# Dynamic Modeling of Free Piston Stirling Generator for Micro Nuclear Reactors

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### 1. Introduction

In recent times, considerable attention is being directed towards the utilization of downsized, modularized, and enhanced safety nuclear energy [1]. Micro-reactors, notably developed and employed globally for applications including space exploration, remote power supply, and military deployments, have already substantiated their viability [2].

Power conversion systems that generate electricity from energy sources play a key role in determining the efficiency of microreactors. Several power conversion systems, including Stirling engines, thermoacoustic, Brayton cycles, thermoelectric, Rankine cycles, and etc. have been proposed. Among these, Stirling generators, particularly free-piston Stirling generators, emerge as a suitable power conversion system considering their high efficiency, technology readiness level, and stability for micro-reactors [3].

The Free Piston Stirling Generator (FPSG) constitutes a complex system comprising thermal, mechanical, fluidic, magnetic, and electrical components. Despite the growing interest and considerable in-depth research undertaken in each of these specific domains, comprehensive model investigations are limited. In this study, a system-level dynamic model is implemented and evaluated to provide a comprehensive understanding of the FPSG system, which hold promising integration potential with micro-reactors [4].

### 2. Free Piston Stirling Generator (FPSG)

The free piston Stirling generator, illustrated in Fig 1, is a system where mechanical elements like power piston, displacer piston, and flywheel are eliminated from the fundamental Stirling configuration.



This generator operates through four distinct processes: isothermal expansion, isochoric cooling, isothermal compression, and isochoric heating. A graphical representation of these processes is provided in Fig 2, illustrating the implementation of these stages through a phase difference between the displacer piston and the power piston [5].



Fig 2. P-V diagram of Stirling Cycle

The configuration of the power piston and generator assembly is composed of a power piston, magnets attached to the power piston, and wire coils surrounding these magnets, as illustrated in Fig 1. Hence, the magnets attached to the power piston induce electromotive force and generate electricity by oscillating within the wire coils. In this process, the magnets experience a Lorentz force as a counteractive force by the induced electromotive force, and this force is determined by the current value calculated by the repetitive motion speed of the power piston. The electromotive force and the current can be obtained from the alternator and load circuit as shown in Fig 3. The piston mechanical equations and electromotive force and current equations of the FPSG are as follows [6], [7]:



model

The equation of the displacer piston is:

$$M_d \ddot{x_d} + c_d \dot{x_d} + K_d x_d = F_d \tag{1}$$

where  $M_d$  is the displacer piston mass.  $x_d$  is the displacer piston displacement.  $c_d$  is the displacer piston damping coefficient.  $K_d$  is the spring constant.  $F_d$  is the

pressure force between the expansion and compression cell.

The equations of the power piston can be obtained:

$$M_p \ddot{x_p} + c_p \dot{x_p} + K_p x_p = F_p + F_e \tag{2}$$

$$F_e = N \frac{d\varphi}{dx_p} \eta_{mag} I_{alt} = BLI_{alt} = K_i I_{alt} \qquad (3)$$

where  $M_p$  is the power piston mass (kg).  $x_p$  is the power piston displacement (m).  $c_p$  is the power piston damping coefficient  $(N \cdot s/m)$ .  $K_p$  is the spring constant (N/m).  $F_p$  is the pressure force between the compression and bounce cell. B is the magnetic induction intensity of the linear generator (T). L is the coil length (m). N is number of turns of the generator winding.  $\phi$  is the magnetic flux (Wb).  $\eta_{mag}$  is generator magnetic efficiency.  $I_{alt}$  is generator current (A).  $K_i$ : alternator current electromagnetic force constant (N/A).

The coils are considered strongly coupled to the magnets, so the electromotive force  $v_{emf}$  and the current  $I_{alt}$  can be calculated as follow:

$$v_{emf} = K_e \cdot \dot{x_p}$$
, where  $K_e = N \frac{d\phi}{dx_p}$  (4)

$$v_{emf} = v_{R_{alt}} + v_{L_{alt}} + v_{C_t} + v_{R_{load}}$$
(5)

$$\frac{dI_{alt}}{dt} = \frac{K_e}{L_{alt}} \dot{x}_p - \frac{R_{alt} + R_{load}}{L_{alt}} I_{alt} - \frac{1}{L_{alt}} v_{C_t}$$
(6)

$$\frac{dv_{C_t}}{dt} = \frac{1}{C_t} I_{alt} \tag{7}$$

where  $K_e$  is the alternator constant  $(V \cdot s/m)$ .  $L_{alt}$  is the generator inductance (H).  $R_{alt}$  is the generator resistance ( $\Omega$ ).  $R_{load}$  is the external load resistance ( $\Omega$ ).  $C_t$  is the tuning capacitance ( $\mu$ F).

### 3. Dynamic modeling of Free Piston Stirling Generator

In this article the dynamic modeling of FPSG is performed using the AMESim (Advanced Modeling Environment for Simulation) program, a commercial simulation tool developed by SIEMENS, as shown in Fig 4. AMESim is multi-domain system-level simulation software used for modeling and analyzing complex engineering systems. It has numerous components that contain equations which can solve engineering problems. Connecting those components and making diverse combination of them, it can design diverse engineering models and simulate various kinds of problems in a short time. Table I shows the main design parameters of the modeled free piston Stirling generator in this paper.

