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2D FDM Structural Analysis Model for Cr-coated ATF cladding: Development and implementation into fuel analysis code

4A, Fuel fabrication, performance & test I
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Research Background [1/4]

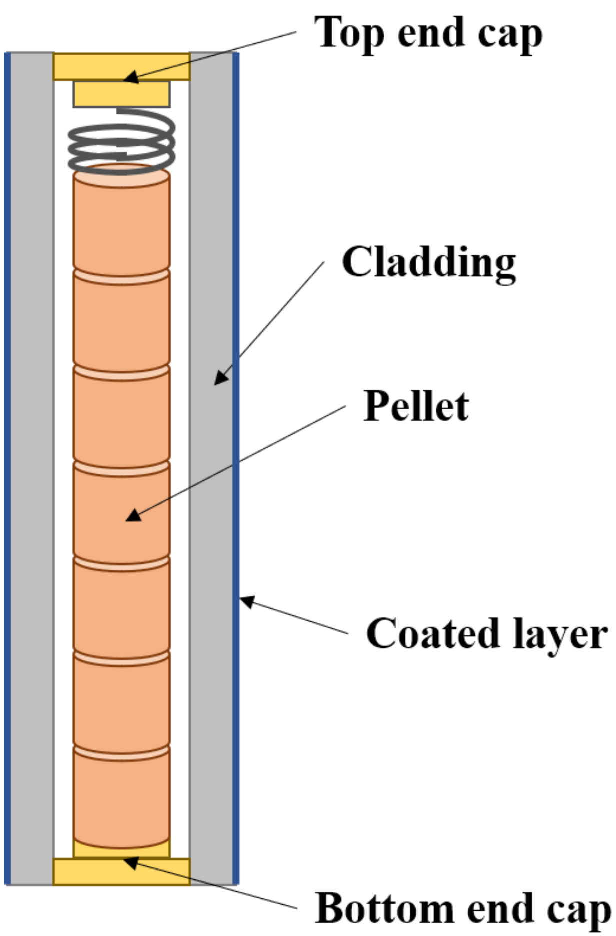
Coated cladding

- **Coated cladding** is studied as a major ATF concept, creating a need for an specialized **analysis model**.

- The coated cladding is composed of **multiple layers**, each made of a **different material**.

- For coated cladding analysis, it is necessary to develop a **high-fidelity structural analysis model** that can simulate various physical phenomena occurring in **multi-layer structures**.

  - Elastic properties difference
  - Creep deformation difference
  - Axial irradiation growth difference
  - Plastic behavior difference
  - Thermal conductivity difference
  - Thermal expansion difference

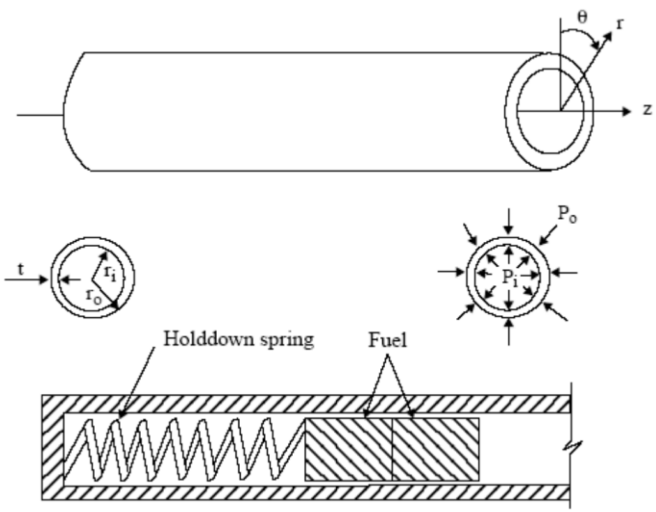


Schematic diagram of coated cladding

Research Background [2/4]

Existing nuclear fuel analysis code: FRAPCON (Simplified point model)

- FRAPCON, developed by the US NRC, is a widely used nuclear fuel code.
- FRACAS, a structural analysis model of FRAPCON, simplifies the analysis by **neglecting axial interactions** and using the **thick wall approximation**.
- While FRACAS is suitable for steady-state analysis of single-layer cladding, it is not suitable for analyzing coated cladding due to its **inability to simulate multi-layer structures**.



Fuel rod geometry and coordinates of FRAPCON

$$\sigma_\theta = \frac{P_i r_i - P_o r_o}{t}$$

$$\sigma_z = \frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2}$$

Thick wall approximation used in FRACAS

Research Background [3/4]

Existing nuclear fuel analysis code: FEM

- As one of the attempts to analyze the multi-layer structure, nuclear fuel analysis codes based on **Finite Element Method (FEM)** are being studied a lot.

- The FEM-based code has a **high-fidelity model**, it is possible to simulate the **coated cladding**, and accurate analysis is possible even in cases where **large deformation** occurs, such as ballooning.

- However, one disadvantage is that the **computational cost** increases when there are numerous meshes or dealing with complex systems.

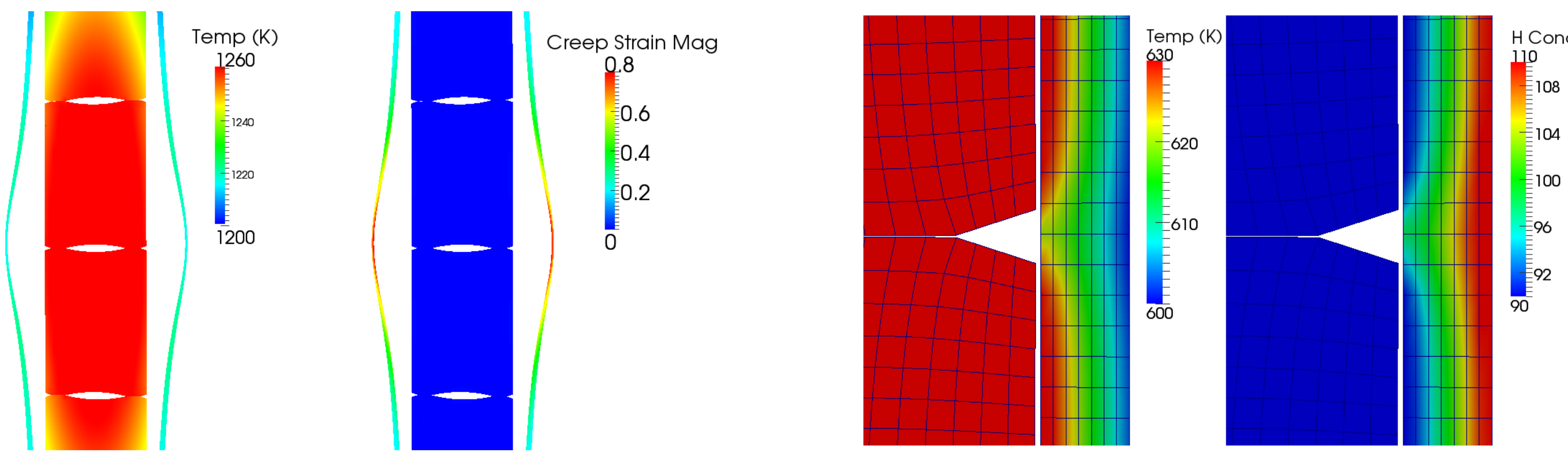
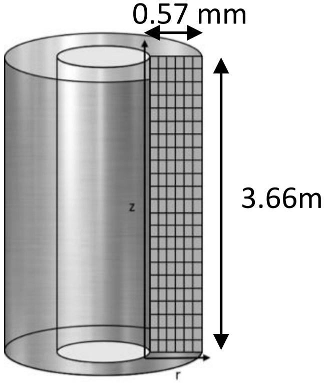


Fig. Cladding analysis of BISON code
https://bison.inl.gov/SitePages/Applications.aspx

Research Background [4/4]

Characteristics of nuclear fuel cladding

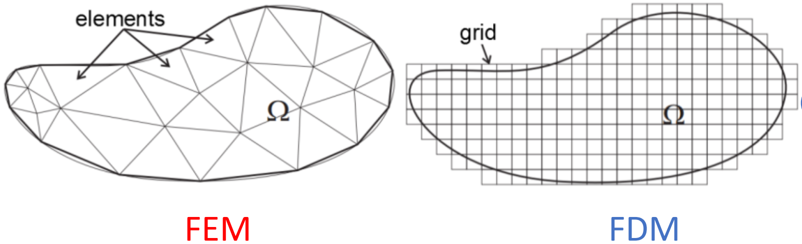
1) **High aspect ratio** (thickness : height = 1 : 6000)
2) **Small deformation under normal operating condition**



Nuclear fuel code development using Finite Difference Method

- FEM code shows excellent performance for **local** analysis, **large deformation** analysis, and **unstructured structure** analysis.
- However, **many axial meshes** increase the **computational cost** of FEM codes, and since cladding under normal operating condition do not deform much, using FEM codes for this type of analysis is **quite inefficient**.
- FDM enables more **fast and efficient** analysis than FEM codes for **long length** simulations under **normal operating conditions**.

