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Development and Stability Analysis of a Simulation Model for Small Modular Reactors using Padé Approximant

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1. Introduction

Small Modular Reactors (SMRs) are increasingly popular due to their smaller size, lower cost, and greater flexibility compared to traditional nuclear power plants. However, ensuring the safe and efficient operation of SMRs under all conditions is critical, as with any nuclear system. Therefore, developing a rapid and precise simulation model for SMRs is essential for the continued growth and success of the nuclear power industry. Tennessee Research has created a simulation model for the Babcock & Wilcox Generation mPower SMR using MATLAB-Simulink. The objective of this study is to evaluate whether the simulation model utilizing the Pade approximant via the Tennessee simulation model maintains accuracy.

2. Previous research and Padé approximant

2.1. Previous study

The system model was presented in the article "*Dynamic Modeling of a Small Modular Reactor for Control and Monitoring*" [1]. The Babcock & Wilcox Generation mPower Small Modular Reactor was used as the basis for developing the system model in the MATLAB environment. Simulations were performed using the ODE solver available in MATLAB-Simulink, with the Runge-Kutta method used for solving the majority of the ODEs. This numerical method is commonly used for solving differential equations, constructing a high-order accurate numerical method using only the function itself without computing higher-order derivatives. Therefore, this study aims to investigate the accuracy of the Padé approximant based on the simulation results obtained using the Runge-Kutta method.

2.2. Padé approximant

The Padé approximant is a method used to approximate a function using a rational function of a specific order in order to find the most accurate approximation. It is considered the best approximation method because it provides a more accurate approximation than polynomial approximations for many functions. Additionally, the Padé approximant is useful for modeling time delay effects in continuous-time systems.

For power series,

$$f(x) = c_0 + c_1 x + c_2 x^2 + \cdots = \sum_{i=1}^{\infty} c_i x^i \qquad (1)$$

The Padé approximant is as follows,

$$PA[M/N]_{f(x)}(x) = \frac{\sum_{i=1}^{M} a_i x^i}{1 + \sum_{i=1}^{N} b_i x^i} \ (M, N > 0) \qquad (2)$$

The following Equation can be obtained through Eq. (1) and Eq. (2).

$$\sum_{i=1}^{\infty} c_i x^i = f(x) = PA[M/N]_{f(x)}(x) + O(x^{M+N+1}) \qquad (3)$$

Reutrning to Eq. (3).

$$(c_0 + c_1 x + c_2 x^2 + \cdots)(1 + b_1 x + b_2 x^2 + \cdots + b_n x^n) = (a_0 + a_1 x + a_2 x^2 + \cdots + a_m x^M) + O(x^{M+N+1}) \qquad (4)$$

We can obtain the polynomial coefficients through the linear algebraic equations system.

$$\begin{aligned} b_N c_{M-N+1} + b_{N-1} c_{M-N+2} + \cdots\ c_{N+1} &= 0 \\ b_N c_{M-N+2} + b_{N-1} c_{M-N+3} + \cdots\ c_{N+2} &= 0 \\ &\vdots \\ b_N c_N + b_{N-1} c_{M+1} + \cdots\ c_{N+M} &= 0 \end{aligned} \qquad (5)$$

Where $c_j = 0$ at $j < 0$, we can determine the coefficients of b_i, and we also determine the coefficients of a_i.

$$\begin{aligned} a_0 &= c_0 \\ a_1 &= c_1 + b_1 c_0 \\ a_2 &= c_2 + b_1 c_1 + c_0 \\ &\vdots \\ a_M &= c_M + \sum_{i=1}^{M} b_i c_{M-i} \end{aligned} \qquad (6)$$

In this paper, we use the Padé approximant for the Taylor series general type. The Padé table is shown in Table I.

Table I: A portion of Padé table

M	N=0	N=1	N=2	N=3
0	$\frac{1}{1}$	$\frac{1}{1-x}$	$\frac{2}{2-2x+x^2}$	$\frac{6}{6-6x+3x^2-x^3}$
1	$\frac{1+x}{1}$	$\frac{2+x}{2-x}$	$\frac{6+2x}{6-4x+x^2}$	$\frac{24+6x}{24-18x+6x^2-x^3}$
2	$\frac{2+2x+x^2}{2}$	$\frac{6+4x+x^2}{6-2x}$	$\frac{12+6x+x^2}{12-6x+x^2}$	$\frac{60+24x+3x^3}{60-36x+9x^3-x^3}$
3	$\frac{6+6x+3x^3+x^3}{6}$	$\frac{24+18x+16x^2}{24-6x}$	$\frac{60+36x+9x^2+x^3}{60-24x+3x^2}$	$\frac{120+60x+12x^2+x^3}{120-60x+12x^2-x^3}$

Choosing an appropriate order can maintain accuracy while maximizing the computational time interval.

