Application of Anderson Acceleration on Neutronics/Thermo-Fluid Coupled Analysis for a Block-type VHTR

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Introduction

- It is advisable to perform a neutronics/thermo-fluid coupled analysis for a VHTR core simulation (e.g. CAPP [1]).
- However, sometimes the convergence of the coupled calculation is slow or the calculation does not converge*.

*Because the simple alternating iteration does not guarantee convergence.

- Some previous studies showed that the Anderson acceleration [2] resolved the convergence problem in light water reactor core analysis [3].

In this study,

- Anderson acceleration has been implemented in a VHTR reactor physics analysis code CAPP.
- This improvement is tested if it resolves the convergence problem in reactor analysis for a block-type VHTR core.

Coupled Analysis in CAPP

- Neutronics (*P*): A multi-group neutron diffusion equation is solved by a finite element method.
- Thermo-fluid analysis (*T*)
  - Coolant temperature: solving the energy-balance equation.
  - Fuel/moderator temperatures: solving the conduction equation.
- Alternating iteration process (symbolic eq.) for coupled analysis:

$$T^{(k+1)} = g_1\left(P^{(k)}\right)$$
$$P^{(k+1)} = g_2\left(T^{(k+1)}\right)$$

- Fixed point iteration form:

$$T^{(k+1)} = g_1\left(g_2\left(T^{(k)}\right)\right) = g\left(T^{(k)}\right)$$

- Simple relaxation method to resolve the convergence issue:

$$T^{(k+1)} = \beta g\left(T^{(k)}\right) + (1-\beta)T^{(k)}$$

- However, if an appropriate relaxation factor is not chosen, it may interfere with convergence.

Simplified Anderson Acceleration

- Anderson acceleration is a general acceleration algorithm for fixed-point iteration and has high versatility.
- Simplified Anderson Acceleration is the case of m=1:

Algorithm: *Simplified Anderson Acceleration* $(x_0, g, m=1)$
Compute $x^{(1)} = g(x^{(0)})$.
For $k = 1, 2, \ldots$, do
  Compute $g(x^{(k)})$.
  Set $f^{(k)} = g\left(x^{(k)}\right) - x^{(k)}$.
  Set $\Delta f^{(k)} = f^{(k)} - f^{(k-1)}$.
  Set $\alpha^{(k)} = \frac{(\Delta f^{(k)})^T f^{(k)}}{(\Delta f^{(k)})^T \Delta f^{(k)}}$.
  Set $x^{(k+1)} = (1-\alpha)g\left(x^{(k)}\right) + \alpha g\left(x^{(k-1)}\right)$.
End for

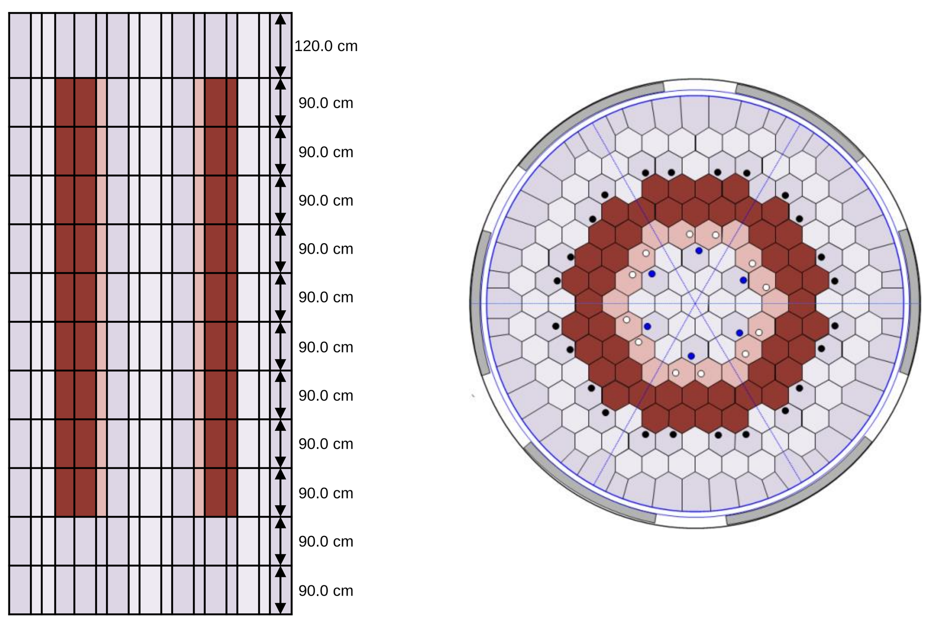
- This algorithm only needs to store the information calculated from the previous step.
- The new variable update is very similar to applying the relaxation factor. However, this automatically determines α.

