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# Critical Flow Prediction by Direct Solving of Characteristic Equations

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## 1. Introduction

SPACE [1] has two kinds of critical flow models. The first one is the mechanistic model developed by Ransom-Trapp and the other one is semi-empirical model developed by Henry-Fauske. Ransom and Trapp developed an analytic two-phase choking criteria based on characteristic analysis of a two phase equations. SPACE which has been adopting three field equations employs two phase equations as follows to develop two-phase choking criteria [2].

$$\frac{\partial \rho_m}{\partial t} + \frac{\partial \left(\rho_m u_m\right)}{\partial x} = 0 \tag{1}$$

$$\begin{aligned} &\alpha_g \rho_g \left( \frac{\partial u_g}{\partial t} + u_g \frac{\partial u_g}{\partial x} \right) + \alpha_g \frac{\partial P}{\partial x} + \\ &C \alpha_g \alpha_l \rho_m \left( \frac{\partial \left(u_g - u_l\right)}{\partial t} + u_l \frac{\partial u_g}{\partial x} - u_g \frac{\partial u_l}{\partial x} \right) = 0 \end{aligned} \tag{2}$$

$$\begin{aligned} &\alpha_l \rho_l \left( \frac{\partial u_l}{\partial t} + u_l \frac{\partial u_l}{\partial x} \right) + \alpha_l \frac{\partial P}{\partial x} + \\ &C \alpha_g \alpha_l \rho_m \left( \frac{\partial \left(u_l - u_g\right)}{\partial t} + u_g \frac{\partial u_l}{\partial x} - u_l \frac{\partial u_g}{\partial x} \right) = 0 \end{aligned} \tag{3}$$

$$\frac{\partial \left(\rho_m s_m\right)}{\partial t} + \frac{\partial \left(\rho_m u_m s_m\right)}{\partial x} = 0 \tag{4}$$

where $\rho$ is the fluid density, $u$ is the velocity, $\alpha$ is the phase volume fraction, P is the fluid pressure, $s$ is the entropy, and $C$ is the virtual mass coefficient. Subscript *m, l, and g* means mixture, liquid phase, and gas phase, respectively.

In order to calculate the choking flow SPACE solves three equations. The first one is the homogeneous equilibrium sound velocity described in section 2 and the second one is the difference equation subtracting the liquid momentum equation from the gas momentum equation and third one is also the difference equation subtracting the droplet momentum equation from the gas momentum equation. This paper deals with the other method in the sonic velocity calculation considering non-condensable gas additionally.

Section 2 describes the derivation of original sonic velocity model implemented in SPACE code and section 3 accounts for the new direct solving method of calculating sonic velocity. Section 4 shows the feasibility of new method of sonic velocity calculation through the simulation of a choked flow problem.

## 2. Conventional sonic velocity model [3]

Above equations can be written in terms of $\alpha_g$, P, $u_g$, and $u_l$ as four quasi-linear, first-order partial differential equations of the form

$$A\left(\overline{U}\right)\frac{\partial \overline{U}}{\partial t} + B\left(\overline{U}\right)\frac{\partial \overline{U}}{\partial x} + C\left(\overline{U}\right) = 0 \tag{5}$$

where $A$ and $B$ are fourth-order square matrices.

The characteristic velocities of system of equations defined in equation (5) are the roots of characteristic polynomial. The choking criteria is defined by $\lambda_j = 0 \; for \;\; j \le 4$ and $\lambda_i \ge 0 \; for \;\; i \ne j$.

$$\left(A\lambda - B\right) = 0 \tag{6}$$

Ransom and Trapp obtained the characteristic roots for $\lambda_{1,2}$ by neglecting the second-order factors of $\left(\lambda - u_g\right) \; and \; \left(\lambda - u_l\right)$. The remaining characteristic roots of $\lambda_{3,4}$ are also obtained by dividing out the quadratic factor containing $\lambda_{1,2}$ , neglecting the remainder, and subsequent factorization of the remaining quadratic terms.

