Success and Limitation of RANS Models for Buoyancy-Driven Flows

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1. Introduction

For nuclear thermal-hydraulic systems, free convection phenomena have been widely used to design circulation systems. In buoyancy-driven flows, if the fluid is colder at the top and warmer at the bottom in an enclosed domain, such as Rayleigh-Bénard convection, there exists potential energy due to the density difference. Turbulent natural convection is the process through which this potential energy is converted into turbulent kinetic energy. Because turbulence mixes fluids so effectively, turbulent heat transfer is much more efficient than typical conductive heat transfer.

To predict turbulent flows, Reynolds-averaged Navier-Stokes (RANS) models are commonly employed in commercial computational fluid dynamics (CFD) software for a variety of engineering applications. During the process of Reynolds averaging, the governing equations are dealt with through time-averaging, with decomposing velocity and temperature fields into their time-averaged and fluctuating components. Subsequently, the effects of turbulence are represented as the unknowns obtained through Reynolds-averaging the non-linear terms, which necessitates modeling.

In the RANS approach, turbulent natural convection is identified as the production of turbulent kinetic energy due to buoyancy, along with its effects on the anisotropies of turbulent heat flux and the Reynolds stress tensor. Previous RANS models proposed incorporating these buoyant effects on turbulence modeling, and this was the subject of active studies from the 1980s through the 2000s. Hanjalić's review [1] provides an excellent summary of the physical assumptions about buoyancy-extended RANS models and their successes. Other important references to consider are Hanjalić's book [2] and the review papers by Choi et al. [3] and Durbin [4].

Buoyancy-RANS models developed to date have been optimized through a physical analysis based on the transport equations of the second moment for fluctuating velocity and temperature. This optimization involves budget analysis using data from direct numerical simulations and term-by-term fitting. Subsequently, the conventional buoyancy-RANS models have primarily been validated for vertical natural convection and similar problems. These models are known to offer more accurate results when compared to simple eddy-diffusivity models that do not account for buoyancy [1].

While Reynolds averaging, a methodology for fluids that simplifies through time-averaging (or ensemble-averaging), is a highly effective tool for predicting turbulent flows at a low computational cost in the industrial context, the loss of flow information resulting from simplification poses an inherent limitation of the RANS modeling technique. Shedding light on this issue, Spalart's review [5] offers a comprehensive analysis of the physical assumptions and their fundamental limitations of the RANS model. However, Spalart's paper did not address issues related to buoyancy-driven flows, as it primarily focused on more general flows, such as shear or wall-bounded flows.

Existing buoyancy-RANS models have well known to be often unstable, and reliable results are not guaranteed unless the model constants are optimized and tailored to the specific problem. To delve into the specifics of this issue, the present paper adopts a perspective similar to Spalart's review, discussing the fundamental limitations of buoyancy-related RANS modeling that have been rarely considered. Here, the issues that have already been mentioned in Spalart's review, or that are not directly related to buoyancy-driven flows, are excluded from the current discussion.

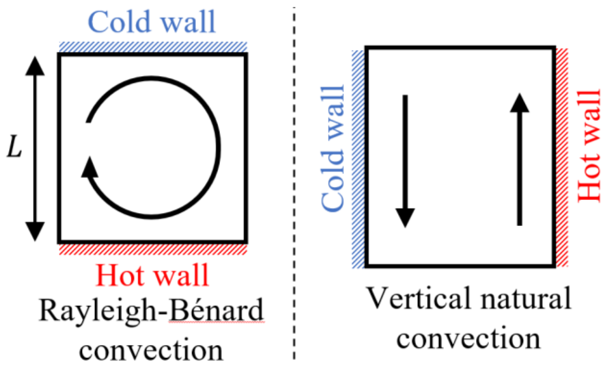

Fig. 1. Schematic of two types of natural convection: Rayleigh-Bénard convection (left) and vertical natural convection (right).