Numerical Results

Test Problem: VHTR-350 Core [4]

- Core thermal power: 350 MWth
- Inlet/outlet coolant temperature: 490/950℃
- VHTR-350 adjusts the criticality by using 24 CRs at the periphery.

<Design parameters of VHTR-350>



<Core configuration of VHTR-350>

Parameters	Values
Coolant pressure (MPa)	7
Active core height (cm)	810
Number of fuel columns	66
Number of axial fuel block layers	9
Top/bottom reflector height (cm)	120/180
Fuel block height (cm)	90
Block pitch (cm)	36.2
Nuclear fuel material	UO_2
U-235 enrichment (w/o)	≤15.5
Burnable absorber material	B_4C
TRISO kernel diameter (μm)	500
Core power density (W/cm^3)	5.83

Case 1: CRs insertion depth: 756 cm

- The conventional coupled analysis failed to converge.

<k_{eff} for VHTR-350 core problem: Case 1>

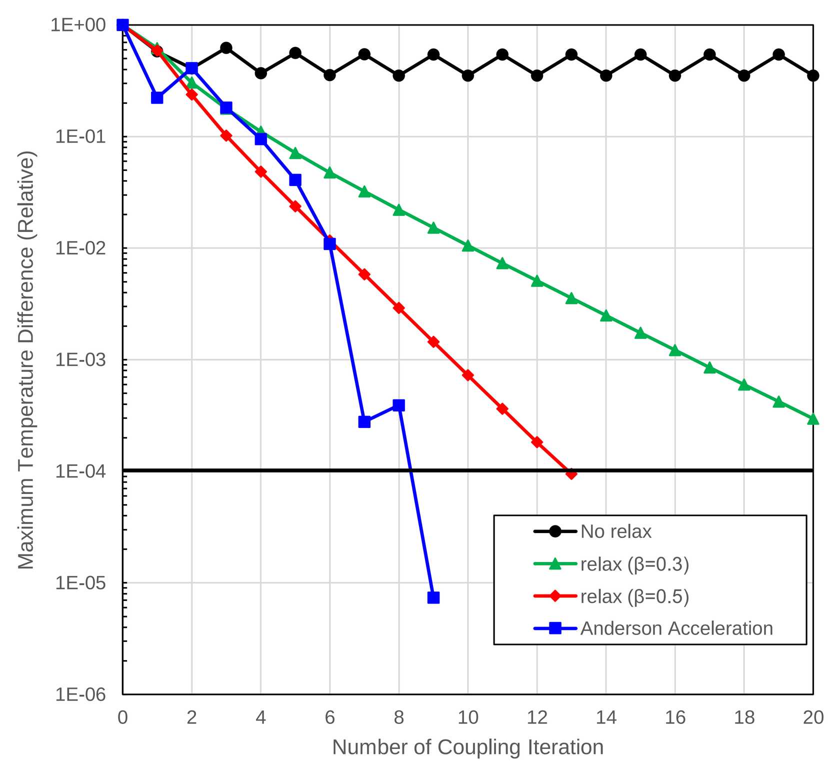
Methods*	k_{eff}	Difference (pcm)
Reference**	0.95652	--
No relaxation	--	--
Relaxation (β=0.3)	0.95660	8
Relaxation (β=0.5)	0.95656	4
Anderson acceleration	0.95651	-1

*Error criteria: 10^{-4} for both maximum power/temperature differences
**Reference: Relaxation (β=0.5) with error criteria=10^{-7}