Good but unnecessarily heavy



Good enough and fast

Schematic diagram of FEM & FDM

L. A. Ferreira et al. *(JMOe), 14 (2015)*

Research Objective and Scope

Research Objective

- **Development of FDM model for steady-state structural analysis of coated cladding**

Target Capability

- Multi-layer analysis
- Plastic deformation
- Creep deformation
- Large deformation

Verification

- Comparative verification with commercial FEM code

Application

- Simulate coated cladding behavior in PWR environment

Model Development: Numerical scheme [1/3]

Governing equations

- **Force equilibrium equation** in axisymmetric cylinder domain:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0$$
$$\frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \tau_{rz}}{\partial r} + \frac{\tau_{zr}}{r} = 0$$

Stress

- **The constitutive equation(Hooke's law)** represents the correlation between stress and strain in a material, and it is mathematically expressed using the stiffness matrix:

$$\begin{bmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \\ \sigma_{zz} \\ \tau_{rz} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 \\ c_{21} & c_{22} & c_{23} & 0 \\ c_{31} & c_{32} & c_{33} & 0 \\ 0 & 0 & 0 & c_{44} \end{bmatrix} \begin{bmatrix} \epsilon_{rr} - \epsilon_{rr}^{p} - \epsilon_{rr}^{c} - \epsilon_{rr}^{th} \\ \epsilon_{\theta\theta} - \epsilon_{\theta\theta}^{p} - \epsilon_{\theta\theta}^{c} - \epsilon_{\theta\theta}^{th} \\ \epsilon_{zz} - \epsilon_{zz}^{p} - \epsilon_{zz}^{c} - \epsilon_{zz}^{th} \\ \gamma_{rz} - \gamma_{rz}^{p} - \gamma_{rz}^{c} \end{bmatrix}$$

Stress ⇔ Strain

- **Kinematic relations** describe the strain and displacement of a material:

$$\begin{cases} \epsilon_{rr} = \dfrac{\partial u_r}{\partial r} \\ \epsilon_{\theta\theta} = \dfrac{u_r}{r} \\ \epsilon_{zz} = \dfrac{\partial u_z}{\partial z} \\ \gamma_{rz} = \dfrac{\partial u_r}{\partial z} + \dfrac{\partial u_z}{\partial r} \end{cases}$$

Strain ⇔ Displacement

- The **equilibrium equation** is initially formulated as a **differential equation for stress**, but it can be **transformed into a differential equation for displacement** by incorporating constitutive equations and kinematic relations.

Model Development: Numerical scheme [2/3]

Discretization

- Discretization process for use in FDM:

$$
\begin{cases}
\dfrac{du}{dr} = \dfrac{u_{i+1,j} - u_{i-1,j}}{2\Delta r} \\
\dfrac{du}{dz} = \dfrac{u_{i,j+1} - u_{i,j-1}}{2\Delta z} \\
\dfrac{d^2u}{dr^2} = \dfrac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{2\Delta r} \\
\dfrac{d^2u}{drdz} = \dfrac{u_{i+1,j+1} - u_{i+1,j-1} - u_{i-1,j+1} + u_{i-1,j-1}}{4\Delta r\Delta z}
\end{cases}
$$

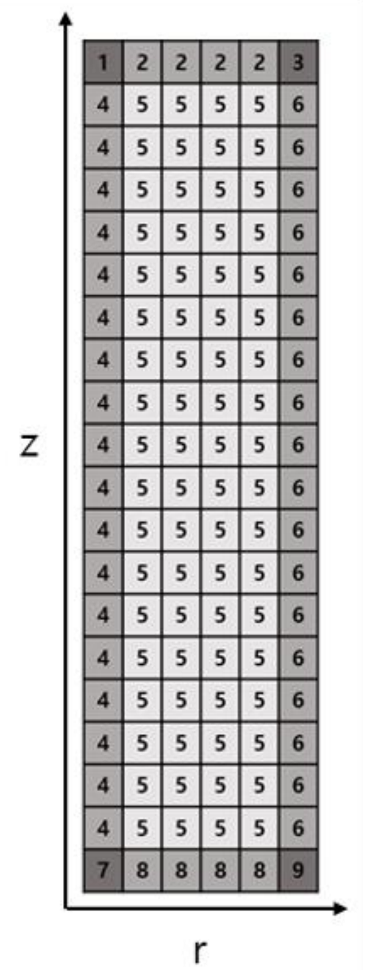
- The equilibrium equation can be expressed as a matrix expression:

$$A \quad X = b$$

$$
\begin{bmatrix}
b_1 & c_1 & & & & 0 \\
a_2 & b_2 & c_2 & & & \\
 & a_3 & b_3 & c_3 & \ddots & \\
 & & a_{mn} & b_{mn} & c_{mn} & \\
 & & \ddots & & \ddots & c_{2mn-1} \\
0 & & & & a_{2mn} & b_{2mn}
\end{bmatrix}
\begin{bmatrix}
u_{1,1} \\ u_{1,2} \\ \vdots \\ u_{m,n} \\ w_{1,1} \\ \vdots \\ w_{m,n}
\end{bmatrix}
=
\begin{bmatrix}
d_1 \\ d_2 \\ \vdots \\ d_{mn} \\ d_{mn+1} \\ \vdots \\ d_{2mn}
\end{bmatrix}
$$

- **Displacement** in all meshes can be calculated **by solving the matrix equation**

$$X = A^{-1}b$$



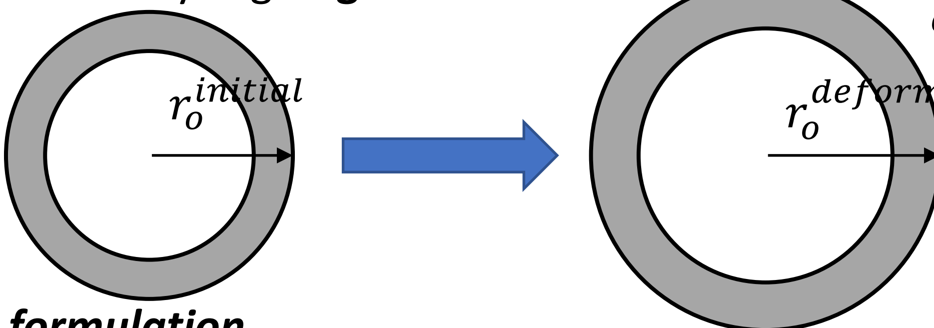
Schematic diagram of discretization

Model Development: Numerical scheme [3/3]

Numerical method for structural analysis

- ***Infinitesimal strain theory***
  : an approach used to analyze **small deformations** in materials.
  : It assumes that the changes in shape and size of a material are **small enough**.
  : not suitable for analyzing **large deformations**.

$$\epsilon_{\theta,o}^{deform} = \frac{u}{r} = \frac{r_o^{deform} - r_o^{initial}}{r_o^{initial}}$$