In a historical context, RANS models for buoyancy were mainly developed until the 2000s. On the other hand, as summarized by Ahlers et al. [6], the current understanding of the turbulent natural convection phenomenon has greatly enhanced due to a wealth of reliable data resulting from advancements in experimental techniques and computational power, as well as a deeper theoretical understanding. Accordingly, this paper utilizes recent knowledge about turbulent thermal convection to comprehensively discuss the fundamental principles in RANS modeling.

2. Paradox of the 'One-Point Model' for Buoyant-Driven Flows

2.1 Nature of Reynolds Stresses and Turbulent Heat Flux in Free Convection

Spalart [5] pointed out that while the concept of Reynolds stress tensor is clear in its definition, its corresponding physical phenomena are not consistent. Even for the same value of Reynolds stress, the shape of the coherent structure and its derived physical behaviors can be different. A strong example cited in Spalart's review is cylinder wakes, where the laminar lumps and turbulent regions of vortex shedding are all simplified to a single value in averaging.

To focus on buoyancy-driven flow, Rayleigh-Bénard convection will now be discussed as a basic natural convection problem. This problem involves a flow between two horizontal plates separated by a distance L, with gravity applied in the vertical direction. The boundary condition for the two walls is isothermal, with the top being cold and the bottom being warm, maintaining a temperature difference Δ. This results in a convection cell circulating throughout the domain at a speed of U and a length similar to L. Then, the turbulent kinetic energy (the trace of Reynolds stress tensor) is found to be proportional to U^2. The dissipation rate of turbulent kinetic energy (corresponding to ε in the k-ε model) is cascaded by U^3/L, where U/L represents the rotation frequency of the convective cell [6].

The steady-state heat flux between the two horizontal plates is consistent across all regions, and its value is much larger than the conductive heat transfer in the case of turbulent flow. This turbulent heat transfer can be estimated by the heat balance equation near the isothermal wall as $\alpha(\Delta/\lambda_\theta)$, where α and λ_θ denote the thermal diffusivity and thermal boundary layer thickness, respectively. The thermal boundary layer thickness of Rayleigh-Bénard convection is influenced by the circulation of a large-scale convection cell with a velocity U and a length L. Both experimentally and theoretically, it has been established that the thermal boundary layer thickness is linked to the length (L) and velocity scale (U) of the large-scale convective cell, following a relationship similar to a Blasius-type boundary layer flow, as $\lambda_\theta \sim (L/U)^{1/2}$ [6]. Consequently, the turbulent heat transfer in Rayleigh-Bénard convection is also governed by the large-scale convection cells, which are unique to this flow.

In summary, the characteristics of Reynolds stress and turbulent heat flux in Rayleigh-Bénard convection are believed to be difficult to generalize to other flows. These observations provide a fundamental explanation for the limited applicability of the conventional convection model. Even if a certain model works effectively for heat flow problems where buoyancy is not considered, it may not be guaranteed to work for natural convection problems.

2.2 Severe Limitations of Locality-Based Models in Predicting Natural Convection

Spalart [5] noted that the algebraic turbulence models developed in the 1960s to 1970s were non-local, while many of the currently used models, such as the k-ε model, are local. The locality of models implies that the algebraic or modeled transport equations being solved are represented using only local information, such as turbulent kinetic energy (k), velocity, temperature, dissipation rate (ε), and their time- or space-derivatives. Conversely, non-local information encompasses parameters like the displacement thickness of the boundary layer, wall distances, and length characteristics of the geometry, reflecting the unique aspects of each problem.

The non-local model is still employed in the XFOIL code, designed for 2D airfoil problems. This model utilizes and generates information specific to the boundary layer characteristics of that airfoil problems. On the other hand, local models like the k-ε model, widely utilized in commercial CFD codes, use local information. The advantage of using local models lies in their adaptability to a broad range of problems, including those with intricate geometries, using a single model.

