Sampling Based Method for Estimating Functional Failure Probability of Passive Safety Systems in Small Modular Reactors

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1. Introduction

Recently, the design and licensing of Small Modular Reactors (SMRs) have been actively pursued worldwide. In South Korea, the i-SMR is being developed by leveraging the advantages of Passive Safety Systems (PSSs), based on the design experience of SMART100. Meanwhile, concerns have been raised regarding the reliability of PSS when performing Probabilistic Safety Assessment (PSA) for SMR designs. Although PSS ensures safety by relying on natural forces and low - driving force, this characteristic makes it susceptible to not only mechanical or component failures but also functional failures. While mechanical failures can be analyzed using conventional assessment methods, functional failures require an analysis of initial conditions and the behavior of thermal-hydraulic variables, necessitating a quantitative evaluation.

To quantify the functional failure probability of PSS, previous studies have employed methods such as REPAS, RMPS, and APSRA. Notably, unlike REPAS and RMPS, the APSRA method employs various techniques and focuses on failure points to generate a failure surface, which is then used to quantify the functional failure probability of PSS [1]. However, since the functional failure probability of PSS is extremely low, the high computational cost of thermal-hydraulic analysis required to generate the failure surface remains a major challenge.

Moreover, once the failure surface is generated, Monte Carlo (MC) simulations can be a powerful tool for estimating the functional failure probability of PSSs. However, in practice, since the functional failure probability of PSSs is often very low, a large number of samples is required to ensure sufficient reliability [2].

This study aims to minimize computational costs while ensuring sufficient reliability in the quantitative assessment of functional failure probability. To achieve this, the 'principle of superposition' will be applied to perform only a small number of simulations based on initial variables, and a predictive technique will be used to estimate the plant's state to generate the failure surface. Additionally, an Importance Sampling-based MC method will be employed to achieve a more efficient failure probability assessment. The case study will focus on the Passive Auxiliary Feedwater System (PAFS) of a pressurized water reactor-type SMR.

2. Previous Study

In previous studies, the quantification of the functional failure probability of PSS aimed to address the issues of time and computational cost associated with predicting the state of a power plant under various initial conditions. To achieve this, it was assumed that the principle of superposition is valid, allowing for a more efficient analysis of system performance variations [3].

Based on this assumption, the power plant system is modeled, with thermal-hydraulic variables V_i influencing system performance $\mathrm{S_i}$ or $f(V_i)$. The overall system state $\mathrm{S_T}$ is defined as the sum of the nominal performance $\mathrm{S_{nominal}}$ and the variations $\Delta \mathrm{S_i}$ caused by individual variable changes. According to the superposition, the total performance variation $\mathrm{S_T}$ can be expressed as the sum of the individual variable variations, as shown in Equation (1).

$$\mathrm{S_T} = \mathrm{S_{nominal}} + \sum_{\mathrm{i}=1}^{\mathrm{n}} \Delta \mathrm{S_i} \qquad (1)$$

Since the uncertainties of key variables related to PSS tend to follow specific probability distributions, this study also establishes the probability distributions of each variable in a similar. Instead of directly computing the probability distribution of all variables, a sensitivity analysis is conducted using the standard deviation σ to quantitatively express the variability of each variable.

To quantify the impact of the standard deviation on system changes, a specific function Equation (2) is applied. This function allows for a systematic assessment of how variations in input parameters influence overall system performance, providing a more efficient approach to analyzing uncertainty in PSS functionality.

$$\Delta \boldsymbol{f}(\boldsymbol{V_i}) = \frac{\boldsymbol{f}(\boldsymbol{\mu}+\boldsymbol{n\sigma})-\boldsymbol{f}(\boldsymbol{\mu}-\boldsymbol{m\sigma})}{(\boldsymbol{n}-\boldsymbol{m})\boldsymbol{\sigma}} \qquad (2)$$

Assuming that power plant performance variations follow the superposition within a specific range, interpolation was utilized to predict performance changes based on existing data. As illustrated in Figure 1, this approach enables performance prediction using three initial variables: Emergency Cooling Tank (ECT) temperature, ECT water level, and PAFS injection pipe diameter. By leveraging interpolation, the study effectively reduces computational costs while maintaining accuracy in estimating system behavior under different initial conditions. In figure 2, the black line represents the performance variation due to a single variable, while the red line indicates the performance variation based on the superposition. By utilizing this approach, a deterministic analysis was conducted for only seven single-variable cases, allowing for the prediction of 729 different scenarios. This significantly reduces computational costs while maintaining accuracy in assessing system performance variations.

