Approaches on Uncertainty Expression in Nuclear Material Accounting

Haneol Lee[a*], Hyun Cheol Lee[a], Jung Youn Choi [a], Hyun Ju Kim [a], Jinho Ryu[a]
[a]*KINAC, 1418 Yuseong-daero, Yuseong-gu, Daejeon, Republic of Korea*
[*]*Corresponding author: haneol@kinac.re.kr*

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1. Introduction

As a technical supporting organization (TSO) of the state system of nuclear accounting for and control (SSAC) of the Republic of Korea (ROK), the Korea Institute of Nuclear non-proliferation and Control (KINAC) has been conducting national safeguards inspection. We plan to include material balance evaluation (MBE) for national safeguards inspection to reinforce the capability of the SSAC. Since the primary process of the MBE is to compare the material balance to its measurement uncertainty, the uncertainty expression method for nuclear material accounting is the most important. We suggest two uncertainty expression methods (ANOVA and GUM) for domestic MBE. We then compare the two methods using the sample analysis results from previous national inspections. The comparison includes the evaluated uncertainty and characteristics of the two methods.

2. Methods and Results

We compared two uncertainty expression methods: analysis of variance (ANOVA) and guide to the expression of uncertainty in measurement (GUM). The International Atomic Energy Agency (IAEA) adopted ANOVA to express measurement uncertainty for the MBE. The International Organization for Standards (ISO) adopted GUM as a standard to express measurement uncertainty. The ANOVA-based uncertainty expression is also known as a top-down method, and the GUM method is also known as a bottom-up method.

2.1 Top-down Approach (ANOVA)

The ANOVA-based uncertainty expression method is applied when the detailed information on the measurement process is limited. Since the IAEA estimates the uncertainty of the accounting systems of all nuclear facilities worldwide, the agency adopts a top-down approach.

The IAEA estimates a facility's random and systematic error using the top-down approach by the following processes [1]:

(1) Establish a database of pair-wise differences for a number of groups of samples.
(2) Let x_{gi} and y_{gi} be the facilities' and inspectors' measurement results for an item whose group is *g* and ID is *i*.
(3) Let m_g and G be the number of items in group *g* and the number of groups.
(4) Let d_{gi} and V_{gd} be the pair-wise difference $x_{gi} - y_{gi}$ and its variance for items in group *g*.
(5) Estimate the random variance ($V_{rd}(=\widehat{\sigma_{rd}}^2)$) of the pair-wise differences using equation (1).
(6) Estimate the systematic variance ($V_{sd}(=\widehat{\sigma_{sd}}^2)$) of the pair-wise differences using equation (2).
(7) Estimate the random variance of a facility ($V_{rx}(=\widehat{\sigma_{rx}}^2)$) based on the Grubbs' method using equation (3).
(8) If impossible ($V_{rx,ry} < 0$), estimate the random variance of a facility based on the CELEX method using equation (5).
(9) Estimate the systematic variance of a facility ($V_{sx}(=\widehat{\sigma_{sx}}^2)$)) based on an assumption ($\widehat{\sigma_{sx}}^2 = \widehat{\sigma_{sy}}^2$).

$$\widehat{\sigma_{rd}}^2 = \frac{\sum_{g=1}^{G}(m_g-1)V_{gd}}{\sum_{g=1}^{G}(m_g-1)} \quad (1)$$

$$\widehat{\sigma_{sd}}^2 = \left(\frac{\sum_{g=1}^{G}\left(\sum_{i=1}^{m_g}\frac{d_{gi}}{m_g}\right)}{G}\right)^2 - \frac{\left(\frac{\sum_{g=1}^{G}\left(\frac{1}{m_g}\right)}{G}\right)\widehat{\sigma_{rd}}^2}{G} \quad (2)$$

$$\widehat{\sigma_{rx}}^2 = \frac{\sum_{g=1}^{G}(m_g-1)\widehat{\sigma_{rgx}}^2}{\sum_{g=1}^{G}(m_g-1)} \quad (3)$$