$$\lambda_{3,4} = u_m + D\left(u_g - u_l\right) \pm a \tag{7}$$

where $u_m$ is the mixture velocity,

$$a = a_{HE}\left[\frac{C\rho_m^2 + \rho_m\left(\alpha_g\rho_l + \alpha_l\rho_g\right)}{C\rho_m^2 + \rho_g\rho_l}\right]^{1/2} \tag{8}$$

$$D = \frac{1}{2}\left[\begin{aligned} &\frac{\left(\alpha_g\rho_l - \alpha_l\rho_g\right)}{C\rho_m + \alpha_g\rho_l + \alpha_l\rho_g} + \frac{\rho_g\rho_l\left(\alpha_l\rho_l - \alpha_g\rho_g\right)}{\rho_m\left(\rho_g\rho_l + C\rho_m^2\right)} \\ &-a_{HE}^2\frac{\rho_m\left(\alpha_g\rho_g^2 S_g^* + \alpha_l\rho_l^2 S_l^*\right)}{\rho_g\rho_l\left(S_g - S_l\right)} \end{aligned}\right] \tag{9}$$

In equation (9) $S_k^* = \frac{dS_k^S}{dP}$, where $k = l$ or $g$.

The final choking velocity is obtained from equations (8), (9) and several assumptions. The first assumption is the sound speed is equal to the homogeneous equilibrium sound speed when the virtual mass coefficient is taken to be infinity($a$=$a_{HE}$). In equation (9), if the third term is neglected and the virtual mass coefficient is taken as zero, the $D$ is as follows;

$$D = \frac{1}{2}\left( \frac{\alpha_g \rho_l - \alpha_l \rho_g}{\alpha_g \rho_l + \alpha_l \rho_g} + \frac{\alpha_l \rho_l - \alpha_g \rho_g}{\rho_m} \right) \tag{10}$$

The equation (11) for the homogeneous equilibrium sound speed is derived by substituting above assumptions into the equation (7).

$$a_{HE} = \frac{\alpha_g \rho_l u_g + \alpha_l \rho_g u_l}{\alpha_g \rho_l + \alpha_l \rho_g} \tag{11}$$

Finally, SPACE solves the equation (11) for the sonic velocity.

## 3. New method for sonic velocity calculation [4]

The new method for sonic velocity is to solve the determinant of matrix of equation (6) directly instead of solving equation (11). And in order to take into account a non-condensable gas effect mass conservation equation for inert gas is added.

$$\frac{\partial\left(\alpha_g \rho_n\right)}{\partial t} + \frac{\partial\left(\alpha_g \rho_n u_g\right)}{\partial x} = 0 \tag{12}$$

where subscript $n$ means non-condensable gas.

Matrices $A$ and $B$ to obtain characteristic root of system of equations defined in the equation (6) numerically can be written as equations (13) and (14) respectively.

$$\begin{bmatrix}
\alpha_g\left[\left.\frac{\partial\rho_n}{\partial T_g}\right|_{p_n}\frac{\partial T_g}{\partial p_v}+\rho_v^*\right]+\alpha_g\rho_l^* & \rho_g-\rho_l & 0 & 0 & \alpha_g\frac{\partial\rho_n}{\partial p_n}+\alpha_l\frac{\partial\rho_l}{\partial p} \\
0 & 0 & \alpha_g\left(\rho_g+C\alpha_l\rho_m\right) & -C\alpha_g\alpha_l\rho_m & 0 \\
0 & 0 & -C\alpha_g\alpha_l\rho_m & \alpha_l\left(\rho_l+C\alpha_g\rho_m\right) & 0 \\
\alpha_g\left[s_v\rho_v^*-\rho_v s_v^*+\frac{\partial T_g}{\partial p_v}\left(\rho_n\frac{\partial s_n}{\partial T_g}+s_n\frac{\partial\rho_n}{\partial T_g}\right)_{p_n}\right]+\alpha_l\left(\rho_l^* s_l+\rho_l s_l^*\right) & \rho_g s_g-\rho_l s_l & 0 & 0 & \alpha_g\left[\rho_n\frac{\partial s_n}{\partial p_n}+s_n\frac{\partial\rho_n}{\partial p_n}\right]_{T_g}+\alpha_l\left(\rho_l\frac{\partial s_l}{\partial p}+s_l\frac{\partial\rho_l}{\partial p}\right)_{T_l} \\
\alpha_g\left.\frac{\partial\rho_n}{\partial T_g}\right|_{p_n}\frac{\partial T_g}{\partial p_v} & \rho_n & 0 & 0 & \alpha_g\left.\frac{\partial\rho_n}{\partial p_n}\right|_{T_g}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial p_v}{\partial t} \\ \frac{\partial\alpha_v}{\partial t} \\ \frac{\partial u_g}{\partial t} \\ \frac{\partial u_l}{\partial t} \\ \frac{\partial p_n}{\partial t}
\end{bmatrix} \tag{13}$$