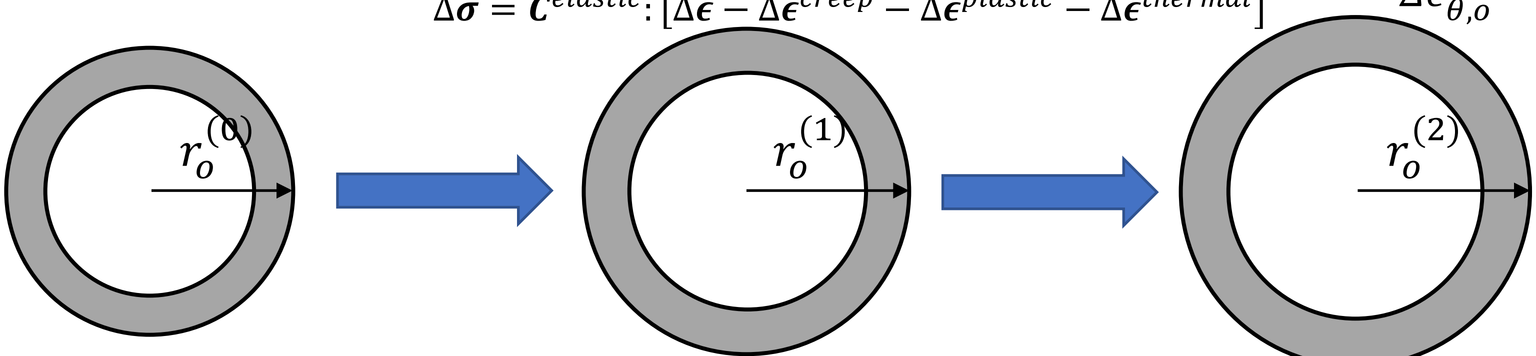


- ***Incremental formulation***
  : Instead of solving the problem in a single step, it breaks it down into a series of **smaller incremental steps**.
  : The solution is updated incrementally until the final deformation is reached.

$$\boldsymbol{\sigma}_{n+1} = \boldsymbol{\sigma}_n + \Delta\boldsymbol{\sigma}$$

$$\Delta\boldsymbol{\sigma} = \boldsymbol{C}^{elastic} : [\Delta\boldsymbol{\epsilon} - \Delta\boldsymbol{\epsilon}^{creep} - \Delta\boldsymbol{\epsilon}^{plastic} - \Delta\boldsymbol{\epsilon}^{thermal}]$$

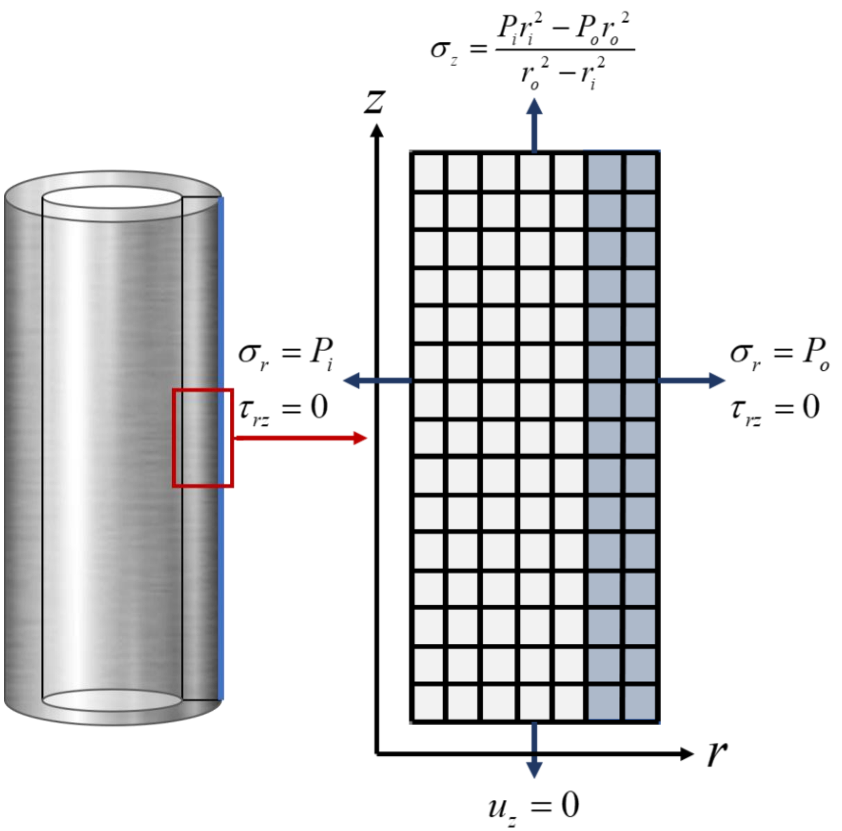
$$\Delta\epsilon_{\theta,o}^{(i+1)} = \frac{r_o^{(i+1)} - r_o^{(i)}}{r_o^{(i)}}$$



Model Development: Boundary conditions [1/2]

Open gap boundary conditions

- Pressure boundary conditions at the inner and outer surface:
$$\begin{cases}\sigma_{rr}(r = r_i) = -P_i \\ \sigma_{rr}(r = r_o) = -P_o\end{cases}$$
- No shear stress due to hydrostatic pressure on the surface in contact with fluid:
$$\begin{cases}\tau_{rz}(r = r_i) = 0 \\ \tau_{rz}(r = r_o) = 0\end{cases}$$
- Bottom of the cladding is fixed axially:
$$u_z(z = 0\,) = 0$$
- The mean axial stress at top in a closed pressurized cladding based on Saint-Venant's principle:
$$\sigma_{zz} = \frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2}$$



Open gap boundary conditions

Model Development: Boundary conditions [2/2]

Closed gap boundary conditions (for PCMI)

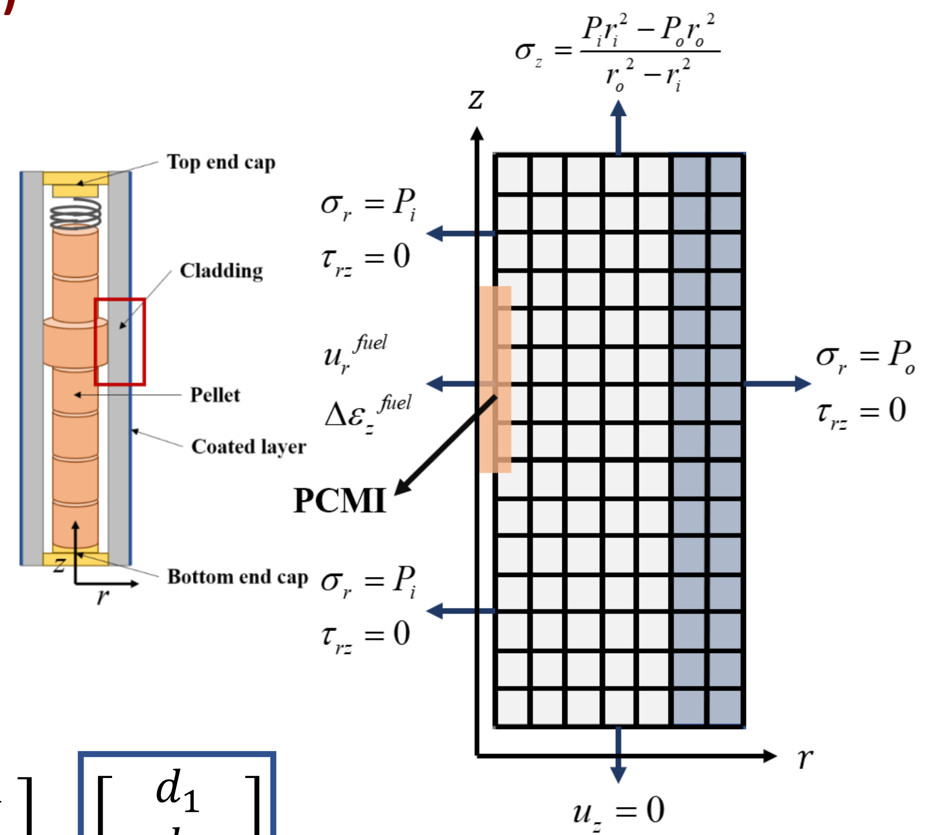
- Pressure boundary conditions at the uncontacted inner surface:

$$\sigma_{rr}(r = r_i) = -P_i$$

- Displacement boundary conditions at the contacted inner surface:

$$u_r^{clad} = u_r^{fuel} - \delta$$
$$\epsilon_z^{clad} - \epsilon_{z,o}^{clad} = \epsilon_z^{fuel} - \epsilon_{z,o}^{fuel}$$

- $AX = b$ matrix size changes when Pellet Cladding Mechanical Interaction (PCMI) occurs.