$$\begin{cases} \widehat{\sigma_{rgx}}^2 = v_{gx} - v_{gxy} \\ v_{gx} = \dfrac{\left(\sum_{i=1}^{m_g} x_{gi}^2 - \frac{\left(\sum_{i=1}^{m_g} x_{gi}\right)^2}{m_g}\right)}{m_g-1} \\ v_{gxy} = \dfrac{\left(\sum_{i=1}^{m_g} x_{gi}y_{gi} - \frac{\left(\sum_{i=1}^{m_g} x_{gi}\right)\left(\sum_{i=1}^{m_g} y_{gi}\right)}{m_g}\right)}{m_g-1} \end{cases} \quad (4)$$

$$\widehat{\sigma_{rx}}^2 = S\frac{I_0}{I_1} \quad (5)$$

$$\begin{cases} S = \frac{\sum_{g=1}^{G}(m_g-1)\widehat{\sigma_{rgx}}^2}{\sum_{g=1}^{G}(m_g-1)} + \frac{\sum_{g=1}^{G}(m_g-1)\widehat{\sigma_{rgy}}^2}{\sum_{g=1}^{G}(m_g-1)} \\ n = \sum_{g=1}^{G}(m_g - 1) \\ S_1^2 = \sum_{g=1}^{G}(m_g - 1)\, v_{gx}/n \\ S_2^2 = \sum_{g=1}^{G}(m_g - 1)\, v_{gy}/n \\ S_{12} = \sum_{g=1}^{G}(m_g - 1)\, v_{gxy}/n \\ Q = (1 + (S_2^2 - S_{12})/(S_1^2 - S_{12}))^{-1} \end{cases} \quad (6)$$

$$\begin{cases} I_0 = \int_0^1 v f(v) dv \\ I_1 = \int_0^1 f(v) dv \\ f(v) = \left(Sv^2 + 2\left(S_{xy} - S_x^2\right)v + S_x^2\right)^{-0.5n} \end{cases} \quad (7)$$

$$\begin{cases} (for -1 \le Q \le 2) \\ \begin{cases} S_x^2 = S_1^2 \\ S_{xy} = S_{12} \end{cases} \\ (for\ Q < -1\ or\ Q > 2) \\ \begin{cases} S_x^2 = 3S\frac{S_1^2}{|S_2^2 - S_1^2|} \\ S_{xy} = \begin{pmatrix} S\frac{(S_1^2 + 2S_2^2)}{(S_1^2 - S_2^2)}\ (for\ S_1^2 > S_2^2) \\ S\frac{(2S_1^2 + S_2^2)}{(S_2^2 - S_1^2)}\ (for\ S_2^2 > S_1^2) \end{pmatrix} \end{cases} \end{cases} \quad (8)$$

$$\widehat{\sigma_{sx}}^2 = \frac{1}{2}\left(\widehat{\sigma_{sd}}^2\right) \quad (9)$$

2.2 Bottom-up Approach (GUM)

The GUM-based uncertainty expression method is applied when detailed information on the measurement process is provided. Since the number of domestic nuclear facilities is limited and national inspectors can ask for information on the facilities' accounting system [2], national inspection considers a bottom-up approach.

The GUM-based method first defines an equation to estimate a measurand X using several measurements ($X = f(x_1, x_2, \dots, x_n)$). It then calculates the combined uncertainty of the measurand (u(X)) using equation (10) [3]. Individual measurement uncertainties ($u(x_i)$) are calculated depending on their type (Type A, B).

$$u(X) = \sum_{i=1}^{n}\left(\frac{\partial X}{\partial x_i}\right)^2 u(x_i)^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\frac{\partial X}{\partial x_i}\frac{\partial X}{\partial x_j}u(x_i)u(x_j)r(x_i, x_j) \quad (10)$$

2.3 Sample Analysis for Comparison

We compared the measurement uncertainty for the two methods using the analysis results from previous inspections (AVOVA) and a standard analysis procedure (GUM). The target accounting process is U concentration analysis using gravimetric analysis (GRAV). The pair-wise difference results for the ANOVA-based uncertainty expression are summarized in Table I. Figure 1 also depicts the fishbone diagram for the GUM-based uncertainty expression. For the GUM method, we defined uncertainty components with correlation as systematic components and uncertainty components independent of each other as random components.

Table I: Pair-wise differences for U concentration analysis.

