Reconstructing Structural Material Profiles using Elastic Full-Waveform Inversion

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1. Introduction

Structural defects (e.g., corrosion, cavity, delamination, etc.) inside nuclear power plant containment buildings have been reported. Inspection results have brought much attention to the importance of developing sophisticated diagnostic methods for evaluating structural integrity. Additionally, identifying the unknown material properties of structures has become increasingly important for continuous maintenance and computational analysis.

This study presents an elastic full-waveform inversion (FWI) approach to reconstruct the material profile of a structure. Inversion has been widely applied and developed in various engineering field, such as non-destructive testing, physical exploration, ground survey, and seismic. By utilizing elastic FWI, cross-sectional image of material profile can be reconstructed, providing essential information for assessing structural integrity.

2. Elastic full-waveform inversion

Elastic FWI estimates the spatial distribution of unknown material properties by applying the absorbed, refracted, and reflected elastic responses into an optimization algorithm. This paper discusses the inverse problem of elastic wave propagation and presents the results of numerical simulations.

2.1 PDE constrained optimization

The inversion problem for reconstructing two Lamé parameters is formulated as a partial differential equation (PDE)-constrained optimization problem, which can be expressed as follows:

Minimize

(1) $$J := {}^{1}\!/_{2} \sum_{i=1}^{N_r} \int_0^T \int_{\Gamma_m} |\mathbf{u}-\mathbf{u}_\mathrm{m}|^2 \delta(\mathbf{x}-\mathbf{x}_i)\, d\Gamma_m\, dt + R(\lambda,\mu)$$

subject to

(2a) $$\boldsymbol{\nabla}\cdot[\mu\{\nabla\mathbf{u}+(\nabla\mathbf{u})^\mathrm{T}\}+\lambda(\nabla\mathbf{u})\mathbf{I}]+\mathbf{f}=\rho\ddot{\mathbf{u}},$$

(2b) $$\boldsymbol{\nabla}\cdot\left(\dot{\mathbf{S}}^\mathrm{T}\widetilde{\boldsymbol{\Lambda}}_\mathrm{e}+\mathbf{S}^\mathrm{T}\widetilde{\boldsymbol{\Lambda}}_\mathrm{p}\right)=\rho(a\ddot{\mathbf{u}}+b\dot{\mathbf{u}}+c\mathbf{u}),$$

(2c) $$\mathbf{D}\colon\left(a\ddot{\mathbf{S}}+b\dot{\mathbf{S}}+c\mathbf{S}\right) = {}^{1}\!/_{2}\left[(\nabla\dot{\mathbf{u}})\widetilde{\boldsymbol{\Lambda}}_\mathrm{e}+\widetilde{\boldsymbol{\Lambda}}_\mathrm{e}(\nabla\dot{\mathbf{u}})^T+(\nabla\mathbf{u})\widetilde{\boldsymbol{\Lambda}}_\mathrm{p}+\widetilde{\boldsymbol{\Lambda}}_\mathrm{p}(\nabla\mathbf{u})^T\right].$$

The optimization problem seeks to minimize the misfit between computed and measured displacement while satisfying the elastic wave equations as constraints. The misfit functional is defined as the L^2 norm of the differences between computed and measured responses. To alleviate the ill-posedness of the inverse problem, a regularization term is introduced, and the Tikhonov regularization scheme is adopted.

2.2 Inverse problem

By applying the Lagrangian multiplier to the constraints of the optimization problem, the objective functional is augmented by the weak-form imposition of the governing PDEs and boundary conditions. Enforcing the stationarity of the Lagrangian functional results in Karush-Kuhn-Tucker (KKT) optimality conditions, which consist of time-dependent state, adjoint, and time-invariant control problem. Material properties are iteratively updated while minimizing the Lagrangian functional using KKT conditions and a reduced-space method. Fig. (1) illustrates the inversion procedure.