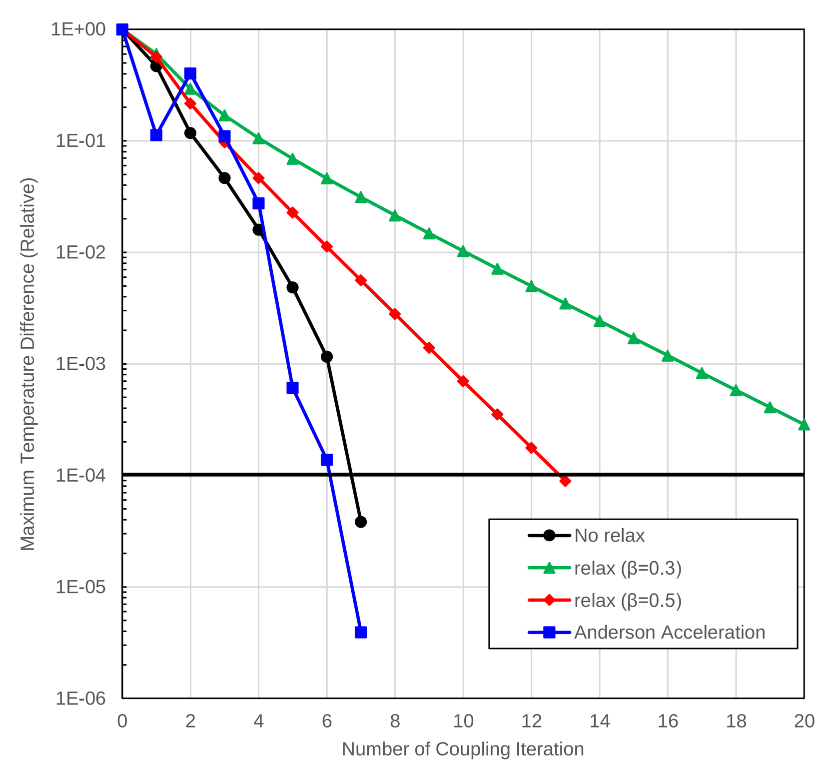
- Relaxation with β=0.3, 0.5: converged / β=0.8: fail.
- Anderson acceleration: converged.
- Convergence speed: Anderson acceleration > Relaxation

Case 2: CRs insertion depth: 60 cm

- The conventional coupled analysis converged well.
- Relaxation methods converged more slowly in case 2.
- Anderson acceleration converged slightly faster than the conventional method.



<Max. Temp. Diff. for Case 1 (765 cm)>



<Max. Temp. Diff. for Case 2 (60 cm)>

References

[1] H. C. Lee, T. Y. Han, C. K. Jo, and J. M. Noh, Development of the HELIOS/CAPP code system for the analysis of pebble type VHTR cores, Annals of Nuclear Energy, Vol. 71, p. 130, 2014.
[2] D. G. Anderson, Iterative Procedures for Nonlinear Integral Equations, Journal of the ACM, Vol. 12, p. 547, 1965.
[3] A. Facchini, J. Lee, and H. G. Joo, Investigation of Anderson acceleration in neutronics-thermal hydraulics coupled direct whole core calculation, Annals of Nuclear Energy, Vol. 153, 108042, 2021.
[4] S. Yuk, C. K. Jo, A Preliminary Study on 950 ℃ VHTR Core Design, Transactions of the Korean Nuclear Society Autumn Meeting, Oct. 22–23, 2020, Changwon, Korea.

Conclusions

- In this study, Anderson acceleration has been implemented to the neutronics/thermo-fluid coupled analysis of a block-type VHTR core analysis.
- It was verified that this method solved the problem that was difficult to converge with the conventional method.
- While the relaxation method should choose the appropriate factor in advance based on the user's experience, the Anderson acceleration method seems to have more advantages because it automatically determines the factor to reduce the error by the formula.
- As future works,
  - Increasing m(>1) of Anderson acceleration
  - Application to other iterative calculations such as critical control rod tip position search.