Group	Item	U conc. (%)		d
		KINAC	Facility	
1	1	[redacted]	[redacted]	1.200E-03
	2	[redacted]	[redacted]	-1.700E-03
	3	[redacted]	[redacted]	6.600E-02
	1	[redacted]	[redacted]	7.700E-03
	2	[redacted]	[redacted]	-1.880E-02
	3	[redacted]	[redacted]	1.110E-02
	1	[redacted]	[redacted]	-6.500E-03
	2	[redacted]	[redacted]	5.800E-03
	3	[redacted]	[redacted]	8.000E-04
	1	[redacted]	[redacted]	-1.090E-02
	2	[redacted]	[redacted]	-1.700E-03
	3	[redacted]	[redacted]	1.260E-02
2	1	[redacted]	[redacted]	4.260E-02
	2	[redacted]	[redacted]	3.030E-02
	1	[redacted]	[redacted]	4.590E-02
	2	[redacted]	[redacted]	3.490E-02
	1	[redacted]	[redacted]	4.740E-02
	2	[redacted]	[redacted]	3.850E-02
	1	[redacted]	[redacted]	4.040E-02
	2	[redacted]	[redacted]	3.080E-02
	1	[redacted]	[redacted]	1.900E-02
	2	[redacted]	[redacted]	2.670E-02
	1	[redacted]	[redacted]	3.420E-02
	2	[redacted]	[redacted]	3.980E-02
	1	[redacted]	[redacted]	2.200E-02
	2	[redacted]	[redacted]	-1.400E-02
	1	[redacted]	[redacted]	-1.200E-02
	2	[redacted]	[redacted]	2.600E-02
	1	[redacted]	[redacted]	9.000E-03
	2	[redacted]	[redacted]	-9.000E-03
3	1	[redacted]	[redacted]	3.700E-03
	2	[redacted]	[redacted]	-4.470E-02
	3	[redacted]	[redacted]	-3.000E-02
	4	[redacted]	[redacted]	-1.200E-03
	1	[redacted]	[redacted]	2.900E-02
	2	[redacted]	[redacted]	-1.520E-02
	3	[redacted]	[redacted]	3.460E-02
	4	[redacted]	[redacted]	-9.700E-03

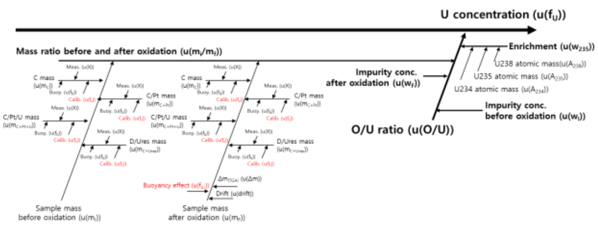

Fig. 1. Fishbone diagram for the U concentration analysis using GRAV.

2.4 Comparison Results

Table II summarizes the calculated relative standard error (for ANOVA) and relative standard uncertainty (for GUM) of U concentration analysis using GRAV. Results indicate the estimated uncertainties are the same for both methods. Therefore, we can apply both methods to evaluate the material balance of a facility depending on the purpose (Table III) [4]. Since the purpose of national inspection includes evaluating facilities' accounting systems as well as diversion detection, the GUM-based uncertainty expression method has more advantages than the ANOVA-based approach.

Table II: Results of relative standard error (ANOVA) and relative standard uncertainty (GUM) for U concentration analysis using GRAV.

ANOVA		GUM	
δ (%)		δ (%)	
0.036		0.038	
δ_r (%)	δ_s (%)	δ_r (%)	δ_s (%)
0.033	0.015	0.038	0.0014

Table III: Characteristics of ANOVA and GUM based uncertainty expression methods

	ANOVA	GUM
Adv.	Can estimate uncertainty with limited information	Can analyze the contribution of individual accounting process
Disadv.	Cannot analyze the uncertainty in detail	Cannot estimate uncertainty without detailed information

3. Conclusions

The KINAC is considering to perform MBE in the national inspection. The uncertainty expression method is a key factor for the MBE. We compared two uncertainty expression methods (ANOVA and GUM) and evaluated the estimated uncertainties using previous inspection results and a standard accounting process for both methods. Results indicated the relative uncertainty for the both methods is consistent. Therefore, the KINAC is considering to adopt the GUM-based uncertainty expression method for domestic MBE considering the uncertainty contribution to the accounting process.

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