Underlying the idea that locally-based modeling can lead to success is the belief that the local physical attributes of turbulence, such as turbulent kinetic energy 'k' and its dissipation rate 'ε', diffuse and dissipate consistently across various flows. However, Spalart pointed out that the actual behavior of local models relies on empirical properties and more closely resembles the successful "mimicking" of representative turbulent flows. To further illustrate Spalart's point, let's examine the k-ε model: The transfer equation for the ε equation isn't derived from statistical turbulence physics governing each term, but rather resembles a 'mimicry' of the typical flows to which turbulence models are often applied. In other words, as described in Pope's book [7], the modeled ε-equation is best understood as being entirely 'empirical'; for any given ε-equation, the model's justification arises from describing how it behaves in scenarios such as decaying turbulence, homogeneous shear flow, log-law regions, and free-stream edges.

In contrast to the 'empirical' modeling approach is the 'systematic' methodology, which formulates transport equations for the ε-equation and models diffusion terms, including respective triple correlations. However, this approach introduces additional unknowns for more complex higher-order moments, and accurately modeling the higher-order terms from which these moments stem is unlikely. (In mathematical terms, a moment signifies a time-averaged value multiplied by various powers of velocity or temperature fluctuation. For instance, turbulent heat flux represents a second moment of velocity and temperature fluctuations' product.) This is because the essence of turbulent phenomena lies in the behavior of coherent structures within three-dimensional flows; even if statistical properties of lower-order moments remain consistent, higher-order moments may differ. Solving governing equations for an nth-order moment introduces an unknown for an (n+1)th-order moment, perpetuating an infinite chain. The extent to which a model should emulate statistical properties of nth-order moments to capture the essence of turbulent flow also depends on the specific problem. Adopting additional models could enable the model to evolve by imposing more "empirical" fits to flow phenomena on the added degrees of freedom, but this undermines the advantage of RANS in efficiently predicting turbulence within time-averaged flows at a small computational cost. The straightforward and accurate approach to predicting coherent structure behavior involves directly 'resolving' it, as seen in large-eddy simulations or direct numerical simulations. Attempting to imitate these flow structures using indirect statistical behavior for higher-order moments using RANS is considered inefficient.

Most of the buoyancy-extended RANS models currently employed are also local, as evidenced by the title of Hanjalić's review paper [1], "One-point closure models for buoyancy-driven turbulent flows," where 'one-point' signifies the locality of a model. For instance, in most commercial software, buoyancy turbulence models are usually integrated by incorporating a buoyancy term into popular models like k-ε or k-ω model.

However, the validity of assuming locality for buoyancy-driven flows has been rarely discussed. As elucidated earlier, the reliability of the conventional k-ε model is established by its capability to replicate representative fluid problems, even while assuming locality. To verify the suitability of applying locality to buoyancy-related issues, it is essential to evaluate whether the assumption of locality can convincingly replicate typical buoyancy problems.

Now, as a counterexample to the modeling principle of locality in buoyancy-driven flows, homogeneous Rayleigh-Bénard convection will be introduced. In this scenario, periodic boundary conditions are imposed on the velocity in all three dimensions of the flow. In contrast, the temperature boundary condition is established to create a temperature difference (denoted as Δ) in a vertical plane located at a distance of L from the direction of gravity. Due to the homogeneous characteristic of the flow, various flow parameters such as turbulent kinetic energy, turbulent heat transfer, dissipation rate, and temperature gradient remain consistent across the entire domain. For the input parameters, the Rayleigh number (Ra) and the Prandtl number (Pr) are defined as follows:

$$\mathrm{Ra} = \frac{g\beta\Delta L^3}{\alpha\nu} \quad \text{and} \quad \mathrm{Pr} = \frac{\nu}{\alpha}.$$

Here, g, β, α, and ν represent gravitational acceleration, the expansion ratio of the Boussinesq approximation, thermal diffusivity, and kinematic viscosity, respectively. The output parameter, known as the Nusselt number (Nu), is defined by the equation:

$$\mathrm{Nu} = \frac{\overline{T'u_z'} - \alpha(\partial T/\partial z)}{\alpha\Delta/L}.$$

Here, $\overline{T'u_z'}$ denotes the turbulent heat flux in the gravitational direction (z-direction). The overline, $\overline{(\ \ )}$, denotes the Reynolds-averaging. In summary, Nu is a function of Ra and Pr [6].

