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Improved J and COD Estimation for Restrained Through-Wall Cracked Pipes

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1. Introduction

The piping system applied to Small Modular Reactors (SMR) is expected to consist of small-diameter pipes due to the narrow space between the reactor vessel (RV) and the containment vessel (CV). The application of small-diameter piping introduces differences in Leak-Before-Break (LBB) assessment results compared to conventional large nuclear power plant piping. In the current SMR LBB evaluation, ensuring leakage detectability with small-diameter piping may require assuming longer crack lengths than those considered in traditional large-scale nuclear power plants.

Previous studies and reports have extensively investigated solutions for predicting the *J*-integral and crack opening displacement (COD) of through-wall cracked (TWC) pipes [1-5]. However, these existing solutions typically consider applicability ranges limited to cracks extending up to half the circumferential length of the pipe ($\theta/\pi<0.5$). For SMR piping LBB assessment, longer cracks may need to be considered due to the small pipe diameter, necessitating the development of *J* and COD estimation solutions for long cracks.

The predicted long cracks can result in significantly high *J*-integral values, which may lead to insufficient LBB safety margins under certain conditions. Moreover, the SMR piping system is expected to be restrained due to the presence of numerous supports. Previous studies have shown that in restrained piping systems, the loading applied to crack section is reduced compared to free-end pipes under the same applied load [6,7]. In conventional LBB assessments, the applied load is typically derived from elastic analyses of the uncracked piping system. However, the compliance of TWC pipe increases, leading to a reduction in the applied load at the same deformation level (COD or rotation). To ensure accurate LBB assessment and secure safety margins, it is essential to consider and quantify the load reduction due to restraint effects, enabling the calculation of *J* values that better reflect actual piping system behavior.

This study extends the existing reference stress-based *J* estimation solutions to long cracks ($\theta/\pi>0.5$). Also, the methodology to quantify the load reduction due to restraint effect was proposed and validated by comparing with finite element (FE) analysis results.

2. Methodology

2.1 Extension of reference stress-based J estimation equation

In the reference stress approach, elastic-plastic *J*-integral can be estimated from [5]

$$\frac{J}{J_e}=\frac{E\varepsilon_{ref}}{\sigma_{ref}}+\frac{1}{2}\frac{L_r^{\,2}\sigma_{ref}}{E\varepsilon_{ref}};\ J_e=\frac{K^2}{E'} \tag{1}$$

where $E'=E/(1-\nu^2)$ for plane strain and $E'=E$ for plane stress; E and ν denote the elastic modulus and Poisson's ratio, respectively. Assuming the TWC pipes under bending moment, the proximity to plastic yielding, represented by the load ratio L_r, is defined as

$$L_r=\frac{\sigma_{ref}}{\sigma_y}=\frac{M}{M_{ref}} \tag{2}$$

where M and M_{ref} denote a applied bending moment and the reference normalizing bending moment; σ_{ref} and σ_y denote the reference stress and the yield (0.2% proof) stress; and ε_{ref} denotes the reference strain at the reference stress $\sigma=\sigma_{ref}$, determined from true stress-strain data of the material. It should be noted that a new reference load (optimized reference moment) M_{OR} was introduced to improve estimation accuracy, given by [5]

$$\begin{gathered}M_{OR}=\gamma(\theta)M_L\\ \gamma(\theta)=0.82+0.75\left(\frac{\theta}{\pi}\right)+0.42\left(\frac{\theta}{\pi}\right)^2\quad \text{for } \frac{\theta}{\pi}\le 0.5\end{gathered} \tag{3}$$

where θ denotes the half-crack angle of TWC pipes. The factor γ in Eq. (3) is a correction factor for the crack length. As mentioned earlier, the longer cracks may need to be considered due to the small pipe diameter for SMR piping LBB assessment. Therefore, the applicability range of the factor γ will be extended for long crack ($\theta/\pi\leq 0.7$) as a function of θ/π and R/t based on FE analysis results.