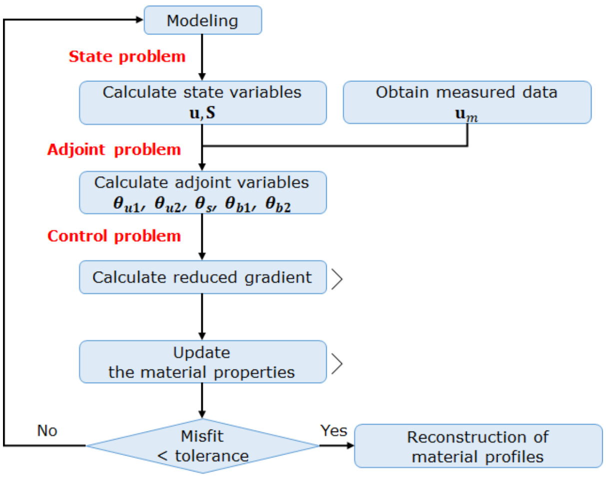


Fig. 1. Inversion procedure

2.3 Numerical simulation

Fig. 2 presents a numerical analysis example in a one-dimensional domain. The target domain consists of multiple layers with five different elastic moduli. The initial guess assumes a constant modulus along the depth.

The inversion result successfully reconstruct the target material profile, capturing sharp variations in material properties with high accuracy.

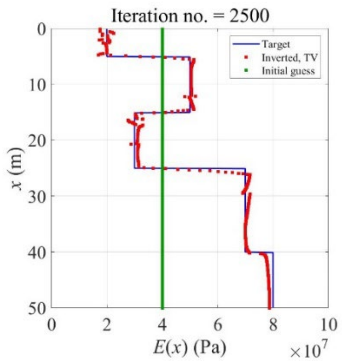


Fig. 2. Reconstructed elastic modulus of one-dimensional numerical analysis

Fig. 3 shows inversion result in a two-dimensional domain. A medium containing a cavity with a side length 0.6 m is considered for numerical analysis. The inversion begins with a homogeneous initial shear modulus estimate of 12 GPa. The inverted shear modulus values are decreased significantly near the cavity, indicating the presence of a cavity. Fig. 4 provides a cross-sectional image of the reconstructed shear modulus. In the cases of medium with a defect, the inversion results effectively delineates the defect's location, size, and shape.

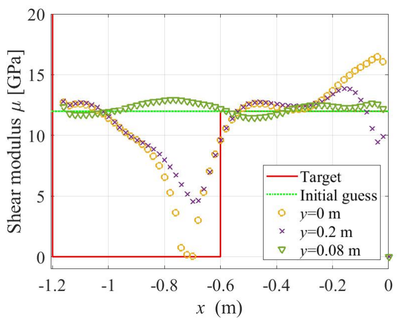
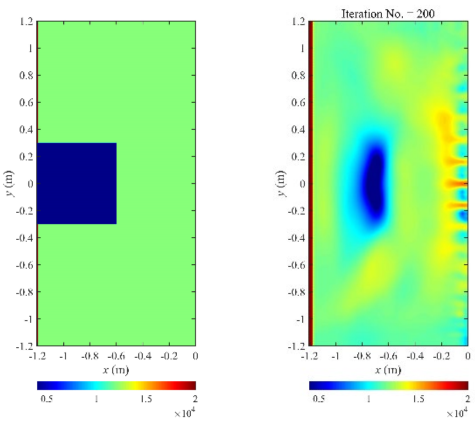


Fig. 3. Reconstructed shear modulus of two-dimensional numerical analysis



(a) Target profile (b) Inversion result

Fig. 4. Cross-sectional image of shear modulus

3. Conclusions

The proposed FWI method for characterizing material properties of structure has been developed and validated using synthetic data. The inversion successfully reconstructs material profiles in both one- and two-dimensional domains. This approach can be applied to structural integrity evaluation and system identification. Furthermore, even in cases where design data, qualification documents for a structure is unavailable, FWI can be utilized to determine its material properties and characteristics.

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