Application of Anderson Acceleration on Neutronics/Thermo-Fluid Coupled Analysis for a Block-type VHTR

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1. Introduction

For a very high temperature gas-cooled reactor (VHTR) core, it is advisable to perform coupled neutronics and thermo-fluid analysis because the power and temperature inside the core vary significantly depending on the location compared to other reactor types. A neutronics code CAPP [1] developed by Korea Atomic Energy Research Institute (KAERI) has a simple thermo-fluid analysis module and can perform coupled neutronics/thermo-fluid analysis by itself. However, sometimes the convergence of the coupled calculation is slow or the calculation does not converge. This is because the simple alternating calculation method between neutronics and thermo-fluid analysis does not guarantee convergence.

Among several previous studies to solve this convergence problem, Anderson acceleration [2] has attracted attention in recent years. Compared to other methods, it is relatively easy to apply and has shown effectiveness in simple problems and light water reactor core analysis [3,4]. In this study, Anderson acceleration has been implemented in the CAPP code. This improvement is tested if it resolves the convergence problem in reactor analysis for VHTR.

2. Methods and Results

2.1 Neutronics/Thermo-Fluid Coupled Analysis in CAPP

The CAPP code is a code developed to analyze the physical phenomena and perform the core design of the block-type VHTR. CAPP uses the finite element method for the multi-group neutron diffusion equation and obtains the solution of the neutron distribution in the core. To obtain the temperature of the helium coolant, a simple energy balance is considered, and to obtain the temperature of the nuclear fuel, moderator, and reflector, 2D and 3D heat conduction problems are solved by the finite volume method. The detailed methodology can be found in the reference [5]. The coupled calculation of CAPP performs neutronics and thermo-fluid analysis alternately, and the results of each are input to the other. Referring to reference [4], this process can be symbolized as follows.

$$\begin{aligned} T^{(k+1)} &= g_1\left(P^{(k)}\right) \\ P^{(k+1)} &= g_2\left(T^{(k+1)}\right) \end{aligned} \tag{1}$$

where k is the index of the iteration step, T is the temperature variable, and P is the power variable. First, the temperature calculation module outputs the temperature by receiving the power as input (function g_1). The neutronics calculation module outputs the power by receiving this temperature as input (function g_2). This power is sent back to the input of the temperature calculation module. This process is repeated until the power and temperature converge. When the function composition in Eq. (1) is performed, a composite function is obtained as follows:

$$T^{(k+1)} = g_1\left(g_2\left(T^{(k)}\right)\right) = g(T^{(k)}) \tag{2}$$

This is in the form of a fixed-point iteration. The fixed-point iteration does not always guarantee convergence, so it can be confirmed that some cases converge slowly or oscillate. The simplest way to resolve this is to introduce a relaxation factor as follows:

$$T^{(k+1)} = \beta g\left(T^{(k)}\right) + (1-\beta)T^{(k)} \tag{3}$$

However, if an appropriate relaxation factor is not chosen, it may interfere with convergence.

2.2 Anderson Acceleration

Anderson acceleration was originally developed to accelerate the iterative method for solving nonlinear equations [2]. Recently, it has been attracting attention again as the usefulness of it in the field of neutronics has been recognized [3,4]. The algorithm of Anderson acceleration is as follows:

Algorithm 1: *Anderson Acceleration* (x_0, g, m)
Compute $x^{(1)} = g(x^{(0)})$.
For $k = 1, 2, \ldots,$ do
  Set $m_k = \min(m, k)$.
  Compute $g(x^{(k)})$.
  Set $f^{(k)} = g\left(x^{(k)}\right) - x^{(k)}$.
  Set $F^{(k)} = [f^{(k-m_k)} \quad \cdots \quad f^{(k)}]$.
  Compute $\alpha^{(k)} = \left[\alpha_0^{(k)} \quad \cdots \quad \alpha_{m_k}^{(k)}\right]^T$
    where $\alpha^{(k)} = argmin_\alpha \left\|F^{(k)}\alpha\right\|_2$ subject to $\sum_{i=0}^{m_k} \alpha_i^{(k)} = 1$.
  Set $x^{(k+1)} = \sum_{i=0}^{m_k} \alpha_i^{(k)} g\left(x^{(k-m_k+i)}\right)$.
End for

This is a general acceleration algorithm for fixed-point iteration and has high versatility. Here, at each step, a minimization problem under constraints must be solved. This can be transformed into an unconstrained least-square problem by a simple transformation. Considering that the neutronics/thermo-fluid coupled analysis converges quickly in many cases, and that the case of *m* =1 is also effective for the light water reactor problems [4], the algorithm can be simplified as follows.