Closed gap boundary conditions

Delete

$$\begin{bmatrix} b_1 & c_1 & & & & & 0 \\ a_2 & b_2 & c_2 & & & & \\ & a_3 & b_3 & c_3 & \ddots & & \\ & & a_{mn} & b_{mn} & c_{mn} & & \\ & & \ddots & & \ddots & c_{2mn-1} & \\ 0 & & & & a_{2mn} & b_{2mn} & \end{bmatrix} \begin{bmatrix} u_{1,1} \\ u_{1,2} \\ \vdots \\ u_{m,n} \\ w_{1,1} \\ \vdots \\ w_{m,n} \end{bmatrix} = \boxed{\begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_{mn} \\ d_{mn+1} \\ \vdots \\ d_{2mn} \end{bmatrix}}$$

Modify

Model Development: Modeling of material behaviors

Plasticity model

① $\boxed{d\epsilon_r^p, d\epsilon_\theta^p, d\epsilon_z^p}$ are assumed

② $d\epsilon^p = \frac{\sqrt{2}}{3}\left[\left(d\epsilon_r^p - d\epsilon_\theta^p\right)^2 + \left(d\epsilon_\theta^p - d\epsilon_z^p\right)^2 + \left(d\epsilon_z^p - d\epsilon_r^p\right)^2\right]^{1/2}$

③ σ_{eff} is obtained from stress-strain curve at the value of $d\epsilon^p$

④ Calculate stress ($\sigma_r, \sigma_\theta, \sigma_z$) with assumed $d\epsilon_r^p, d\epsilon_\theta^p, d\epsilon_z^p$

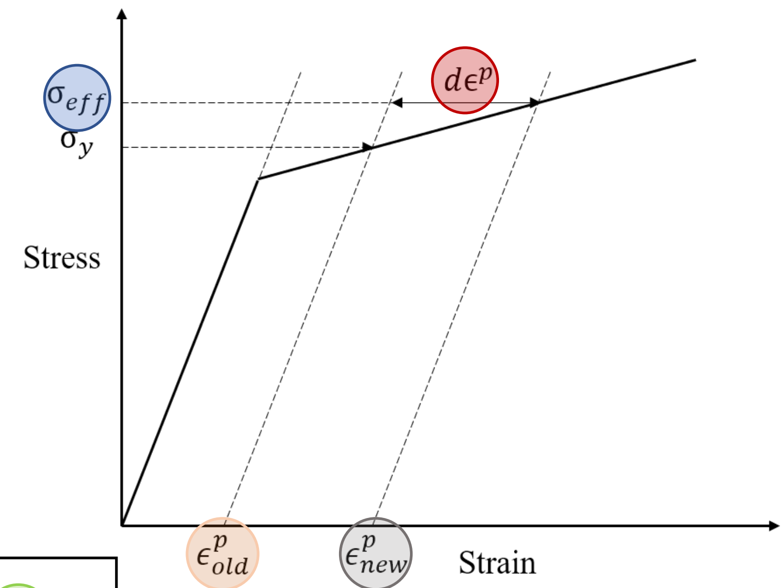
⑥ $d\epsilon_i^p = \frac{3}{2}\frac{d\epsilon^p}{\sigma_{eff}}\left[\sigma_i - \frac{1}{3}(\sigma_r + \sigma_\theta + \sigma_z)\right], \mathrm{i} = \mathrm{r}, \theta, z$

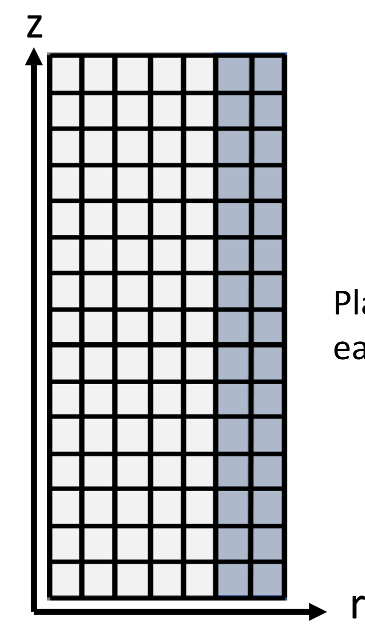
Prandtl-Reuss flow rule

⑦ $d\epsilon_i^p$ converge?

No → New $d\epsilon_i^p$

Yes → Iteration end





Plasticity for each node

Model Verification: Multi-layer & creep & plastic deformation

- ***Purpose***

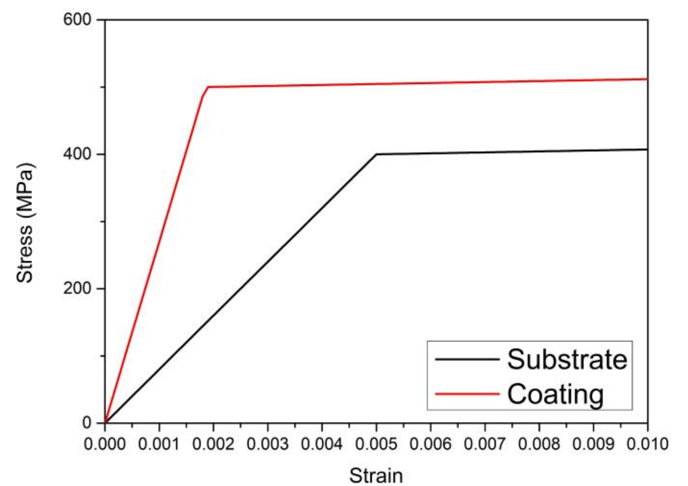
  : Comparative verification with commercial FEM codes in the case of multi-layer, plastic deformation, and creep deformation

Simulation condition

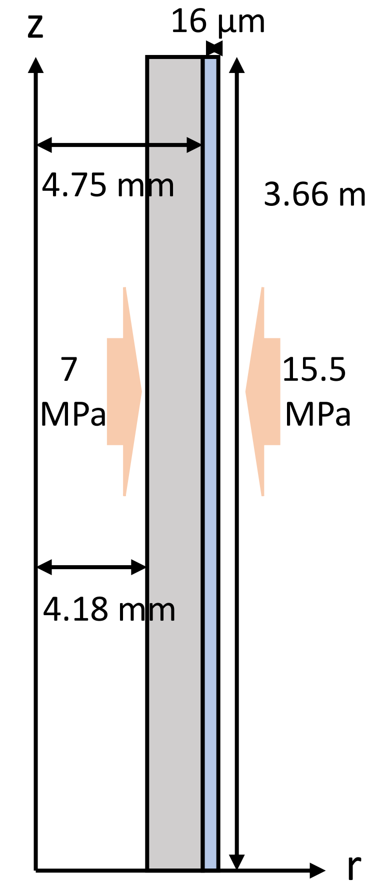
	Conditions	
Internal pressure (MPa)	7	
Outer pressure (MPa)	15.5	
Simulation time (days)	7	
Temperature (°C)	22.5	
	Substrate	**Coating**
Elastic modulus (GPa)	80	270
Poisson's ratio	0.3	0.21
Creep deformation	O	X

Creep correlation (Norton)

$$\dot{\epsilon}^{creep} = 2.0 \times 10^{-17} (\sigma_{vm})^{5.0}$$



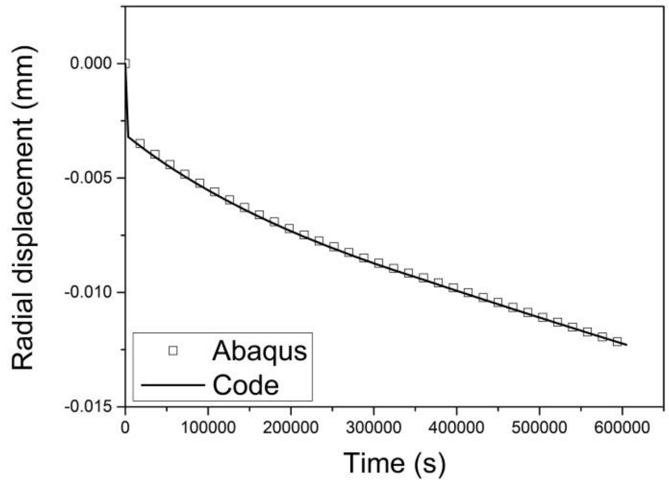
Stress strain curve of substrate and coating layer