$$\begin{bmatrix}
u_g\alpha_g\left[\left.\frac{\partial\rho_n}{\partial T_g}\right|_{p_n}\frac{\partial T_g}{\partial p_v}+\rho_v^*\right]+u_l\alpha_g\rho_l^* & u_g\rho_g-u_l\rho_l & \alpha_g\rho_g & \alpha_l\rho_l & u_g\alpha_g\left.\frac{\partial\rho_n}{\partial p_n}\right|_{T_g}+u_l\alpha_l\left.\frac{\partial\rho_l}{\partial p}\right|_{T_l} \\
\alpha_g & 0 & \alpha_g\left(u_g\rho_g+Cu_l\alpha_l\rho_m\right) & -Cu_g\alpha_g\alpha_l\rho_m & \alpha_g \\
\alpha_l & 0 & -Cu_l\alpha_g\alpha_l\rho_m & \alpha_l\left(u_l\rho_l+Cu_g\alpha_g\rho_m\right) & \alpha_l \\
u_g\alpha_g\left[s_v\rho_v^*-\rho_v s_v^*+\frac{\partial T_g}{\partial p_v}\left(\rho_n\frac{\partial s_n}{\partial T_g}+s_n\frac{\partial\rho_n}{\partial T_g}\right)_{p_n}\right]+u_l\alpha_l\left(\rho_l^* s_l+\rho_l s_l^*\right) & u_g\rho_g s_g-u_l\rho_l s_l & \alpha_g\rho_g s & \alpha_l\rho_l s_l & u_g\alpha_g\left[\rho_n\frac{\partial s_n}{\partial p_n}+s_n\frac{\partial\rho_n}{\partial p_n}\right]_{T_g}+u_l\alpha_l\left(\rho_l\frac{\partial s_l}{\partial p}+s_l\frac{\partial\rho_l}{\partial p}\right)_{T_l} \\
u_g\alpha_g\left.\frac{\partial\rho_n}{\partial T_g}\right|_{p_n}\frac{\partial T_g}{\partial p_v} & u_g\rho_n & \alpha_g\rho_n & 0 & u_g\alpha_g\left.\frac{\partial\rho_n}{\partial p_n}\right|_{T_g}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial p_v}{\partial t} \\ \frac{\partial\alpha_v}{\partial t} \\ \frac{\partial u_g}{\partial t} \\ \frac{\partial u_l}{\partial t} \\ \frac{\partial p_n}{\partial t}
\end{bmatrix} \tag{14}$$

$$\rho_v^* = \left(\frac{\partial\rho_v}{\partial p_v}\right)_{T_g} + \left(\frac{\partial\rho_v}{\partial T_g}\right)_{p_v}\frac{dT_g}{dp_v},\ \rho_l^* = \left(\frac{\partial\rho_l}{\partial p}\right)_{T_l} + \left(\frac{\partial\rho_l}{\partial T_l}\right)_{p}\frac{dT_l}{dp},\ s_v^* = \left(\frac{\partial s_v}{\partial p_v}\right)_{T_g} + \left(\frac{\partial s_v}{\partial T_g}\right)_{p_v}\frac{dT_g}{dp_v},\ s_l^* = \left(\frac{\partial s_l}{\partial p}\right)_{T_l} + \left(\frac{\partial s_l}{\partial T_l}\right)_{p}\frac{dT_l}{dp}$$

Subscript $v$: steam phase

The characteristic polynomial of above system of equations has the fifth order and the equation (6) is extremely difficult to solve analytically. So we solve the equation numerically. This method is easy to take into account the effect of non-condensable gas and to extend to two-fluid, 6-equation model.