Algorithm 1-1: *Simplified Anderson Acceleration* $(x_0, g, m = 1)$
Compute $x^{(1)} = g(x^{(0)})$.
For $k = 1, 2, \ldots$, do
  Compute $g(x^{(k)})$.
  Set $f^{(k)} = g\left(x^{(k)}\right) - x^{(k)}$.
  Set $\Delta f^{(k)} = f^{(k)} - f^{(k-1)}$.
  Set $\alpha^{(k)} = \frac{(\Delta f^{(k)})^T f^{(k)}}{(\Delta f^{(k)})^T \Delta f^{(k)}}$.
  Set $x^{(k+1)} = (1-\alpha) g\left(x^{(k)}\right) + \alpha g(x^{(k-1)})$.
End for

This algorithm only needs to store the information calculated from the previous step because *m*=1. Also, the solution of the minimization problem is expressed in a simple formula. The new variable update is very similar to applying the relaxation factor. However, this algorithm is better than the simple relaxation method because it automatically determines α in the direction of minimizing the residual vector.

2.3 Numerical Results

To verify the convergence performance of the coupled calculation with Anderson acceleration, a core problem that is difficult to converge with the conventional method is analyzed. The problem presented in this paper is the VHTR-350 core problem, and the geometric structure of the core is shown in Figure 1. VHTR-350 is a block-type high-temperature gas reactor with a core outlet coolant temperature of 950 ℃ and a thermal output of 350 MWth. A detailed description of VHTR-350 can be found in reference [6].

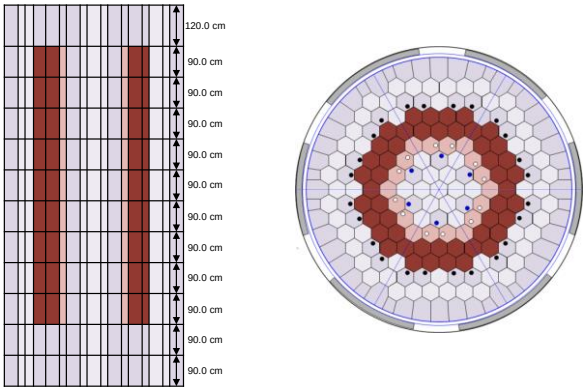

Fig. 1. Core configuration of VHTR-350 [6].

VHTR-350 adjusts the criticality by using 24 control rods at the periphery. To find the insertion depth of the control rods that achieve criticality, CAPP repeated the steady-state calculation by changing the depth (corresponding to the search for the critical boron concentration in light water reactors). However, when all the peripheral control rods were inserted 756 cm from the top of the problem, the conventional coupled analysis failed to converge. To resolve this, coupled calculations with relaxation factor and Anderson acceleration were performed and compared. Here, the convergence was achieved when both the maximum power difference and the maximum temperature difference with the previous iteration were satisfied with 10^{-4}. Reference is obtained by setting the error criteria of the calculation to 10^{-7} with 0.5 relaxation factor.

Table I compares the effective multiplication factors obtained using each method. For the conventional coupled calculation without relaxation factor, the effective multiplication factor could not be known because it did not converge. It was shown that the cases of using 0.5 and 0.3 for the relaxation method (0.8 did not converge) converged, and they showed a difference of several pcm from the reference. Finally, it is confirmed that the convergence using Anderson acceleration, and the difference from the reference was also the smallest.

Table I: Multiplication Factor Results for VHTR-350 Core Problem.