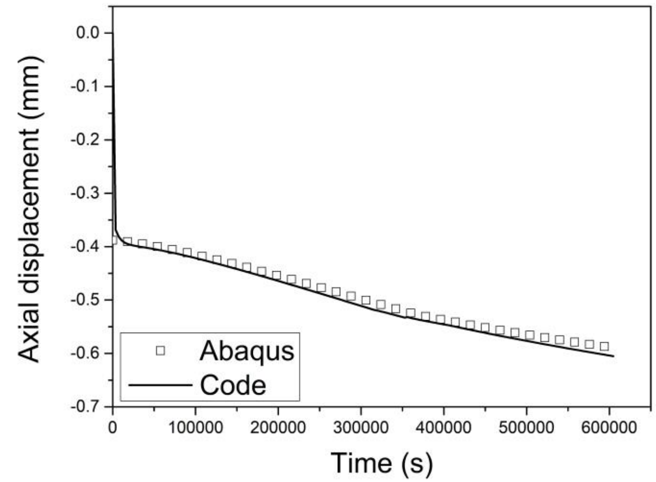
Cladding geometry used for verification

Model Verification: Multi-layer & creep & plastic deformation

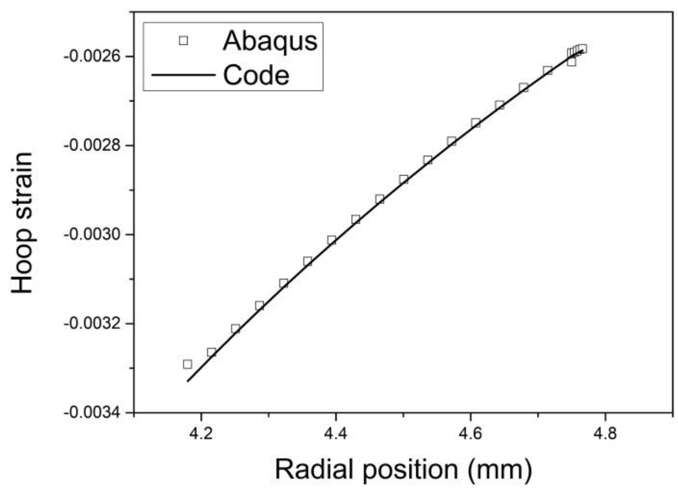
Radial displacement (z = 1.83 m)



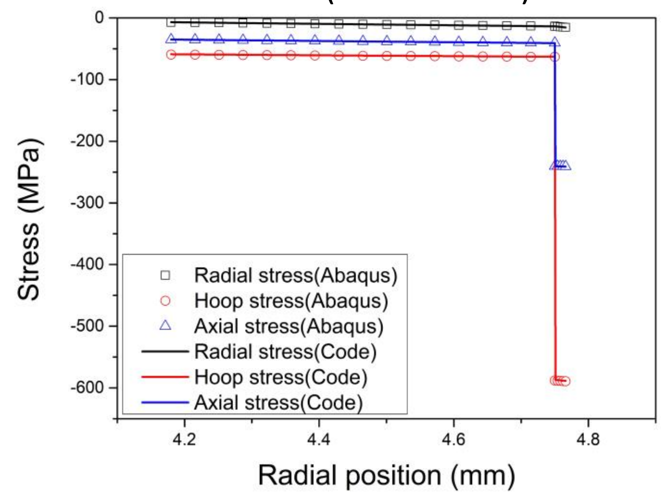
Axial displacement (z = 1.83 m)



Hoop strain (t = 604800 s)



Stress (t = 604800 s)



- The radial displacement, axial displacement, strain, and stress all show excellent agreement with the results obtained from FEM analysis.

Model Verification: Large deformation

- ***Purpose***

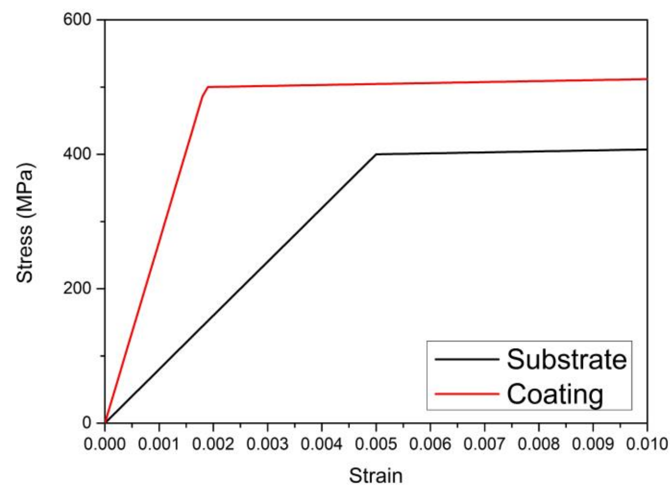
: To compare the difference between the calculated results of the infinitesimal strain theory and incremental formulation with commercial FEM calculations.

Simulation condition

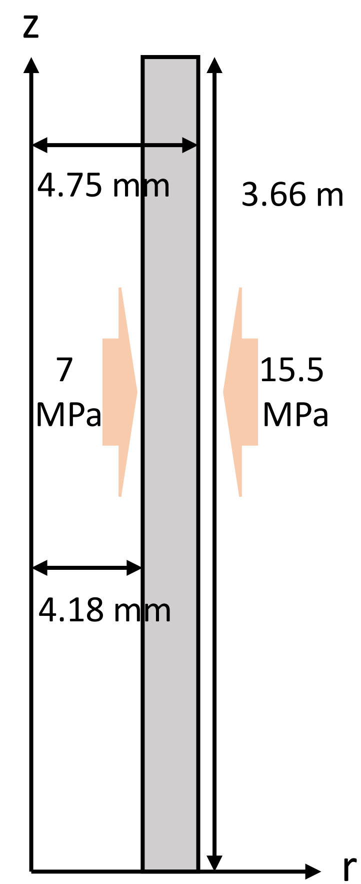
	Conditions
Internal pressure (MPa)	7
Outer pressure (MPa)	15.5
Simulation time (days)	7
Temperature (°C)	22.5
	Substrate
Elastic modulus (GPa)	80
Poisson's ratio	0.3
Creep deformation	O

Creep correlation (Norton)