For the initial guessing of velocities we have to calculate the homogeneous equilibrium sound speed ($a_{HE}$) at the throat and $a_{HE}$ is calculated from the maximum mass flux corresponding throat conditions which is determined by the iteration. Initial throat pressure is assumed to be equal to that predicted by ideal gas expansion.

$$p_f = p_c\left(\frac{2}{\gamma_g+1}\right)^{\gamma_g/(\gamma_g-1)} \tag{15}$$

where, $p_f$ is the throat(face) pressure, $p_c$ is the stagnation upstream(cell) pressure, and $\gamma_g$ is the specific heat ratio. Then the throat quality is calculated by assuming a constant entropy expansion from the stagnation condition.

$$x_f = \frac{\rho_{g,f}\left(s_m^s - s_{l,f}\right)}{\rho_{g,f}\left(s_{v,f} - s_{l,f}\right) + \rho_{n,f}\left(s_{n,f} - s_{n,c}^s - s_{v,f} + s_{v,c}^s\right)} \quad (16)$$

where, superscript $s$ means saturation state. Once the throat quality has been determined, a total mixture enthalpy and mixture density at the throat are calculated as follows;

$$h_{m,f} = x_f h_{g,f} + \left(1 - x_f\right) h_{l,f} \quad (17)$$

$$\rho_{m,f} = \frac{\rho_{l,f}\rho_{g,f}}{x_f\left(\rho_{l,f} - \rho_{g,f}\right) + \rho_{g,f}} \quad (18)$$

Using a mixture total enthalpy at stagnation, equation (17), and (18) the mass flux for a throat pressure of $p_f$ can be calculated by equation (19).

$$G_{m,f} = \rho_{m,f}\sqrt{2\left(h_{m,c} - h_{m,f}\right)} \quad (19)$$

where, $h_{m,c}$ is the stagnation enthalpy.

To obtain maximum mass flux the pressure is slightly adjusted and the calculation from equation (16) to (19) is repeated after the check of $dG_{m,f}/dp_f < \epsilon$.

The maximum mass flux is used to calculate the $a_{HE}$.

$$a_{HE} = G_{\max}/\rho_{m,f} \quad (20)$$

The sound speed of equation (20) is used to obtain for the final gas velocity satisfying $|A\lambda\text{-}B|=0$ where $A$ is equation (13) and $B$ is equation (14).

Equations (13) and (14) include two phasic velocities and the liquid velocity is set based on the slip ratio which is differed with the flow regime[5].

$$S_r = \frac{C_0\left(1-\alpha_{g,f}\right)u_l + u_{gj}}{\left(1-\alpha_{g,f}C_0\right)u_l} \quad (21)$$

where, $S_r$ is slip ratio, $C_0$ is distribution parameter, $u_{gj}$ is drift velocity, and $\alpha_{g,f}$ is void fraction at the throat.