In the context of homogeneous Rayleigh-Bénard convection, the dependence of Nu on Ra and Pr has been measured using the scaling relation:

$$\mathrm{Nu} \sim \mathrm{Ra}^{1/2}\mathrm{Pr}^{1/2}.$$

This scaling relation implies that the flow exists within the fully turbulent regime, meaning that its turbulent heat flux is no longer influenced by kinematic viscosity and thermal diffusivity [6]. By substituting the definitions of dimensionless numbers into the scaling relation when Nu is much larger than one, the turbulent heat flux can be expressed as:

$$\overline{T'u_z'} = g^{\frac{1}{2}}\beta^{\frac{1}{2}}(\Delta/L)^{\frac{3}{2}}L^2.$$

Here, g, β, and (Δ/L) (representing the homogeneous temperature gradient) correspond to local information that can be utilized in ordinary local models. However, L represents a non-local value signifying the total distance between the two horizontal periodic isothermal walls. It is evident that a local model inherently cannot accurately predict the turbulent heat flux, $\overline{T'u_z'}$, in the context of homogeneous Rayleigh-Bénard convection without considering the non-local variable L.

It is widely recognized that the time-averaging assumption of RANS modeling can result in a significant loss of turbulent flow information. Conversely, the fact that the principle of locality can also lead to substantial information loss is frequently overlooked, largely due to the consistent success of the local models (e.g. k-ε model) in numerous problems. However, the counterexample of homogeneous Rayleigh-Bénard convection serves as a reminder that the locality assumption inherent in modeling can render it incapable of providing a comprehensive prediction for natural convective flow.

3. Limitations in Predicting Buoyancy-Driven Flows

In this section, significant limitations of the conventional models are introduced, which can lead to a substantial loss of predictability when applying the model to arbitrary buoyancy-driven flows. These limitations are connected to the fundamental paradoxes outlined in Section 2.

3.1 Rayleigh-Bénard Convection: Predictable Only with Unstable RANS

In the review paper by Hanjalić [1], it was emphasized that none of the currently employed RANS models can accurately predict the steady-state turbulent heat transfer of Rayleigh-Bénard Convection. As an alternative approach, Hanjalić suggested utilizing unsteady RANS (URANS). Similarly, the review paper by Choi et al. also employed URANS for Rayleigh-Bénard Convection. However, there is insufficient theoretical grounding to ascertain the validity of URANS.

The process of modeling time-dependent flow is often understood in terms of the turbulent energy spectrum. In turbulent flows, it is universally observed that a large eddy breaks down into smaller-scale eddies, and these smaller eddies tend to exhibit more isotropic behavior. This phenomenon is referred to as an energy cascade, representing the transfer of energy from the large-scale motions to the smaller scales. Within the inertial subrange of turbulence scales, the rate of energy transfer equals the dissipation rate of turbulent energy. The basic principle of large-eddy simulation (LES) is applying a spatial filter within the inertial subrange and modeling the sub-filter turbulent effects based on the universality of the energy cascade.

In the energy spectrum for the turbulent kinetic energy of Rayleigh-Bénard convection, a typical buoyancy-driven flow, the inertial subrange is equally observed. The effect of buoyancy on turbulence, known as the Bolgiano-Obukhov scaling, is observed to be limited at length scales longer than the inertial subrange [8]. These observations seem to provide theoretical support that LES, which models the sub-filter effects in the inertial subrange, can still be valid in buoyancy-driven flows. On the other hand, in the case of URANS, such a theoretical justification becomes notably challenging.

3.2 Lack of Justification for ε-Equation (or ω-Equation) in Two-Equation Models for Buoyancy-Driven Flows

"While the buoyancy effects on the generation of turbulent kinetic energy are relatively well understood, the effect on its dissipation rate is less clear." This description is a direct reference from the ANSYS Fluent theoretical manual [9], where the buoyancy extension term for the k-ε model is elaborated.