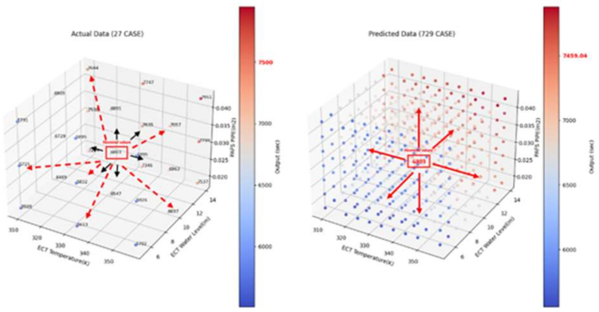

Figure. 1 Performance variation prediction with superposition

3. Methodology

3.1. Generate a failure surface

In the previous study, the principle of superposition and interpolation were utilized to predict the power plant state based on initial variables. Building upon this, the present study aims to generate a failure surface.

In the functional failure analysis of the PSS, the failure surface serves as a useful tool to enhance computational efficiency and determine success or failure. However, due to the nature of the PSS, directly applying an Event Tree (ET), which classifies states into a simple binary success/failure, is challenging. Therefore, applying traditional PSA methods requires establishing a reasonable margin. The way the failure surface is defined significantly influences uncertainty, which poses limitations in algorithmic representation and generalization. Thus, this study seeks to explain the failure surface generation process using an example with two initial variables and to derive a more reasonable failure surface by incorporating the validated principle of superposition as a margin.

The performance variation is predicted, and the failure surface is described using two variables, V_1 and V_2. If the principle of superposition is verified for the observed values of V_1 and V_2 and is determined to be within an acceptable error range, the predicted values for V_1 and V_2 are estimated using the principle of superposition.

Next, when generating predicted values through probabilistic analysis based on observed values from deterministic analysis, the sensitivity range corresponding to the standard deviation of the variable V_i is used, as shown in Figure 2. In the example involving two variables, the deterministic analysis sensitivity is evaluated using the nominal value and its variations, specifically at $\mu - m\sigma$, μ, $\mu + n\sigma$ which are represented by the black lines. Meanwhile, in the probabilistic analysis, if k represents the sensitivity interval, the interval is determined based on the analyst's judgment, using a range of $\pm k\sigma$, while also utilizing interpolation for estimation.

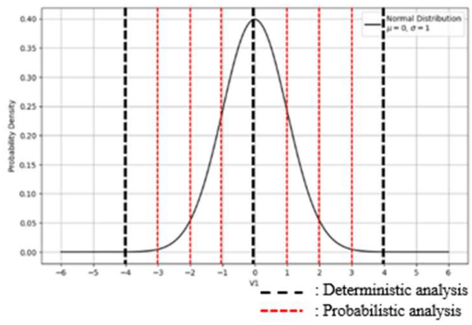

Figure. 2 PDF of variables according to the analysis interval

This method involves using actual simulation cases (deterministic analysis) and predicted cases (probabilistic analysis) while utilizing sensitivity intervals in the variable space to generate a more realistic failure surface, as illustrated in Figure 3. As the number of predicted cases increases, this approach mitigates conservatism, leading to a more practical failure surface.

However, while reducing conservatism can help in minimizing uncertainty, it may pose limitations in defining the failure surface. Additionally, since this method is based on the failure surface generation from the superposition, as established in previous research, the values used to validate the principle of superposition will be applied to define the failure surface margin.