$$\dot{\epsilon}^{creep} = 2.0 \times 10^{-17}(\sigma_{vm})^{5.0}$$

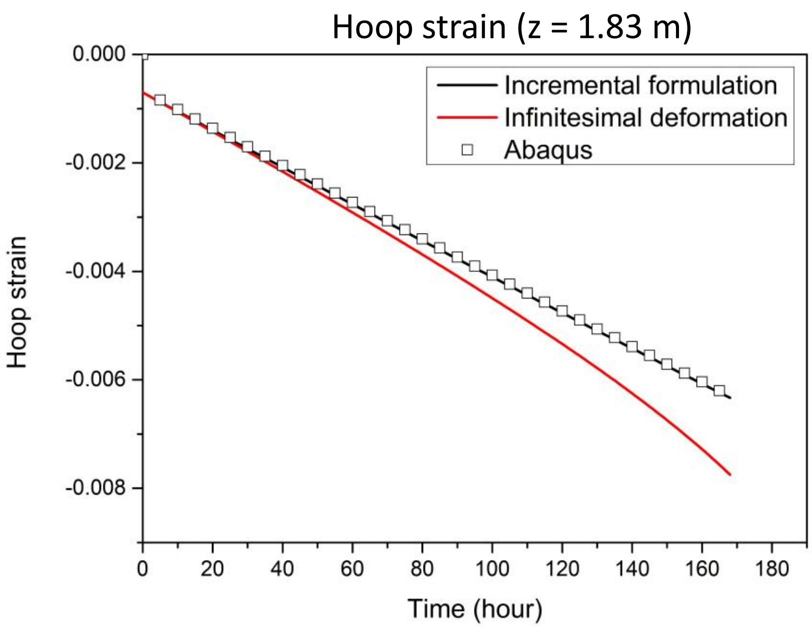


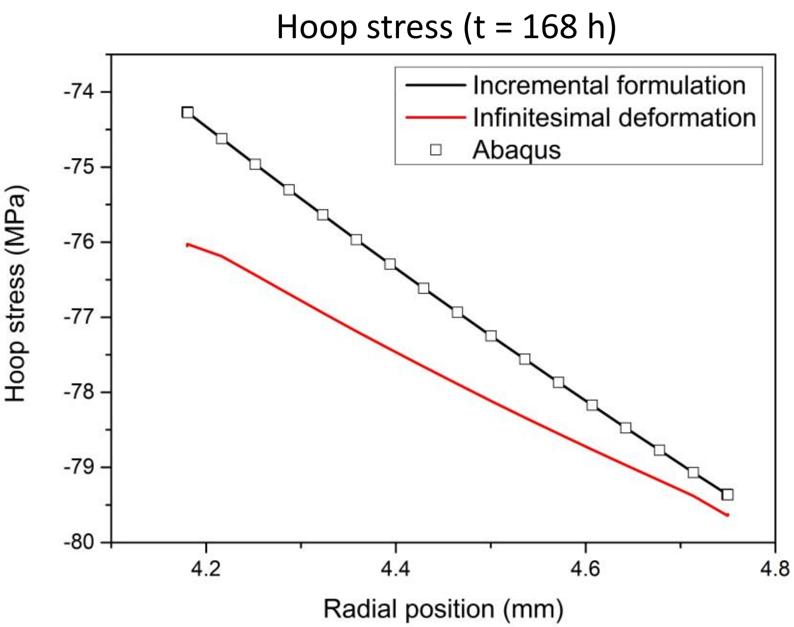
Stress strain curve of substrate and coating layer



Cladding geometry used for verification

Model Verification: Large deformation





- At the beginning of the simulation, the hoop strain calculated by the two methodologies and the hoop strain calculated by Abaqus are in good agreement.
- As time goes on, there are differences between the results from the infinitesimal strain theory and Abaqus, as well as small differences in stress.

Model Verification: Large deformation

- ***Purpose***

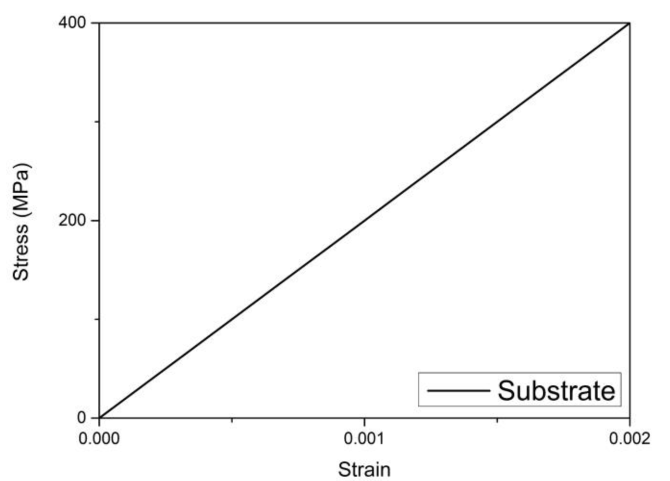
  : Comparative verification with commercial FEM codes in the case of hoop strain is larger than 1%

Simulation condition

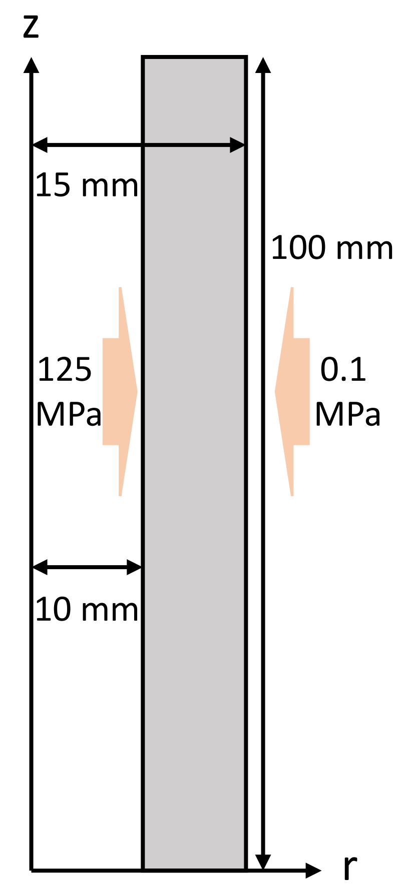
	Conditions
Internal pressure (MPa)	125
Outer pressure (MPa)	0.1
Simulation time (sec)	360
Temperature (°C)	22.5
Elastic modulus (GPa)	200
Poisson's ratio	0.3
Creep deformation	O

Creep correlation (Norton)

$$\dot{\epsilon}^{creep} = 2.0 \times 10^{-17} (\sigma_{vm})^{5.0}$$



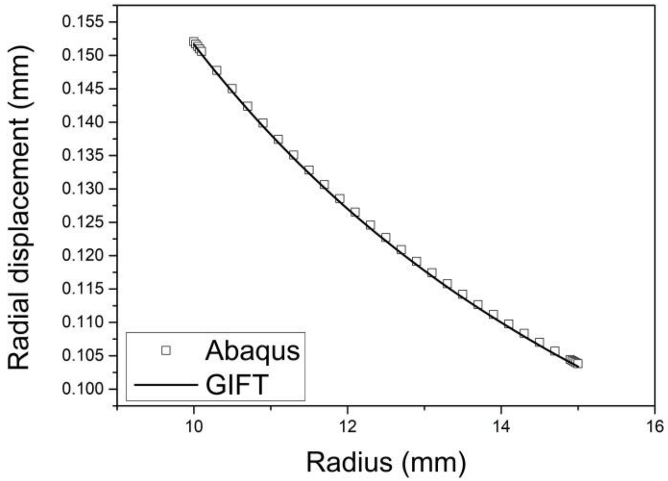
Stress strain curve of substrate layer



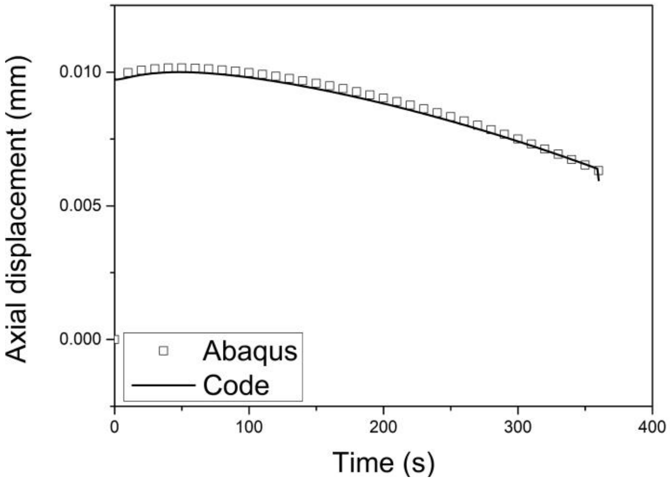
Cladding geometry used for verification

Model Verification: Large deformation

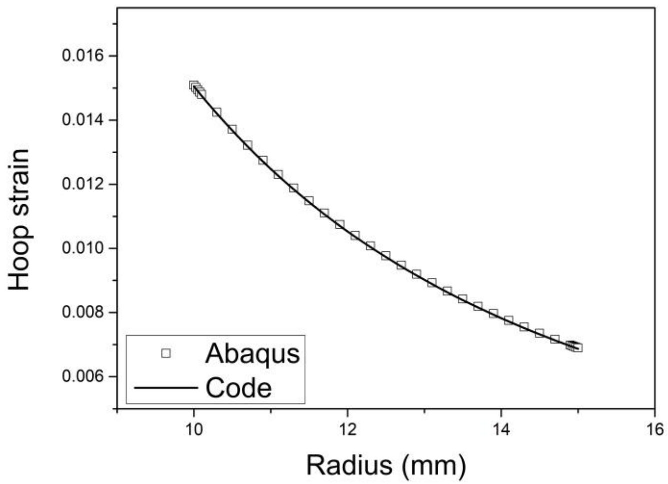
Radial displacement (z = 1.83 m)