First of all, the transport equation for turbulent kinetic energy (k) can be precisely derived from the Reynolds-averaged governing equations, wherein two types of production terms for turbulent kinetic energy are identified: those arising from the velocity gradient and those due to buoyancy. In the absence of buoyancy, the 'production term' for the ε-epsilon equation is modeled as the turbulence production term divided by the turbulence timescale (τ=k/ε). Currently, most used models handle the turbulence production term due to buoyancy and the turbulence generation term due to velocity gradient in the ε-equation in the same manner.

In Section 2.2, the empirical understanding of the ε-equation has been demonstrated. However, this empirical exploration of the buoyancy problem has been seldom studied, despite the completely distinct physical mechanisms behind turbulence production caused by buoyancy and velocity gradient.

For instance, in the conventional k-ε model, k- and ε -equation are given by [1,2]:

$$\frac{Dk}{Dt} = \left[\overline{u_i'u_j'}\frac{\partial U_i}{\partial x_j}\right] + [-\beta g_i \overline{T'u_i'}] + \left(\nu + \frac{\nu_t}{\sigma_k}\right)\nabla^2 k,$$

$$\frac{D\varepsilon}{Dt} = C_1\frac{\varepsilon}{k}\left[\overline{u_i'u_j'}\frac{\partial U_i}{\partial x_j}\right] + \mathrm{C_g}\frac{\varepsilon}{k}[-\beta g_i \overline{T'u_i'}] - C_2\frac{\varepsilon^2}{k} + \left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\nabla^2 \varepsilon.$$

Here, the terms enclosed in the first and second square brackets signify the turbulent kinetic energy productions due to velocity gradients and buoyancy, respectively.

It is known that the suitable value of $\mathrm{C_g}$ differs depending on the type of flow. Rodi [10] pointed out that $\mathrm{C_g}$ is close to 1 in vertical natural convection and close to 0 in a horizontal boundary layer. In this context, Henkes et al. [11] proposed the damping function $\mathrm{C_g} = \tanh|v/u|$, where v and u represent the mean flow velocity parallel to the gravitational vector and the mean flow velocity perpendicular to the gravitational vector, respectively. However, it is obvious that this model can not account for situations where the mean flow is zero. Furthermore, the suitable values of $\mathrm{C_g}$ reported in the literature are all problem-specific. Even for the same geometry, consistent results are not guaranteed when the Rayleigh or Prandtl number changes.

3.3 Lack of Guaranteed Rayleigh-Prandtl Number Scaling for Predicted RANS Results

For both Rayleigh-Bénard and vertical natural convection, two dimensionless numbers—the Rayleigh number (Ra) and the Prandtl number (Pr)—determine the flow characteristics of the problem. Additionally, to differentiate between the natural and forced convection regimes, the Richardson number (Ri) is defined to express the ratio of buoyancy and the rate of shear.

RANS models developed for solving buoyancy-driven flows before the 2000s were primarily validated to ensure accurate fitting of heat transfer and velocity profiles in specific vertical natural convection problems. This process involved computing correct budget equations based on direct numerical simulation data for a given flow and then verifying the a priori accuracy of each model term.

However, a RANS model is a complex system of interconnected nonlinear models. Consequently, the modeling approach means that errors in any of the turbulence variables can lead to overall incorrect results. For example, no matter how accurate the algebraic model for turbulent heat transfer and Reynolds stress is, if the epsilon distribution derived from the epsilon equation is unrealistic, then Reynolds stress and turbulent heat transfer, which include the turbulent component of epsilon, will also produce inaccurate results. From this perspective, adding intricate functions to the sub-model to match a detailed heat transfer profile and verifying it with an accurate a priori test doesn't significantly impact the model's overall accuracy when applied to arbitrary flows.

Before delving into the detailed distribution of heat transfer, the first thing to consider is how the average turbulent heat transfer varies for a given boundary condition. Due to the scarcity of available experimental data and the complexity of the model, previous RANS models for buoyancy-driven flows have rarely been validated how the Nusselt number changes across a wide range of Ra-Pr-Ri parameters. Before attempting to predict detailed turbulence variable profiles, it is believed that further dimensional analysis is required to understand how average turbulent heat transfer varies over parameter ranges.