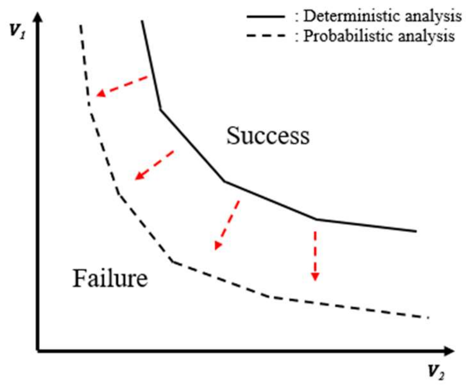

Figure. 3 Example of failure surface with two variables

3.2. Quantification with the Importance Sampling

Once the failure surface is generated, MC sampling is performed to randomly sample a comprehensive set of variables. The functional failure probability $\boldsymbol{P_{failure}}$ obtained through the MC simulation is expressed as follows in Equation (3).

$$\boldsymbol{P_{failure}} = \frac{1}{N}\sum_{K=1}^{N} \boldsymbol{I}\,(\boldsymbol{S_K} \geq \boldsymbol{S_{failure}}) \qquad (3)$$

In the quantification process, random sampling was performed based on the distributions of the previously defined variables and the generated failure surface. Given the characteristics of PSS and the specific case study, the probability of failure was found to be very small, making it difficult to determine the convergence region for probability estimation. Therefore, Importance Sampling was employed to focus on specific regions.

Importance sampling is a method used to efficiently estimate extremely small failure probabilities. While conventional MC simulation evaluates results through random sampling that directly follows the original variable distributions, Importance Sampling modifies the parameters of the initial variable distributions to concentrate samples near the failure surface. The sampled results are then adjusted by applying weights to correct the estimation.

Equation (4) represents the failure probability estimation formula, illustrating the process of computing failure probability using weighted Importance Sampling. The failure probability is calculated as the weighted sum of failure occurrences divided by the total sum of weights. In other words, the probability of the failure region is corrected based on the results sampled from the importance distribution. The numerator accumulates the weights of the samples corresponding to the failure region, while the denominator sums the total weights of all samples, thereby enabling the estimation of the failure probability in the actual probability distribution.

$$\mathbf{P_{failure}} = \frac{\sum_{i=1}^{10^n} \mathbf{weight_i \cdot failure_i}}{\sum_{i=1}^{10^n} \mathbf{weight_i}} \qquad (4)$$

Equation (5) represents the formula for calculating the weight ω(x) in Importance Sampling, which is derived from the original probability distribution and the importance distribution's Probability Density Function (PDF).

Here, the numerator is the PDF of the original probability distribution $\boldsymbol{N(\mu, \sigma^2)}$ before modification, while the denominator is the PDF of the modified probability distribution $\boldsymbol{N(\mu, \sigma_i^2)}$ after the change. By taking the ratio of these PDFs, the probabilistic contribution of each sample under the original distribution is adjusted accordingly.

$$\boldsymbol{\omega}(\mathbf{x}) = \frac{\frac{1}{\sqrt{2\pi\sigma^2}} exp(-\frac{-(x-\mu)^2}{2\sigma^2})}{\frac{1}{\sqrt{2\pi\sigma^2}} exp(-\frac{-(x-\mu)^2}{2\sigma_i^2})} \qquad (5)$$

4. Case Study

The method presented in Chapter 3 was applied to the case study. The PSS analyzed in this study is the PAFS of a hypothetical SMR model using a helical heat exchanger, and the MARS-KS thermal-hydraulic code was used. To apply the existing binary ET, this study defines a specific scenario of the Complete Loss Of Flow (CLOF) accident, in which the steam generator pressure fails to reach a specific pressure within a certain time [4].

The key initial variables were selected based on the PAFS of APR+, including the ECT temperature, ECT water level, and PAFS injection pipe diameter [5]. A truncated normal distribution, where the probability density is defined only within a specific range, was used. The variability of these three parameters was defined as -4 sigma, nominal value, and +4 sigma. By combining the number of variables and sensitivity analysis levels, a total of 27 cases were analyzed.

The observed values obtained from the deterministic analysis were compared with the predicted values from the probabilistic analysis, and the Root Mean Square Error (RMSE) and the R^2 were calculated. This analysis confirmed that the principle of superposition is reasonably maintained within a specific range. Using this approach, 729 probabilistic cases were predicted, which were then used to generate a failure surface.

