

## Equilibrium Core Loading Pattern Search for APR1400 using Simulated Annealing Algorithm

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### 1. Introduction

Traditional optimization methods have shown limitations in achieving optimal equilibrium core configurations due to the complex and nonlinear nature of the problem. Finding the ideal subset of features is classified as an NP-Hard problem [1]. Simulated Annealing (SA), inspired by the annealing procedure used in metallurgy, is a heuristic optimization algorithm that was developed to handle difficult optimization problems [2]. Its ability to escape local optima and explore a large solution space makes it a promising approach for nuclear reactor equilibrium core optimization.

In this research, a nuclear design process is proposed to achieve an optimized Loading Pattern (LP) for equilibrium cycle of APR1400 with 17 by 17 FAs and WABA (Wet Annular Burnable Absorber). This design process aims to optimize fuel batch selection and loading pattern search for a given energy requirement using a code developed in Python programming language called COSUME (CONsecutive SURvival METHod). In this research, several factors are considered to optimize loading pattern during the optimization process, including Power Peaking Factor (PPF), Critical Boron Concentration (CBC), fuel enrichment and several Fuel Assemblies (FA) in a batch.

One of the challenges addressed is Central Processing Unit (CPU) time needed for SA iterations. Finding an LP with full depletion mode takes about 200 minutes of CPU time. To reduce CPU time, a zero-depletion mode with an acceleration scheme is integrated into the code and takes only about 30 minutes to finish searching LPs. By employing both zero-depletion and full depletion modes, equilibrium core LPs for cycle lengths, 17.5, 19.5 and 23.5 GWD/t are investigated.

### 2. Methods and Results

The steps taken in this study before reaching the equilibrium cycle are as follows:

1. Estimation of core average enrichment (w-t % of <sup>235</sup>U) and number of BA rods in a core at beginning of Cycle (BOC) using FMNG.
2. Optimization of batch average enrichment and number of fuel assemblies (FA) per batch using Evolutionary Algorithms (EA).
3. Loading pattern determination using Simulated Annealing.

#### 2.1 Core Average Enrichment and Total Number of WABA Rods in a Core

Fuel Management Net Graph (FMNG) for the initial core is developed based on a previous study [3] to estimate initial core parameters.

To construct an FMNG for the reload cycle, MASTER-3 [4] is used to estimate feed fuel enrichment for the following cycles. To generate FMNG for the reload cycle, a total of 9 different calculations were obtained using MASTER-3, with three different cycle lengths (17.5, 19.5, and 23.5 GWD/t), and three different BA numbers (4, 5, and 6 per FA). Fig. 1 shows the R-FMNG (Reload cycle FMNG) obtained using the linear interpolation method.

Since core average enrichment and total number of Burnable Absorber (BA) to meet the requested cycle length and CBC at BOC are obtained from R-FMNG, the remaining parameters to determine is fresh FA enrichment for a reload cycle. Fresh fuel enrichment is determined using Equation (1).

$$(1) e_f = \frac{e_{avg} \times a - e_b \times (a-b)}{b}$$

Where,

- $e_{avg}$  = Core average enrichment,
- $e_f$  = Fresh FA enrichment,
- $e_b$  = Burnt fuel enrichment,
- $a$  = Total number of FAs in a core,
- $b$  = Number of fresh FAs

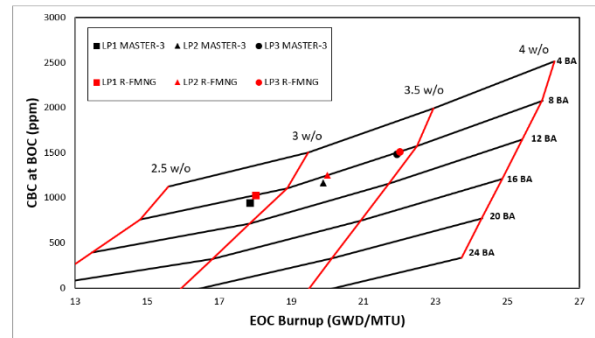


Fig. 1. Fuel Management Net Graph for Reload Core

To validate R-FMNG, test LPs are selected with cycle lengths of 18.0 GWD/t, 20.0 GWD/t and 22.0 GWD/t. The predicted cycle lengths by R-FMNG are compared with MASTER-3 results. Red color is estimation of R-FMNG and black color is MASTER-3 results. In Fig. 1, R-FMNG provides consistent results in estimating core parameters for reload cycles.