4. New Modification for the ε-Equation Based on Rayleigh-Prandtl-Number Scaling of Thermal Convection

The content of this section is being prepared for publication in the Journal of Fluid Mechanics under the title "A Reynolds-averaged Navier–Stokes Closure Model Based on Rayleigh–Prandtl-Number Scaling of Natural Convection," and the same content will be presented at the American Physical Society's 76th Annual Meeting of the Division of Fluid Dynamics.

3.1 Rayleigh-Prandtl-Number Scaling Theory of Thermal Convection

A fundamental contradiction in RANS model research is that as a model is enhanced to encompass new flow phenomena, its complexity may increase while its generality decreases. This is why many of the RANS models still in use today were developed in the 1980s and 1990s.

So, are there no fresh ideas that can significantly enhance current models without sacrificing their 'simplicity'? If we broaden our horizons beyond RANS modeling research, numerous experimental and theoretical advancements have taken place in natural convective turbulence from the 2000s to the present. One of these is the scaling theory in thermal convection by Grossmann and Lohse, highlighted in the review paper by Ahlers et al. [6]. This theory describes the variation of scaling laws of Nusselt and Reynolds numbers across a wide Ra and Pr parameter space of Rayleigh-Bénard convection.

The Ra-Pr scaling theory has successfully described the power-law tendency of $Nu{\sim}Ra^{b}Pr^{c}$, which depends on the parameter range in all phase spaces for Ra and Pr numbers of Rayleigh-Bennard convection. In the Ra-Pr scaling theory, eight theoretical scale-regimes are distinguished, depending on whether thermal and kinetic dissipation rates dominate in the boundary or bulk regions, as well as whether the thermal or kinetic boundary layer is thick. For example, assuming that thermal and kinetic dissipation rates are dominant in the bulk region and that the kinetic boundary layer thickness is significant, a theoretical power law of $Nu{\sim}Ra^{1/2}Pr^{1/2}$ can be estimated.

Furthermore, the Ra-Pr scaling theory has been applied to understand the overall power law of Ra-Pr number scaling for various other thermal convection flows, such as vertical natural convection, volumetrically heated convection, homogeneous Rayleigh-Bénard convection, and turbulent electroconvection.

These theoretical studies are appealing because they explain a wide range of experimental data using a simple combination of scaling laws. The fact that a simpler scaling theory, compared to the RANS model, can effectively predict natural convective heat transfer trends gives us hope that proposing new models based on this theoretical knowledge might be possible.

3.2 Analysis of the Conventional Two-Equation Model Based on Dynamical Systems Theory

New modeling work begins with a detailed analysis of how conventional k-ε models specifically predict turbulent heat transfer in Rayleigh-Bénard convection. In the present analysis based on the dynamical systems theory, it is found that conventional buoyancy-RANS models incorrectly predict turbulent heat transfer and turbulent heat fluxes to grow infinitely and continuously, rather than converging to a finite steady-state value, for Rayleigh number conditions above a certain threshold in Rayleigh-Bénard convection. Through the present analysis, it is found that because conventional models treat the buoyancy term in the ε-equation the same way as the mechanical term, they incorrectly predict turbulent heat transfer between two horizontal plates with a constant temperature difference to grow exponentially over time.

The reason for this incorrect prediction is due to the locality of the model mentioned in Section 2.2. If a conventional model is not provided with the length information (L) of a large-scale convection cell in natural convection, it cannot predict a finite specific value of heat transfer. In other words, the output of the model system cannot converge to a finite specific non-zero value.

In summary, conventional models can inaccurately predict the turbulent heat flux to increase infinitely over time under high Ra conditions in Rayleigh-Bénard convection. This suggests that conventional models are also at risk of producing incorrect results in various natural convection problems involving significant buoyancy-induced turbulent kinetic energy generation.