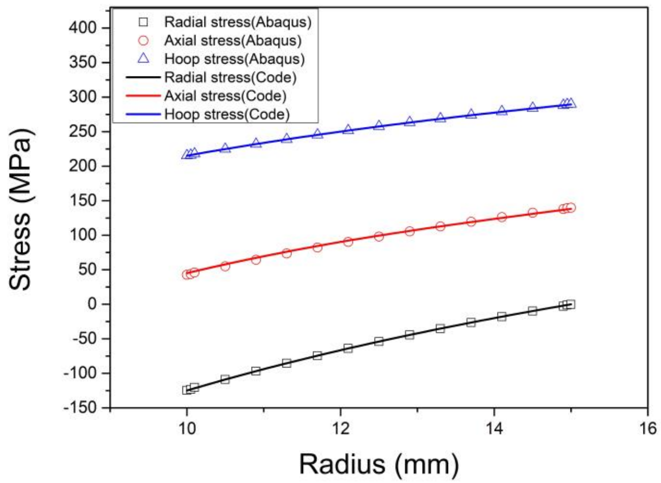
Axial displacement (z = 1.83 m)



Hoop strain (t = 604800 s)



Stress (t = 604800 s)



- High agreement with FEM calculation results in displacement or stress even in cases where the hoop strain exceeds 1%

Model Application: Case study

- ***Purpose***

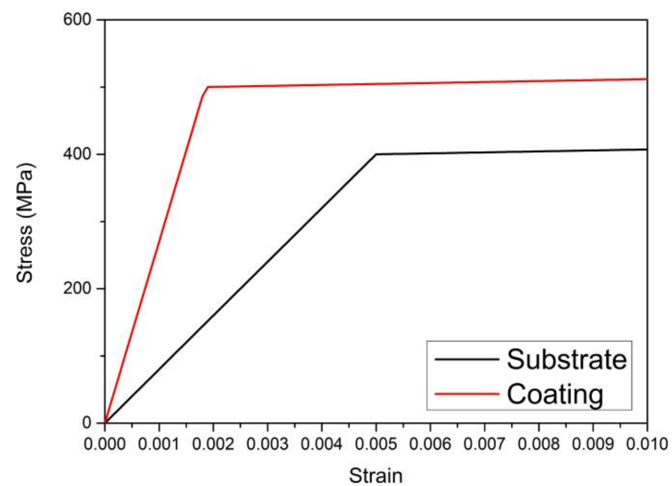
: To verify the reduction of stress in the substrate and the decrease in creep deformation when introducing a coating layer

Simulation condition

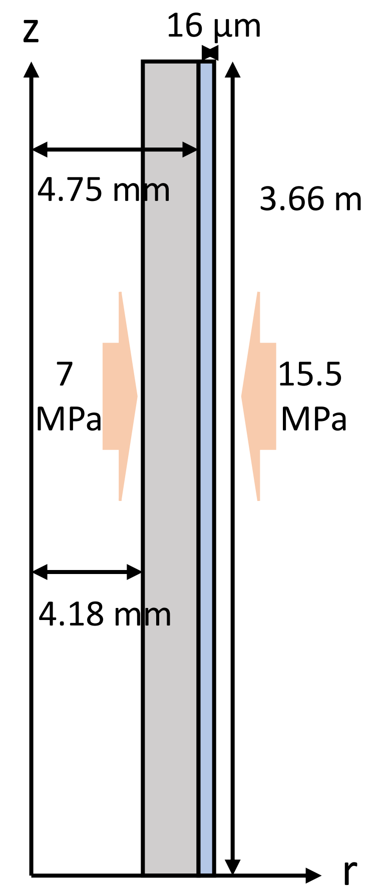
	Conditions	
Internal pressure (MPa)	7	
Outer pressure (MPa)	15.5	
Simulation time (days)	7	
Temperature (°C)	22.5	
	Substrate	**Coating**
Elastic modulus (GPa)	80	270
Poisson's ratio	0.3	0.21
Creep deformation	O	X

Creep correlation (FRAPCON)

$$\dot{\epsilon}_H = \frac{52\cdot\epsilon_P^S\cdot\dot{\epsilon}_{th}^{1/2}}{2\cdot t^{1/2}}\exp\left(-52\cdot\sqrt{\dot{\epsilon}_{th}\cdot t}\right)+\dot{\epsilon}_{th}$$