## 2.2 Optimization of Batch Average Enrichment and Number of Assemblies Per Batch

FMNG provides the required average core enrichment value for the desired cycle length, however, the number of FAs and enrichment in each fuel batch are still part of the optimization process to be determined. Evolutionary algorithm (EA) is selected as an optimization tool to find an optimal FA number and enrichment in each fuel batch. Computational optimization methods called EA are based on the concepts of genetics and natural selection. EA maintains a population of potential solutions and iteratively improves these solutions over generations. In an EA, certain functions must be defined; in our case, the main objective function is to minimize the fuel cost as given in Equation (2) [5]. The constraints and objective function are listed as follows:

1. The core-average enrichment is close to a value estimated by FMNG.
2. The total number of FAs in the core does not exceed 241, that is,  $N_1 + N_2 + N_3 = 241$ .

$$(2) \text{ Objective Function} = \sum_{i=1}^{batch} N_i \times f_i(C_1, C_2, C_3, C_4)$$

Where:

$C_1$ : ore purchase cost.

$C_2$ : conversion cost.

$C_3$ : enrichment cost.

$C_4$ : fabrication cost.

$N_i$ : Number of FA in a batch  $i$ .

The core average enrichment required for each case are obtained by using FMNG. Using these values, the batch specifications are calculated by EA to minimize the fuel cost for the initial cycle, as shown in Table I.

Table I: Initial cycle batch specifications by EA.

Cases	Cycle Length (GWD/t)	Total Number of BA rods	Average Enrichment (%) / Total number of Assemblies		
			Bath A	Batch B	Batch C
Case 1	17.5	1920	1.75/ 81	3.05/ 88	3.55/ 72
Case 2	19.5	1920	1.92/ 81	3.20/ 80	3.70/ 80
Case 3	23.5	1920	2.05/ 81	3.60/ 80	4.10/ 80

The fuel assembly configurations used for cases 1, 2 and 3 are shown in Fig. 2. Batches A, B and C are loaded in cycle 1 and batch D is in cycle 2, and batch E is used in cycle 3 and further cycles. Average enrichment values of batch D and E vary depending on the batch size (Total number of fresh FA to be loaded). The average core enrichment required to reach the target

cycle length is correlated with the enrichment of the burnt fuel, and average feed batch enrichment is obtained, as mentioned in the Reload FMNG section.

