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A Study on the Bayesian Approach to the Conditional Probability of Oil Fire Severities in Fire Probabilistic Safety Assessment

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1. Introduction

Fire severity factors, which are conditional probabilities to quantitatively measure how severely the fire will impact on the target, play an important role in evaluating the fire-induced core damage frequency (CDF) in fire probabilistic safety assessments (PSAs). NUREG/CR-6850 [1], which proposes an overall methodology for fire PSAs, briefly describes how to assign severity factors to various ignition sources such as fixed, transients, and pump oil spill fires in nuclear power plants (NPPs).

According to [1, 2, 3], the severity factors of oil fires are proposed to be assigned based on the fire severity levels (e.g., Small, Large, and Very large) using conditional probabilities calculated through the classical inference method such as the maximum likelihood estimation (MLE), which is known as a *frequentist* approach. However, the *Bayesian* approach has generally been shown to be a more reliable than the classical inference in avoiding unstable estimation results when the observations are rare. In practice, the Bayesian approach is widely used in PSA for calculating initiating event frequencies and component reliability data [4].

Therefore, this paper attempts to employ the Bayesian statistical inference to calculate the conditional probabilities for oil fire severities and compare them with the results obtained through the MLE method. In addition, one of the calculated probabilities is applied to a simplified fire PSA model to figure out the differences in CDF results.

2. Severity Factor for Pump Oil Fires

2.1 Example of oil fires from general pumps and severity factors based on the MLE method

In the case of oil fires from general pumps, the severity levels are divided into *Small*, *Large*, and *Very Large*. The conditional probability for each severity level directly represents the severity factor.

According to the analysis of oil fires involving general pumps from the EPRI fire events database (FEDB), a total of 21 oil fire events occurred, of which three were classified as severe events, categorized as either *Large* or *Very Large*.

Therefore, based on the classical inference, the conditional probabilities of each severity level can be simply calculated as shown in Table I.

Table I. Severity factors for oil fires from general pumps [3]

Severity level	No. events	Severity factor**
Small	18.5*	18.5/21
Large	1.5*	1.5/21
Very Large	1	1/21

* Among the two cases which were classified as *Large*, one case was counted as 0.5 due to uncertainty, and the remaining 0.5 was assigned to *Small* category.

** MLE: $\theta_i = \frac{x_i}{n}$, where n is the total number of events, x_i is the number of i-th severity level events, and θ_i is the probability of i-th severity level

2.2 Bayesian approach to the severity factor for oil fires

In this paper, the Bayesian inference was performed on the conditional probability of various oil spill fires as well as general pump oil fires described in Sec. 2.1.

2.2.1 Likelihood function

In general, the severity levels of oil fires are classified into two or three categories [1, 2, 3]. Therefore, to describe the probability of the number of occurrences in each category out of n trials, a binomial or multinomial distribution can typically be used as the probability distribution. Since the multinomial distribution is a generalization of the binomial distribution, the type of the likelihood function can be simply determined based on the number of the severity levels, denoted as k in this paper. For example, when the number of severity level is 2, a binomial distribution can be nominated as a likelihood function. Otherwise, a multinomial distribution will be used.

2.2.2 Prior distribution

Prior distributions should also be determined to evaluate the conditional probability for fire severity using the Bayesian inference. In this study, the Dirichlet distribution was used as a prior distribution as shown in Eq. (1) since it is conjugate to the multinomial distribution [5].

$$p(\theta_1, \theta_2 \dots, \theta_k) = Dirichlet(\alpha_1, \alpha_2, \dots, \alpha_k) \quad \text{Eq. (1)}$$

Where θ_i is the probability of i-th category in the multinomial distribution, $\alpha_1, \alpha_2, \dots, \alpha_k$ are the parameters of the Dirichlet distribution.

Consequently, the parameters of the Dirichlet distribution were assumed as shown in Table II to represent two priors: Jeffreys' prior and a uniform prior distribution, respectively.

2.2.3 Posterior distribution

When the likelihood function and prior distribution are given as in 2.2.1 and 2.2.2, the posterior distribution is as follows:

$$p[\boldsymbol{\theta}|x] = Dirichlet(\alpha_1 + x_1, \dots, \alpha_k + x_k) \quad \text{Eq. (2)}$$

Where $p[\boldsymbol{\theta}|x]$ is the posterior distribution of $\boldsymbol{\theta}$ when the observations x is given and $\boldsymbol{\theta}$ is $\theta_1, \theta_2 \dots, \theta_k$. Therefore, the probability of i-th severity level is as follows:

$$E[\theta_i|x] = \frac{x_i + \alpha_i}{n + \alpha_0} \quad \text{Eq. (3)}$$

Where $\alpha_0 = \sum_{i=1}^{k} \alpha_k$. The descriptions of the Bayesian inference when the Jeffreys and uniform prior distribution are assumed are summarized in Table II.

