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Development of Reduced Order Model without Offline Stage

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1. Introduction

Reduced-order models (ROMs) have been developed to reduce the computational costs, in that a compact model with less dimensionality would be constructed and used for subsequent calculations. The most well-known method is a projection-based method, in which the influential subspaces are extracted and the system of equations is projected into the active subspace to reduce the dimensionality. [1,2] First, the full-order model (FOM) would be executed for collecting snapshot samples for basis identification and a reduced order model (ROM) would be constructed via a projection process of the FOM equations onto the low-dimensional subspace spanned by the reduced basis, which is called an '***offline***' stage. Second, for an '***online***' stage, the constructed ROM would be used in lieu of the FOM for subsequent calculations (especially, many query problems, e.g., design optimization and uncertainty quantification).

It is important to note that the reduced order modeling requires the multiple executions of the computationally expensive high-fidelity model at several points in the input-parameter space to extract the basis information. Therefore, for very large problems, i.e., CFD computation with a large number of meshes, the offline stage itself would require inhibitive computational cost and hinder the applicability of ROM. In this study, the reduced order modeling methods without offline stage has been suggested. Instead of constructing the basis at the offline stage once and for all and using the ROM for all subsequent calculations, the basis would be recalculated during the online stage calculations with additional FOM calculations. The methodology has been examined with two separate effect test benchmark problems using SPACE [3].

2. Reduced Order Modeling Algorithm

2.1 Conventional ROM Approach (with Offline stage)

Let a system of equations:

$$\vec{y} = \mathbf{A}\vec{x} \tag{1}$$

where,

$\vec{y} \in \mathbb{R}^n$, $\vec{x} \in \mathbb{R}^n$, $\mathbf{A} \in \mathbb{R}^{n \times n}$.

The state solution $\vec{x}$ can be calculated by solving the equation as:

$$\vec{x} = \mathbf{A}^{-1}\vec{y} \tag{2}$$

The reduced order model can be constructed by basis transformation by:

$$\vec{x} \simeq \mathbf{Q}\mathbf{Q}^T\vec{x} = \mathbf{Q}\vec{x}_r \tag{3}$$

Note that the reduced basis matrix $\mathbf{Q} \in \mathbb{R}^{n \times r}$ can be constructed by snapshot method, i.e., offline stage:

$$\mathbf{QR} = \begin{bmatrix} \vec{x}^{(1)} & \cdots & \vec{x}^{(r)} \end{bmatrix} \tag{4}$$

Then, the reduced order model of Eq. (1) can be constructed by projection:

$$\begin{aligned} \vec{y} = \mathbf{A}\vec{x} \quad &\Rightarrow \quad \vec{y} = \mathbf{A}\mathbf{Q}\mathbf{Q}^T\vec{x} \\ &\Rightarrow \quad \mathbf{Q}^T\vec{y} = \mathbf{Q}^T\mathbf{A}\mathbf{Q}\mathbf{Q}^T\vec{x} \\ &\Rightarrow \quad \vec{y}_r = \mathbf{A}_r\vec{x}_r \end{aligned} \tag{5}$$

After the ROM constructed, the $\vec{x}$ can be calculated with reduced computational cost in online stage:

$$\vec{y}_r = \mathbf{A}_r\vec{x}_r \quad \Rightarrow \quad \vec{x}_r = \mathbf{A}_r^{-1}\vec{y}_r \tag{6}$$

The state solution can be reconstructed into the original dimension by projection,

$$\vec{x} = \mathbf{Q}\vec{x}_r \tag{7}$$

It is important to note that the original matrix system of equations should be solved several times for constructing the basis.

2.2 On-the-Fly ROM Approach (without Offline Stage)

The proposed method assumes that matrix $\mathbf{A}$ and the vector $\vec{y}$ would not be changed dramatically in a very short period of time in a transient; thus, the basis function would not be varied significantly. Instead of constructing the basis at the offline stage, a small number of FOM solutions at online stage would be used for the construction of basis. Comparing the accuracy of ROM with respect to FOM, the additional basis could be added by using additional FOM solution.