## 3.1 Modeling of Free Piston Stirling Engine (FPSE)

For thermal and fluidic modeling of working fluid, QSFM (Quasi Steady Flow Model) modified by Urieli is used, which accounts for the piston motion caused by the expansion and compression of working fluid by heating and cooling [8].



Fig 4. FPSG modeling with AMESim

Table I. Main o	design	parameters	of the	modeled	FPSG
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Parameter	Value	
Туре	γ	
Heater temperature, K	900	
Cooler temperature, K	300	
Mean effective pressure MPa	15	
Working gas	Helium	
Working Frequency, Hz	8	
Regenerator length, cm	10	
Regenerator diameter, cm	2.566	
Regenerator heat exchange area, m <sup>2</sup>	0.2	
Displacer mass, kg	2	
Displacer diameter, cm	5	
Power piston mass, kg	1	
Power piston diameter, cm	5	

As shown in Fig 5, the AMESim components are constructed to solve QSFM, equation (1), and equation (2) (except the term  $F_e$ ), for modeling mechanical motion of pistons by dividing the space inside the FPSG system into four cells (expansion & heater cell, regenerator cell, compression & cooler cell, and bounce cell) to calculate the energy transfer of the working fluid in each cell. The results are as shown in Fig 6.

The movement of the displacer piston enables the working fluid to reciprocate through the heater & expansion cell, regenerator, cooler & compression cell, effectively transferring thermal energy to the power piston. The pressure changes in the working fluid due to this thermal energy push the power piston into the bounce cell. As the power piston acts, the pressure inside the bounce cell increases, functioning like a spring and pushing the power piston out again. This results in undamped repetitive motion of the power piston. The displacer and power pistons have repetitive motion, being not damped, of amplitude of 4cm and approximately 1.299cm respectively.

Especially, for the implementation of the isochoric process, which is the key to the Stirling cycle, the power piston is operated by the displacer piston with a phase difference as shown in Fig 7.



Fig 5. Free Piston Stirling Engine (FPSE) Model





Fig 7. Phase difference between displacer and power piston

A regenerator component, being also solved by the QSFM, is added to original FPSE model as shown in the Fig 8 to evaluate whether the efficiency of the system is improved.

The regenerator stores thermal energy at an intermediate temperature between the heater and cooler temperatures. As a result, thermal energy is more effectively transferred between the heat source and the heat sink when moving from the heat source to the heat sink and vice versa, compared to a scenario without a regenerator. This allows for better temperature separation between the two sides.

In Fig 9, an enhanced efficiency of approximately 26.03% is observed due to the addition of a regenerator, attributed to an increase in the amplitude of the power piston from approximately 1.299cm to 1.637cm.



Fig 8. FPSE model with regenerator



Fig 9. Amplitude of displacer and power piston with regenerator

### 3.2 Magnetic and Electrical Modeling of Generator

Once the expansion and compression of the working fluid and the resulting piston motion, have been modeled, the next step is to model the damping of the power piston caused by the counteracting force induced by the generator. This modeling is necessary to calculate the induced electromotive force and current in the generator, which can be used to calculate the electrical energy generated within the generator.

To implement this, the generator components were configured as shown in Fig 10 to solve the equation (3)  $\sim$  (7), and finally the equation (2). As a result, the motion of the displacer and power piston was effectively damped, as demonstrated by the graphs in Figs 11 and 12. The amplitude of the displacer and power pistons is damped into approximately 1.007cm from 4cm and 0.462cm from 1.637cm, respectively.

As shown in Figure 12, the velocity and amplitude of the power piston are reduced by Lorenz force, which leads to a decrease in pressure in each cell. This results in a slight displacement of the oscillation center in the direction of zero displacement.



Fig 10. Generator Model for FPSG System



3.3 Integrated modeling without start-up condition

The models simulated above are designed to facilitate the easy observation of thermal energy and piston motion behaviors by calculating steady-state conditions in advance. So, these models involve initiating displacer piston with a steady-state reference motion speed as the starting condition. Figs 13 and 14 represent the modeled behaviors of the displacer and power piston, respectively, over time after applying only thermal energy to the heater without any initial start-up conditions. As time progresses, it can be observed that the applied thermal energy gradually increases the amplitude, eventually reaching a steady-state condition.





Fig 14. Amplitude of power piston without start-up condition

### 4. Summary

This study develops a dynamic simulation model for FPSG system and conducts preliminary qualitative analysis. The thermal, fluidic, mechanical, magnetic, and electrical components of the FPSG are individually modeled using the AMESim software and evaluated through qualitative assessments. Subsequently, these components are integrated into a unified system, and successful achievement of the steady-state condition without the need for start-up conditions is qualitatively confirmed. Further research will focus on enhancing the analysis quantitatively and integrating the model with transient analysis models of micro-reactors to advance its practical applicability.

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