Table II. Descriptions of the Bayesian inference to assign the severity factors for various oil fires when $k = 3$

	(1)*	**(2)***
Likelihood function	Multinomial distribution	
Prior dist.	Dirichlet$(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$	Dirichlet$(1, 1, 1)$
$\boldsymbol{E[\theta_i\|x]}$**	$\frac{x_i + 0.5}{n + 1.5}$	$\frac{x_i + 1}{n + 3}$

* (1): Jeffreys noninformative prior, (2): uniform prior
** Where, k is the number of severity levels, n is the total number of events, and x_i is the number of i-th severity level events.

As a result, the conditional probabilities for each ignition source through the Bayesian inference using Table II are presented in Table III. It should be noted that although not oil fires, additional cases (e.g., hydrogen fires from turbine generator) were also analyzed in this paper since they used similar method of assigning severity factor to oil fires [1].

Table III. Severity factors depending on the inference methods

Ignition source	**Severity level**	**No. events from FEDB**	**MLE**	**(1)**	**(2)**
General Pump (Oil fires)	Small	18.5	0.881	0.844	0.813
	Large	1.5	0.071	0.089	0.104
	Very large	1	0.048	0.067	0.083
	Total	21			
Main Feed Water Pump* (Oil fires)	Small	13.5	1.000	0.966	0.935
	Large	0	0.000	0.031	0.058
	Very large	0	0.000	0.003	0.006
	Total	13.5			
Turbine Generator (Oil fires)	Small	19	0.950	0.929	0.909
	Severe	1	0.050	0.071	0.091
	Total	20			
Turbine Generator** (Hydrogen fires)	Small	11	0.846	0.821	0.800
	Severe	2	0.154	0.179	0.200
	Total	13			
Turbine Generator** (All challenging fires)	Small	37	0.974	0.962	0.950
	Severe	1	0.026	0.038	0.050
	Total	38			

* In the case of MWFP [2], the Bayesian inference already has been employed to assign probabilities to severity levels that have not been observed such as *Large* or *Very large* since the MLE method fails to produce the probability for *Large* or *Very large* category of MWFP severity level.
** Although not oil fires, analysis were also conducted for ignition sources that have a similar method of assigning severity factors.

As shown in Table III, it can be observed that more conditional probabilities were assigned to the more severe categories such as *Large* or *Very large* when the probability was estimated through Bayesian inference. Therefore, it is necessary to figure out how these changes impact the fire-induced CDF quantification results.

3. Sensitivity Analysis: differences in CDFs

Using the results in Table III, a simple fire PSA model was employed to perform a sensitivity analysis on the impact of classical and Bayesian methods on the CDF. The fire PSA model used in this study is a simple model that only includes the essential service water (ESW) intake structure as a fire compartment, which generally contains a scenario for general pump oil spill fires (e.g., ESW pump). Table IV shows the sensitivity analysis results using the severity factors for general pump oil fires presented in Table III.

Table IV. Differences in CDFs using the severity factors for general pump oil fires

MLE	(1)	(2)
1 (base)	1.02	1.03

As a result, it was confirmed that incorporating the severity factors through Bayesian approach leads to a 2-3% increase in fire-induced CDF of ESW fire compartment compared to the classical method. However, it should be noted that only one oil fire scenario and a single fire compartment was considered in this sensitivity analysis. Therefore, it is expected that there will be a meaningful change in fire-induced CDF when all oil spill fire scenarios are considered using the severity factors calculated in Table III.

4. Conclusion

In this study, the Bayesian approach was employed to calculate the severity factors for oil fires and compared with the existing MLE estimates. As a result, it was observed that more probabilities tend to be assigned to the more severe categories such as *Very large* when the Bayesian inference was used. Using a simplified fire PSA model, this change led to an approximately 2% increase in the fire-induced CDF of a specific fire compartment. Therefore, in a fully developed fire PSA model, severity factors should be carefully selected to enhance the results of risk assessment due to fires.

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