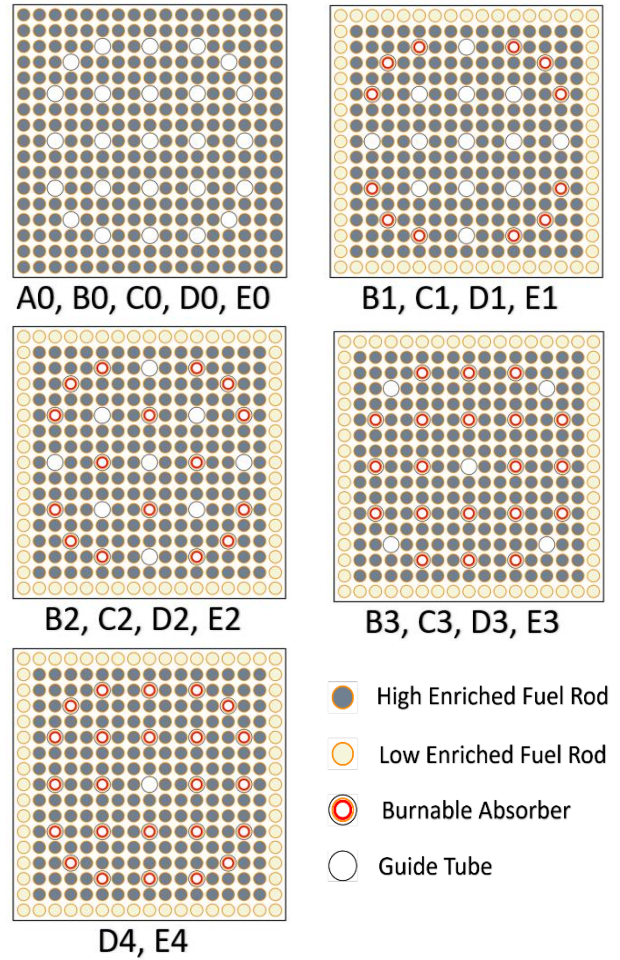


Fig. 2. Enrichment zoning and BA rod locations for FAs used in equilibrium core models

## 2.3 Acceleration Scheme

An issue that needs to be addressed is the CPU time required for SA iterations. Computation time of Zero-depletion mode and full depletion mode is compared using a computer with AMD Ryzen 7 5800H with Radeon Graphics @ 3.20 GHz and the results are shown in in Table II.

Instead of performing full-depletion, zero-depletion mode is to run MASTER-3 only for BOC and SA evaluates nuclear parameters of BOC, such as pin peaking factor, to select candidate LPs. After selecting candidate LPs with peaking factor lower than design limits, the best LP with lowest peaking factor is determined by depleting candidate LPs consecutively until End of Cycle (EOC).

Table II shows three cycle comparison using 100 Fresh FAs for a cycle length of 19.5 GWD/t for both full depletion and zero-depletion modes. In Table II, zero-depletion mode has completed LP search with about 6 times faster in CPU time. Computation time of

zero-depletion mode is six times faster in CPU time than full depletion mode.

Table II: Comparison of CPU time and total number of LPs for full depletion and zero-depletion modes

Cycle ID	CPU Time (minutes)	
	Full Depletion Mode	Zero-Depletion Mode
Cycle 1	146	26
Cycle 2	196	30
Cycle 3	209	30
Total	551	86

### 2.4 Simulated Annealing Algorithm

The algorithm implemented in the code, COSUME, is SA and the objective function is defined as  $\Delta F_{xy}$  which is pin peaking factor difference between LPs.

In this research, the acceptance probability (P),  $e^{-\frac{\Delta E}{kT}}$  Equation (3). In COSUME, a simple version of this equation is used by omitting the Boltzmann constant (k) and introducing cooling parameter ( $\alpha$ ) [6]. The cooling parameter,  $\alpha$ , in Equation (4) is a part of the cooling schedule, which is a predetermined constant that reduces temperature with each iteration step. P and  $T_n$  equations are defined as follows, respectively:

$$(3) P = e^{-\frac{\Delta F_{xy}}{T_n}}$$

$$(4) T_n = T_{n-1}\alpha$$

where,

$\Delta F_{xy} = \text{Max. } F_{xy}$  difference of the last two iteration steps

$\alpha = \text{cooling parameter } (0 < \alpha < 1)$

The algorithm in the code is set to minimize the Maximum  $F_{xy}$  value. If a new LP is worse according to the objective function,  $-\Delta F_{xy}$ , an old LP is kept the same [7]. On the other hand, if a new LP is better and the probability is higher than the random number, the new LP is accepted. Otherwise, the old LP is kept. The stopping criterion is predetermined to activate at the point where the SA process has completely cooled down and no improvement is possible.

### 2.5 Equilibrium Core Design

In this study, three equilibrium cores are investigated with different cycle lengths, 17.5, 19.5 and 23.5 GWD/t. COSUME is used to determine LPs for each cycle of each case as shown in Table III. Cycle 1 is an initial cycle. Cycles 2 and 3 are considered transient cycles. The LP obtained in the third cycle is used in the following cycles if they meet the design requirements. In this way, equilibrium core state is achieved at cycle 9 and all LPs in Table III show good agreement with design requirements in peaking factor, cycle length. The design requirements used in the LP search are maximum pin peaking factor less than 1.55 and cycle

length equal to utility requirement. For the cycle length of 19.5 GWD/t, convergence of  $F_{xy}$ , probability progression and Equilibrium Cycle LP are shown in figure 3, 4 and 5, respectively.