$$C_0 = 1.0,\quad u_{gj} = 1.53\alpha_{l,f}^2\left[\frac{\sigma g \Delta\rho}{\rho_l^2}\right]^{0.25} \quad \text{for } \alpha_{g,f} \le 0.2 \quad (22)$$

$$C_0 = 1.2 - 0.2\sqrt{\frac{\rho_{g,f}}{\rho_{l,f}}},\quad u_{gj} = 0.37\left[\frac{g\Delta\rho D}{\rho_{l,f}}\right]^{0.5} \quad \text{for } 0.2 \le \alpha_{g,f} < 0.51 \quad (23)$$

$$C_0 = 1.2 - 0.2\sqrt{\frac{\rho_{g,f}}{\rho_{l,f}}},\quad u_{gj} = \sqrt{2}\left[\frac{\sigma g \Delta\rho}{\rho_l^2}\right]^{0.25} \quad \text{for } 0.51 \le \alpha_{g,f} < 0.75 \quad (24)$$

$$C_0 = 1.0,\quad u_{gj} = \left(1-\alpha_{g,f}\right) \Big/ \left[\alpha_{g,f} + \frac{1+75\alpha_{l,f}\rho_g}{\sqrt{\alpha_{g,f}\rho_{l,f}}}\right]^{0.5} \quad \text{for } 0.75 \le \alpha_{g,f} < 0.999 \quad (25)$$

**4. Simulation for Marviken test 15**

The Marviken test 15 [6] was selected to validate the new method of sonic speed calculation in the SPACE code. The critical discharge of subcooled liquid, two-phase mixture, and steam was simulated at the Marviken facility of which pressure vessel was made as a full-scale Boiling Water Reactor. The inner diameter and height of the vessel are 5.22m and 24.55m, respectively. The total volume is about 420 m$^3$.

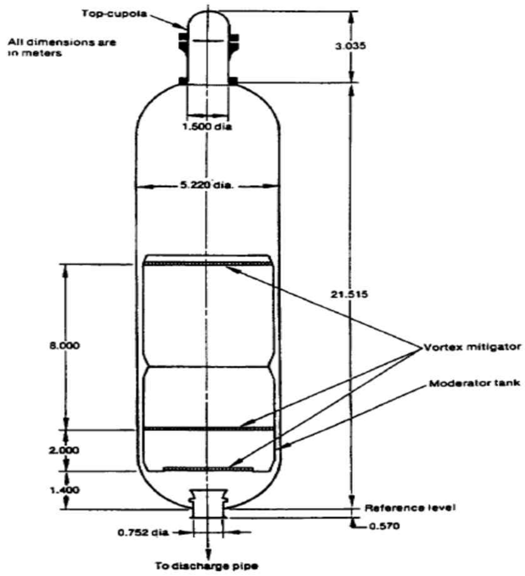


Fig. 1 The vessel schematic of Marviken facility

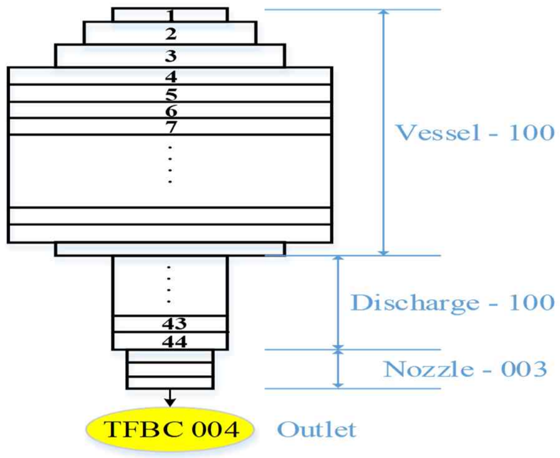


Fig. 2 SPACE Nodalization for Marviken test 15

The diameter and length of the test nozzle are 0.5 m and 1.81 m, respectively. The data from the test include the pressure and temperature of steam dome (PIPE 100-01 of Fig. 2) and the discharge flow rate (TFBC 004 of Fig. 2). The water level was initially located at the center of third volume of pipe component 100. The water of vessel upper part was almost saturated and the lower part was filled with water of about 30 °C subcooling.

The subcooling at the inlet nozzle was initially about 70 °C and after the bursting of rupture disks it decreased gradually until saturated conditions were established at 25 seconds. Then two-phase flow discharge was maintained until 57 seconds.