For illustration, assume only one snapshot would be used for basis. Then, Eq. (5) would be a scalar product which could be computed very fast. In case that FOM and ROM are alternatively solved, the computational costs would be significantly reduced compared to the original FOM calculations.

3. Numerical Demonstration

The proposed method has been applied to thermal hydraulic system analysis code, SPACE. The numerical solution method of SPACE shown in Fig. 1 has been applied to the three field balance equations discretized spatially on a staggered grid and solved employing a semi implicit time advancement method. As shown in the figure, On-the-Fly ROM has been implemented in pressure matrix solver to compute the pressure matrix with the FOM/ROM hybrid method.

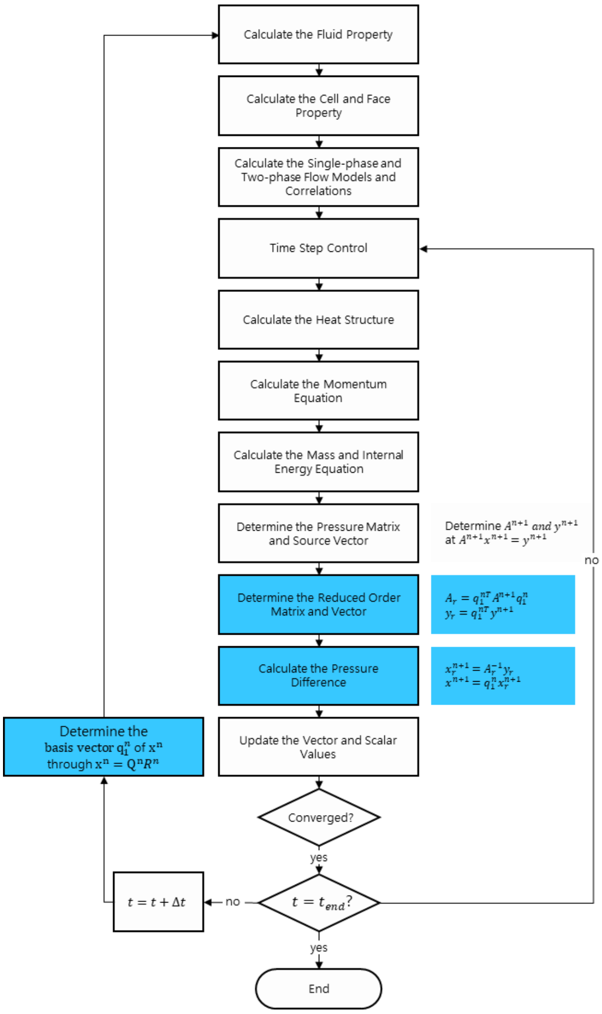

Figure 1. Algorithm for On-the-Fly ROM

3.1 MIT Pressurizer [4]

The objective of the MIT pressurizer test is to investigate the heat transfer process occurring in a pressurizer. For this test, subcooled liquid is injected into a pressurizer partially full of saturated water. The injection is terminated at 40 s. A balance between the steam condensation and compression determines pressure in the test section. Fig. 4 depicts the results FOM/ROM hybrid calculations along with the FOM stand-alone calculation. The figure shows that as the number of ROM calculations decreases, i.e., the frequency of using ROM solver decreases, the calculated pressure agrees well with the FOM solution.

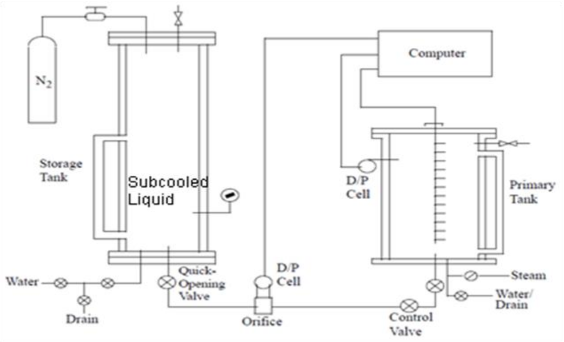

Figure 2. Schematic of Experimental Facility of MIT Pressurizer [4]