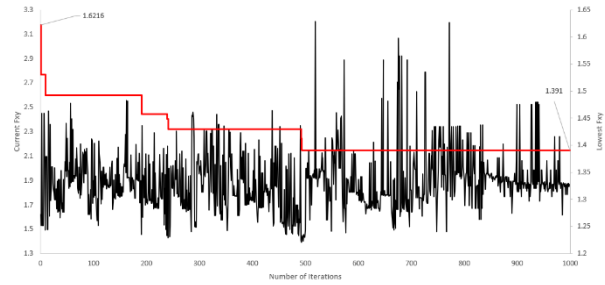


Fig. 3. Convergence of best Fxy

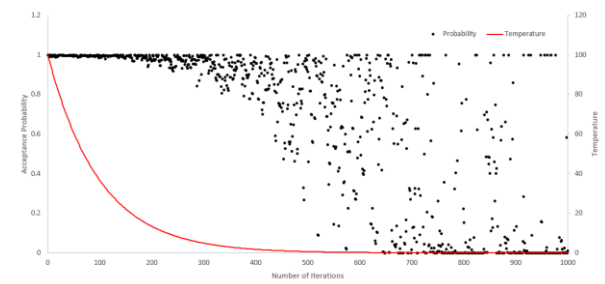


Fig. 4. Probability progression according to temperature during SA iterations

	J	K	L	M	N	P	R	S	T	
9	J2	K2	L2	K0	L2	K2	L2	K2	L0	
10	K2	L2	J2	L2	J2	L2	J2	L2	K2	
11	L2	J2	K2	K2	L2	J0	L2	K2	L0	
12	K0	L2	K2	L2	J2	L2	K2	L1	K2	
13	L2	J2	L2	J2	K2	K1	L2	K2		
14	K2	L2	J0	L2	K1	L2	K0	K2		
15	L2	J2	L2	K2	L2	K0	K2		J	Twice Burnt
16	K2	L2	K2	L1	K2	K2			K	ONCE Burnt
17	L0	K2	L0	K2					L	FRESH

Fig. 5. Equilibrium Cycle Loading Pattern for 19.5 GWD/t cycle length

### 3. Conclusions

In this research, a nuclear design methodologies aiming to meet design requirements is proposed and a computer code, COSUME, is developed. Three equilibrium core models are investigated to validate the nuclear design methodologies. Three equilibrium core models are 17.5, 19.5 and 23.5 GWD/t with different feed fuel numbers ranging from 88 to 104. As shown in Table III, the maximum  $F_{xy}$  values are less than the

design requirement of pin peaking factor, 1.55 and the cycle lengths calculated by MASTER-3 shows good agreement with target cycle lengths. Zero-depletion mode combined with SA reduces CPU time compared with full depletion mode.

As a result, the application of the proposed method in this study is possible to generate many feasible LPs for the initial and reload cycles, as well as to reach an equilibrium cycle in a systematic way.

Table III. Equilibrium Core Parameters

cycle	1	2	3	4	5	6	7	8	9	Cases
Cycle Length (GWD/t)	17.91	17.60	17.62	18.29	18.03	18.07	18.09	18.07	18.08	1
CBC at BOC (ppm)	767	910	854	928	907	911	912	911	911	
Maximum Fxy	1.47	1.42	1.44	1.48	1.48	1.46	1.47	1.47	1.47	
Cycle Length (GWD/t)	19.53	19.75	19.66	19.71	19.66	19.63	19.66	19.65	19.65	2
CBC at BOC (ppm)	1007	1089	1085	1109	1101	1099	1101	1100	1100	
Maximum Fxy	1.46	1.43	1.41	1.43	1.43	1.43	1.43	1.43	1.43	
Cycle Length (GWD/t)	23.72	23.53	23.92	23.77	23.62	23.71	23.69	23.69	23.69	3
CBC at BOC (ppm)	1401	1629	1609	1621	1599	1610	1607	1607	1607	
Maximum Fxy	1.46	1.51	1.50	1.41	1.41	1.41	1.41	1.41	1.41	

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