2.2 Quantification of load reduction for restrained pipe due to pressure-induced bending effect

Fig. 1 shows the schematics of TWC section and the free-body diagram of a restrained TWC pipe under axial tension equivalent to internal pressure. The TWC pipe could rotate by the bending moment M_c and shear force V_c. The rotation induced by deflection and internal pressure was not considered, as small deformation was assumed. Using the free-body diagram, the rotation of a restrained TWC pipe ϕ_c under axial tension equivalent to internal pressure T can be expressed as follows:

$$\phi_c = \phi_{c,1} + \phi_{c,2} \tag{4}$$

$$\phi_{c,1} = \phi_{M_c,1} - \phi_{V_c,1}\ ;\ \phi_{c,2} = \phi_{M_c,2} + \phi_{V_c,2}$$

$$\phi_{M_c,1} = \frac{x}{EI} M_c\ ;\ \phi_{M_c,2} = \frac{L-x}{EI} M_c \tag{5}$$

$$\phi_{V_c,1} = \frac{x^2}{2EI} V_c\ ;\ \phi_{V_c,2} = \frac{(L-x)^2}{2EI} V_c$$

$$\phi_c = \frac{L}{EI} M_c - \frac{(L-x)^2 - x^2}{2EI} V_c \tag{6}$$

where subscript 1 and 2 denote the section 1 and 2 as shown in Fig. 1. By compatibility condition, the shear force V_c can be expressed in terms of M_c. Then, the rotation of a restrained through-wall cracked pipe is given as follows:

$$\phi_c = \left\{ \frac{L}{EI} - \frac{3}{4EI} \frac{\left[(L-x)^2 - x^2\right]^2}{(L-x)^3 + x^3} \right\} M_c = F\left(\frac{x}{L}\right) \times M_c \tag{7}$$

where x and L denote the distance of TWC from the restrained section and pipe length, respectively. The rotation solution of TWC pipe is given in GE/ERP solution [4].

$$\phi_c = F_T TR - F_M M_c \tag{8}$$

where F_T and F_M denote the compliance for TWC pipe under axial tension and bending moment, respectively and are given in GE/EPRI solution. By solving the simultaneous equations with the previously derived rotation equation Eq. (7), the applied moment at crack section M_c can be obtained

$$M_c = F_T TR / \left[F_T + C_M\right] \tag{9}$$

The comparison of estimated M_c using the proposed equation with FE results for various θ/π and x/L. It was confirmed that the proposed equation shows good agreement with FE results.

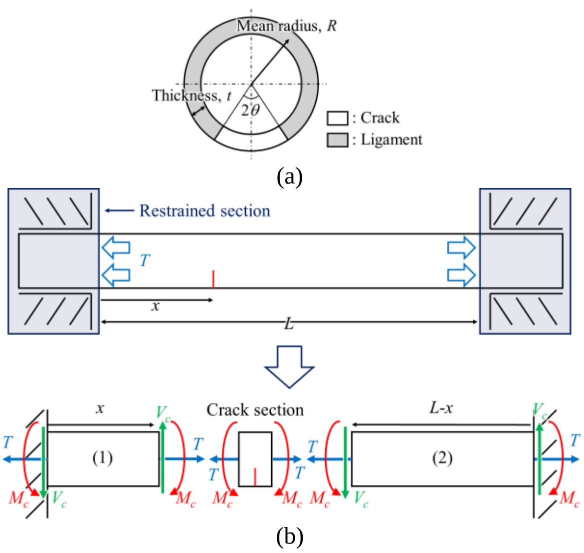

Fig. 1. (a) Schematics of crack section of TWC pipe and (b) free body diagram of restrained TWC pipe under axial tension equivalent to internal pressure.

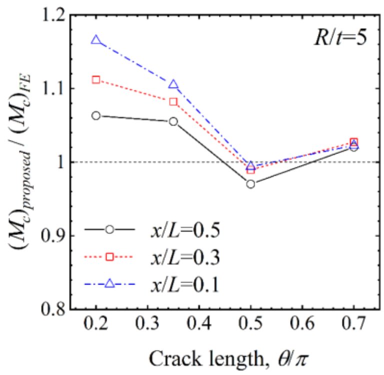

Fig. 2. Comparison of estimated M_c using the proposed method with FE results for various θ/π and x/L.

3. Conclusions

The SMR piping system is installed in the narrow space between RV and CV, requiring the application of small-diameter piping and being restrained by numerous supports. In this study, two approaches were proposed for the LBB assessment of the small-diameter piping system in SMR: (1) The existing reference stress based J solution was extended to predict J for $\theta/\pi \leq 0.7$ and (2) a predictive equation was developed to estimate the load reduction in a restrained piping system due to the presence of supports.

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