Methods	k_{eff}	Difference (pcm)
Reference	0.95652	--
No relaxation	--	--
Relaxation (β=0.3)	0.95660	8
Relaxation (β=0.5)	0.95656	4
Anderson acceleration	0.95651	-1

Figure 2 shows the convergence performance for each convergence strategy. The comparison criterion is the maximum temperature difference from the previous iteration. In the conventional coupled calculation without relaxation factor, the maximum temperature difference oscillates and does not converge even as the iteration progresses. When using relaxation factor, it was shown that the error continued to decrease at a constant rate. However, the problem remains that the convergence rate varies depending on the relaxation factor value and the appropriate relaxation factor cannot be known in advance. Finally, when using Anderson acceleration, the error decrease rate changes as the alpha value changes, but it can be confirmed that the convergence speed is faster than using relaxation factor.

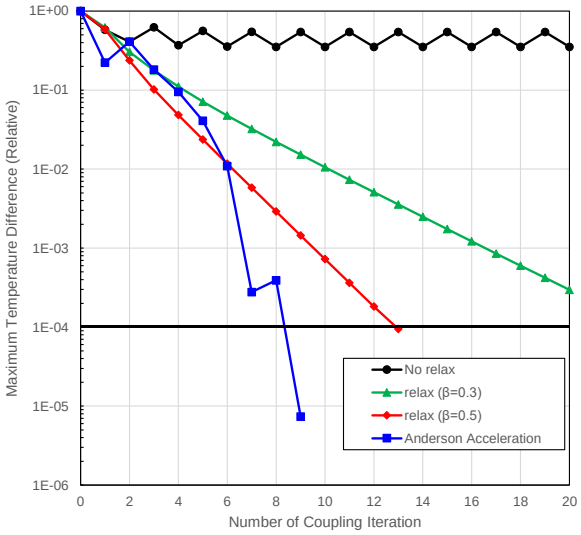

Fig. 2. Maximum temperature difference for various iterative methods (control rods were inserted 756 cm from the top of the problem).

The methods were also compared for the case where conventional fixed-point iteration can also converge. Figure 3 shows the convergence performance of each method for the case where the outer control rods are inserted 60 cm into the VHTR-350 core. It can be seen that the fixed-point iteration, which oscillated in the previous case, converges well this time, while the relaxation method converges more slowly in this case. The Anderson acceleration converges slightly faster than the fixed-point iteration. In a well-converged problem like this, the Anderson acceleration seems to be more appropriate, as the relaxation method actually worsens the convergence.

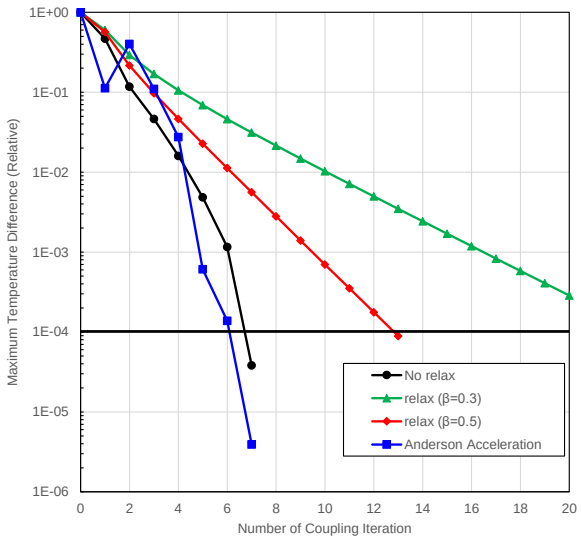

Fig. 3. Maximum temperature difference for various iterative methods (control rods were inserted 60 cm from the top of the problem).

3. Conclusions

In this study, Anderson acceleration, which has been effectively applied to fixed point iteration in various fields, especially the neutronics/thermal-hydraulic coupled analysis of light water reactors, has been implemented to the neutronics/thermo-fluid coupled analysis of VHTR. It was verified that this method solved the problem that was difficult to converge with the conventional method. While the relaxation method to solve the problem that does not converge normally has a problem that the appropriate relaxation factor should be determined in advance based on the user's experience, the Anderson acceleration method seems to have more advantages because it automatically determines the factor to reduce the error by the formula.

This study started to solve the problem of difficulty in convergence during the critical control rod search of VHTR core. It would be a further study to supplement the research by performing an analysis of the cost/effectiveness depending on the size of m and whether Anderson acceleration is effective for more divergent cases. In addition, the application of it to other iterative calculations such as critical control rod position search methods can be another future work.

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