the effectiveness of the History-Based Batch sampling approach in power variance estimation.

3. Conclusions

In this study, we applied the History-Based Batch Method to the iDTMC framework to improve variance estimation. The iDTMC method, while enhancing computational efficiency through deterministic acceleration and truncation, introduces strong cycle-wise correlations, leading to underestimation of the real variance when using conventional variance estimators. By incorporating History-Based Batch sampling, we mitigated these correlations and enabled more accurate variance estimation.

The numerical results obtained from the SMR problem demonstrate that the eigenvalues calculated using iDTMC with History-Based Batch sampling remain within 2σ of the real variance, confirming the reliability of the proposed approach. Moreover, variance estimation using first-order perturbation theory exhibited significant improvements over conventional methods, with only a slight underestimation observed across active cycles. For the power distribution variance, regardless of the number of history-based batches used, the estimated values were within a 5% error compared to the real variance.

However, the proposed method also has some limitations. First, since iDTMC operates on a fine-mesh scale, a massive number of histories is required to tally p-FMFD parameters accurately. The use of History-Based Batch sampling further reduces the number of histories per batch, necessitating an even greater total number of histories for precise variance estimation. Second, because both weight-corrected and uncorrected parameters must be tallied, the method demands significantly more memory compared to conventional approaches. These factors increase computational and memory requirements, potentially limiting the feasibility of the method for large-scale simulations.

Despite these challenges, the History-Based Batch method proves to be particularly effective in highly cycle-wise correlated problems such as iDTMC, where conventional variance estimators fail to provide reliable uncertainty quantification. These findings demonstrate that History-Based Batch sampling enables improved and reliable variance estimation within the iDTMC framework while maintaining consistency in eigenvalue calculations.

Future work will focus on analyzing the correlation between p-FMFD parameters, also at the node-wise level, to verify the independence assumption used in this study. This analysis will further justify the reliability of the proposed variance estimation method and may lead to improved modeling of parameter dependencies.

ACKNOWLEDGEMENT

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