Figure 3 shows the mass flow rates from two kinds of calculations and the measured data. In the calculations all discharge coefficients are set to 1.0. The transition from subcooled single phase liquid flow to two phase flow at about 25 seconds is clear on Figure 3. The conventional method using sound speed equation of (11) predicts lower than the experimental data in the span of two phase discharge, however the new method solving the eigen values for system of equations directly gives much better agreements with measured data. This means equation (11) has some error in the prediction of sound speed and the numerically direct solving of equation (6) gives more accurate sound speed. Figure 4 compares the predicted sonic speeds. The speed from the new method is definitely higher than its conventional method in the two phase discharge region.

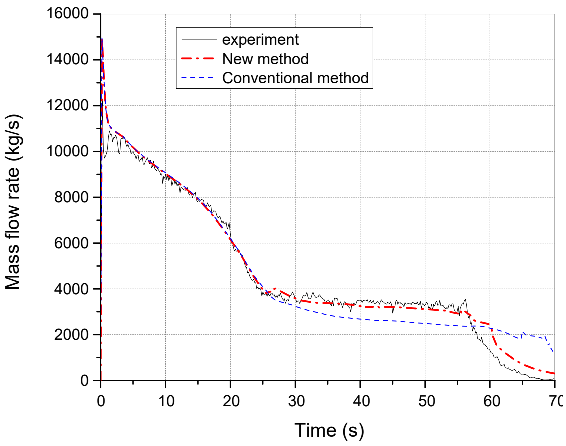


Fig. 3. Calculated and measured mass flow rates.

Pressure behaviors in the upper dome are shown in Figures 5. In the experiment, the pressure is rapidly decreased to about 4.3 MPa and increased to 4.8 MPa due to flashing of water in the upper part of vessel. However the SPACE code does not predict the phenomenon appropriately and the pressures in the calculations are slightly low compared with the experimental data. Depressurization around 56 seconds is also exact in the new method.

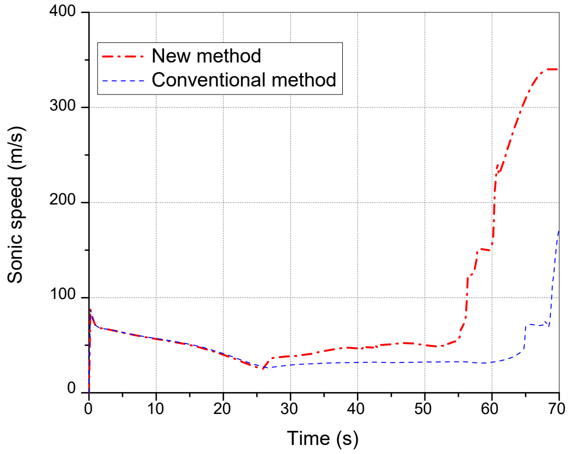


Fig. 4 comparison of sonic speed calculations

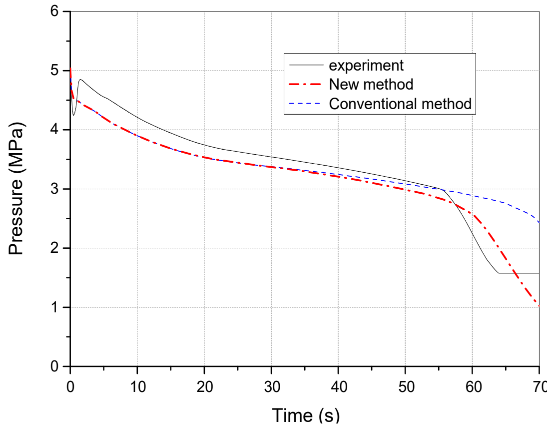


Fig. 5 Calculated and measured pressure at dome

## 5. Conclusion

The numerically direct solving method for sound speed was proposed for improving critical flow model of SPACE code. The sonic speed is obtained by solving determinant of the matrices of system of equations directly. The Marviken test 15 was used to evaluate the appropriateness of the proposed method. The SPACE code using the new method represents much better prediction compared with using conventional method in the two phase discharge region.

**ACKNOWLEDGMENT**

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