Various methods can be used to define a failure surface. In this case study, the Convex Hull approach was employed to generate the smallest convex polygon or convex polyhedron that encloses the boundary point set. The process of generating the convex hull, a reasonable margin for success/failure was applied by using the R^2 value as the failure criterion. The points that constitute the convex polygon are shown in Table 1.

Table 1. Boundary points for Convex Hull

Point	ECT_T (K)	ECT_L (m)	$PAFS_A$ (m^2)
1	357.15	12.8	0.42918
2	357.15	13.9	0.42918
3	309.15	13.9	0.42918
4	309.15	12.8	0.42918
5	357.15	12.8	0.018394
6	357.15	13.9	0.018394
7	348.15	13.9	0.018394
8	348.15	12.8	0.018394
9	348.15	12.8	0.26594
10	348.15	13.9	0.26594
11	309.15	13.9	0.26594
12	309.15	12.8	0.26594

In other words, the failure surface is defined by constructing the minimal convex shape that encloses the given 12 boundary points, as shown in Figure 4.

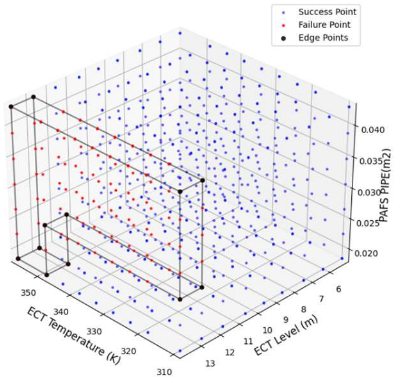

Figure 4. Failure surface with Convex Hull

To perform Importance Sampling in the quantification process based on the failure surface, the nominal values and distributions of the initially defined variables were adjusted, as shown in Table 2, so that more samples are positioned near the failure surface. Subsequently, weights were applied to correct the adjusted sampling distribution.

Table 2. Parameter modification for Importance Sampling

Variables	Random sampling	Importance sampling
$\boldsymbol{ECT_T}$	$(313.15, 6^2)$	N $(333.15, 12^2)$
$\boldsymbol{ECT_L}$	N $(9.5, 1.2^2)$	N $(11.05, 1.2^2)$
$\boldsymbol{PIPE_A}$	N $(0.030656, 0.00305^2)$	N $(0.030656, 0.006108^2)$

Using this approach, Importance Sampling was performed 10^8 times to estimate the failure probability of the PAFS in a hypothetical SMR under a specific CLOF accident, with a total of 30 iterations conducted. As a result, the functional failure probability of the PAFS was estimated to be 8.33×10^{-11} within the 95% confidence interval of 2.85×10^{-11}, 1.53×10^{-10}. This result is a calibrated value for the case study, so caution is needed in its interpretation.

5. Conclusion

This study focuses on generating failure surfaces necessary for quantifying the functional failure probability of the PSS by leveraging superposition, thereby reducing computational time in the quantification process, and applying Importance Sampling to minimize sampling time while ensuring sufficient reliability. By utilizing the proposed approach, it is expected that a Boolean logic-based Fault Tree (FT) can be applied within a reasonable computational cost, similar to existing international methodologies, as illustrated in Figure 5.

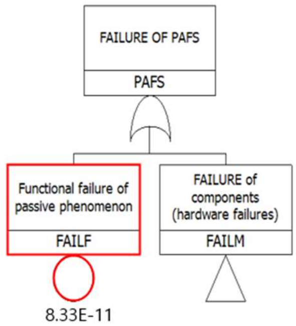

Figure 5. Application of functional failure to FT

However, this study has limitations in considering uncertainties during the failure surface generation process. Additionally, if the importance distribution significantly differs from the actual distribution, bias may occur during the sampling process, posing a risk of reduced accuracy in the results. Furthermore, quantifying the failure probability of the PSS remains a challenging issue due to variations in thermal-hydraulic conditions depending on the accident type. Moreover, there is currently no established method to compare and validate the quantified results of functional failure probability. A standardized methodology for practical application is lacking, and while this study adopted a conservative approach based on existing methodologies, its significance lies in presenting an efficient approach for estimating the functional failure probability of SMRs.

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