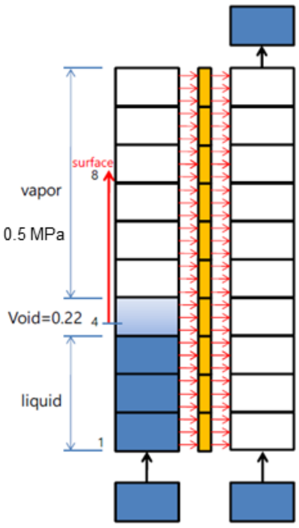

Figure 3. SPACE Modeling of MIT Pressurizer

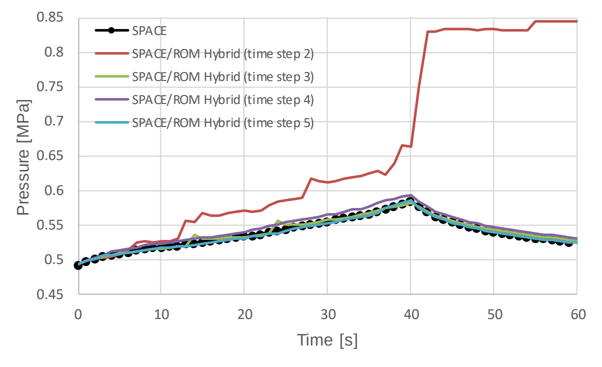

Figure 4. SPACE Calculation vs. On-the-Fly ROM

3.2 FEBA 214 [5]

The objective of FEBA program is to obtain an insight into most important heat transfer mechanisms during reflood phase of LOCA. For this test, 5x5 electrically heated rods are utilized for a number of test series. FEBA 214 test is performed under the test conditions presented in Table I utilizing rod bundle of

3.9 m heated length with full decay heat rod power. Fig. 6 shows the results FOM/ROM hybrid calculations along with the FOM stand-alone calculation. An excellent agreement between the FOM/ROM hybrid calculations and the FOM stand-alone calculation was observed.

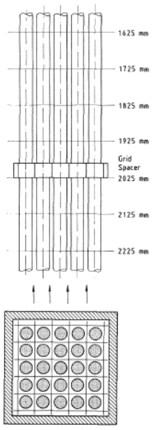

Figure 5. Experimental Facility of FEBA [5]

Table I. Test Condition

Test No.	Inlet Velocity [cm/sec]	System Pressure [bar]	Feedwater Temperature [C]		Bundle Power [kW]	
			0-30 sec	end	0 sec	Transient
214	5.8	4.1	45	37	200	120% ANS

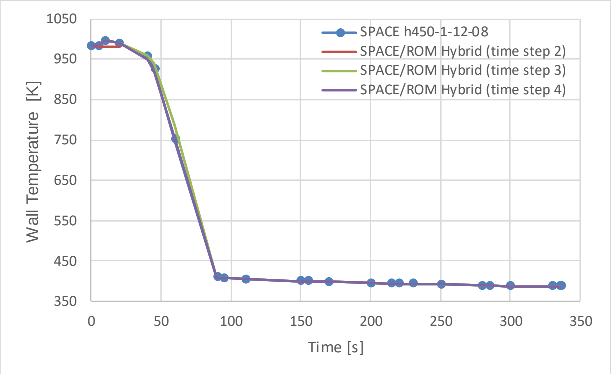

Figure 6. SPACE Calculation vs. On-the-Fly ROM

4. Conclusion

Instead of constructing the basis at the offline stage once and for all and using the ROM for all subsequent calculations, the basis was recalculated during the online stage calculations with additional FOM calculations. On-the-Fly ROM has been implemented in SPACE to calculate the pressure matrix with small number of basis and tested with two benchmark problems: MIT pressurizer and FEBA 214. The overall results present promising for application to time dependent thermal hydraulic analysis. The methodology will be further improved by incorporating an algorithm for estimating a large variation in state solutions. Therefore, the frequency to solve the FOM will be adjusted automatically without compromising the accuracy and the computational savings.

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