3.3 Modification for the ε-Equation

The theoretical studies in various natural convection problems, to which the Ra-Pr scaling theory has been applied as described in Section 3.1, all employ a combination of the same scaling laws to comprehend the heat transfer phenomenon.

To apply this universality to the RANS model for natural convective flows, a new model is devised by incorporating the length scale, L, of the large-scale coherent flow that might exist in each problem. The objective of this modeling is to replicate the power law of natural convection as predicted by the Ra-Pr scaling theory.

As mentioned in Section 3.2., the conventional ε-equation is given by:

$$\frac{D\varepsilon}{Dt} = C_1 \frac{\varepsilon}{k}\left[\overline{u_i' u_j'} \frac{\partial U_i}{\partial x_j}\right] + \mathrm{C_g} \frac{\varepsilon}{k}\left[-\beta g_i \overline{T' u_i'}\right] - C_2 \frac{\varepsilon^2}{k} + \left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\nabla^2 \varepsilon.$$

The new model modifies only one term associated with buoyancy in the ε-equation:

$$\frac{D\varepsilon}{Dt} = C_1 \frac{\varepsilon}{k}\left[\overline{u_i' u_j'} \frac{\partial U_i}{\partial x_j}\right] + \mathrm{C_g'} \varepsilon^{\gamma_1} \left(\frac{\varepsilon}{k} max\left[-\hat{g}_i \overline{T' u_i'}, 0\right]\right)^{\gamma_2} - C_2 \frac{\varepsilon^2}{k} + \left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\nabla^2 \varepsilon$$

In the above equation, γ_1 and γ_2 are dimensionless model constants that determine the stability of the model. $\mathrm{C_g'}$ is a nondimensional model constant that incorporates the characteristic length L and influences the overall magnitude of the Nu value predicted by the model.

In the case of homogeneous Rayleigh-Benard convection, the new model can exactly predict the theoretical power law of $Nu \sim Nu_0 Ra^{1/2} Pr^{1/2}$ as a steady-state solution. Here, $\mathrm{C_g'}$ is exactly derived as the function of Nu_0, γ_1, and γ_2.

The newly proposed model is capable of reproducing the Nu dependence across a wide range of Ra and Pr parameters in Rayleigh-Bénard convection, closely approximating actual experimental results. Specifically, values of $\gamma_1 = -1.5$ and $\gamma_2 = 4$ are chosen to align with experimental observations of $\mathrm{Nu} \sim \mathrm{Ra}^{0.3}$ over the range of $10^7 \leq Ra \leq 10^{12}$.

4. Conclusions

In this paper, the fundamental limitations of RANS models for buoyancy problems and the resulting errors have been discussed. It's important to note that the purpose of this paper is not to argue that these errors render conventional models worthless, but rather to clarify the scope of problems for which they can be used.

For instance, problems resembling vertical natural convection in channels are the primary focus of most conventional models. Predictions for such problems can reasonably be expected to achieve some level of success using current buoyancy models. However, flows dominated by laminar-like convection cells with nearly negligible mean velocity fields, such as Rayleigh-Bénard convection, cannot be accurately predicted by conventional RANS models in steady-state manners.

It is believed that the criteria for identifying severe limitations also indicate "which problems to prioritize" in modeling research. Problems currently labeled as severe limitations make it impossible to guarantee model convergence and a unique solution before determining its accuracy. On the other hand, methods seeking to slightly improve an conventional model through a priori validation do not significantly impact the applicability of the model. Thus, addressing severe limitations should be a primary concern to ensure the model's general usability, while refining model accuracy should be tackled in subsequent steps.

Furthermore, as a preliminary step towards addressing the universal buoyancy issue, this study has developed a new model capable of reproducing the theoretical scaling power law of Rayleigh-Bénard convection across a broad range of Ra and Pr parameters. Drawing from the principles of the Ra-Pr scaling theory, we anticipate that these modeling principles can be similarly applied to predict the behavior of most other natural convective flows. This represents an essential subject for future research.

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