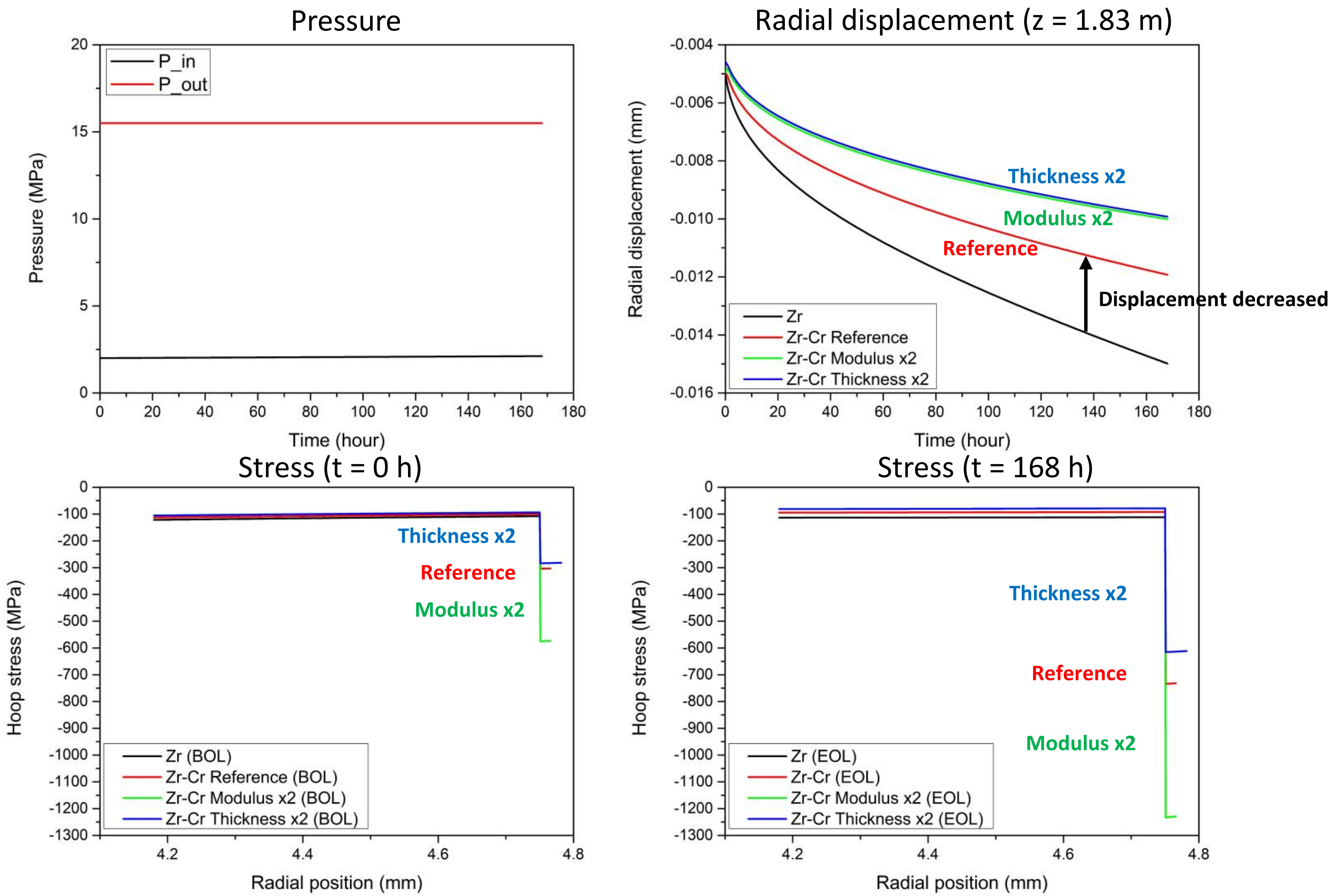


Stress strain curve of substrate and coating layer



Cladding geometry used for verification

Model Application: Case study

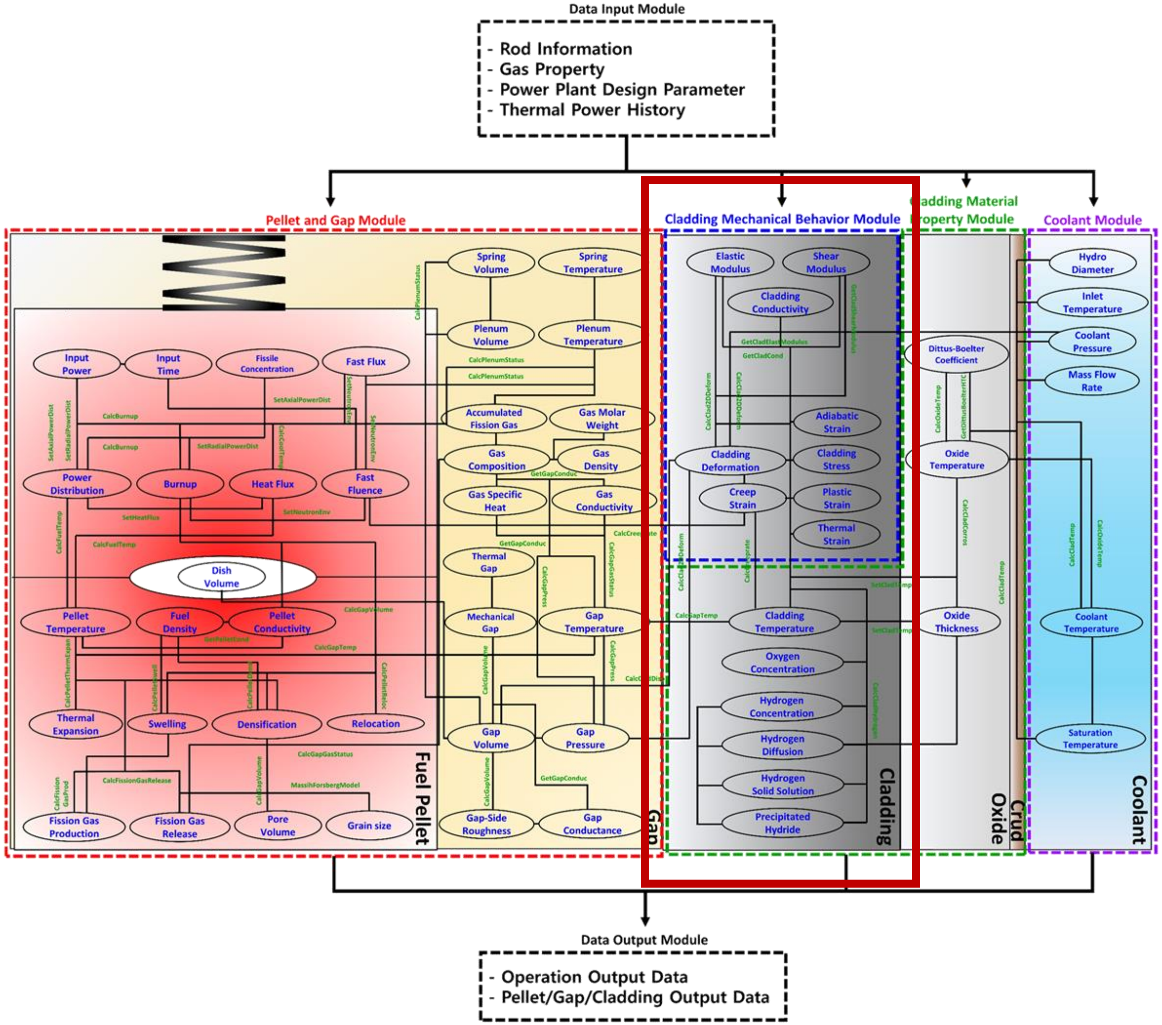


- The coating layer has a diminishing effect on creep deformation, which is enhanced with higher modulus or greater thickness of the layer.
- The coating layer helps alleviate substrate stress.

Model Application: GIFT

GIFT

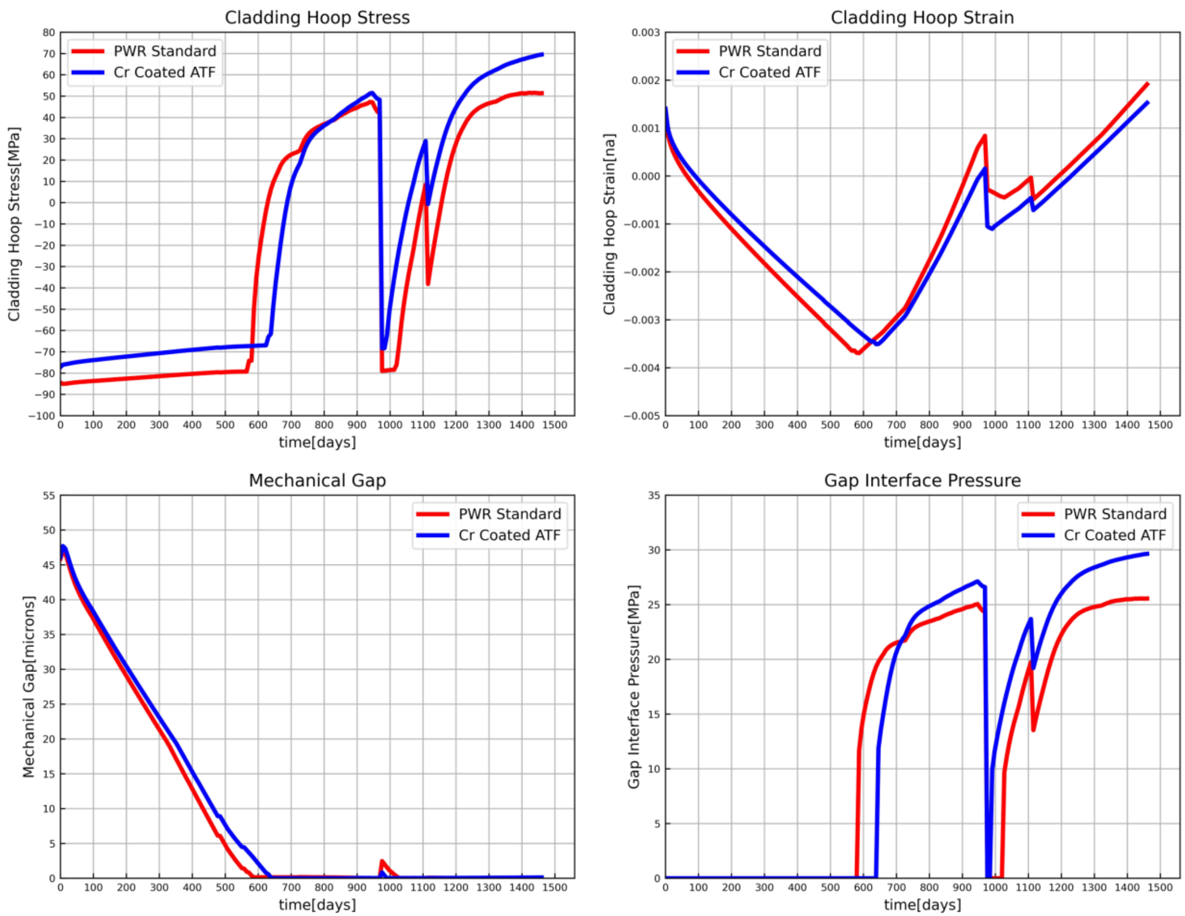
- ***LWR Nuclear Fuel Performance Analysis Code***
  : simulates normal operation and the entire spent fuel period in the reactor



Model Application: GIFT

Comparison between PWR standard fuel and ATF

- ***Change of Stress/Strain Distribution***
  : due to coating layer, less stress and less creep on Zr body, and late gap closure in ATF



Conclusion

- **Nuclear fuel cladding structure analysis code using 2D FDM has been developed.**

  1) The developed code is a versatile tool capable of simulating multi-layer, plastic deformation, creep deformation, and PCMI.

  2) It is fast and accurate high fidelity model that can analyze complex phenomena that can occur in the coated cladding in normal operation condition

  3) Verification completed with commercial FEM code with various test cases

Thank you for listening!

Acknowledgement

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