3. Simulation

When considering the following state-space equation,

$$\frac{dx}{dt} = Ax(t) + Bf(t) \quad (7)$$

Using the Padé approximant, Eq. (7) can express as follows,

$$x(t + \triangle t) = e^{A\triangle t} + (e^{A\triangle t} - I)A^{-1}Bf \quad (8)$$

3.1. Simulation model

When the reactor is operating at a steady state, at time *t=0*, the system experiences a positive reactivity insertion of 10 cents step and we illustrate the system's response in a transient simulation.

In order to obtain simple simulation results, we only simulated the reactor core model. The inlet temperature of the core was determined using the graph from Ref. [1]

3.2. Result

Without the T-avg Controller operating, the simulation was performed with a time step of $\Delta t = 0.1\ sec$ and the results are as follows:

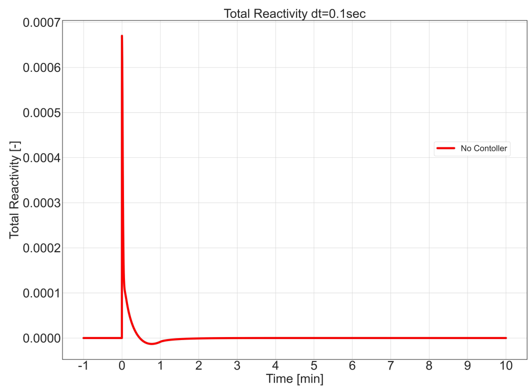

Fig. 1. Total Reactivity (Δ*t=0.1 sec*)

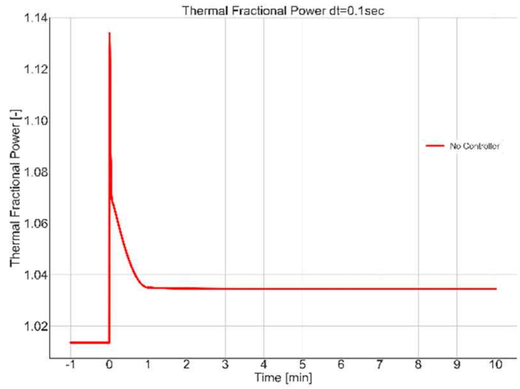

Fig. 2. Thermal Fractional Power (Δt=0.1 sec)

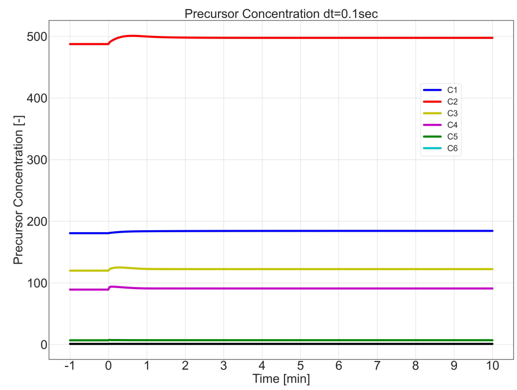

Fig. 3. Precursor Concentration (Δt=0.1 sec)

In Fig. 1, 2, and 3, Total Reactivity, Thermal Fractional Power, and Precursor Concentration are represented, respectively.

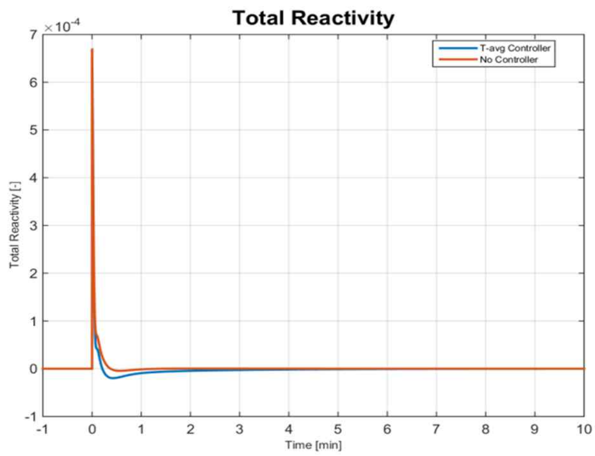

Fig. 4. Total Reactivity in Ref. [1]

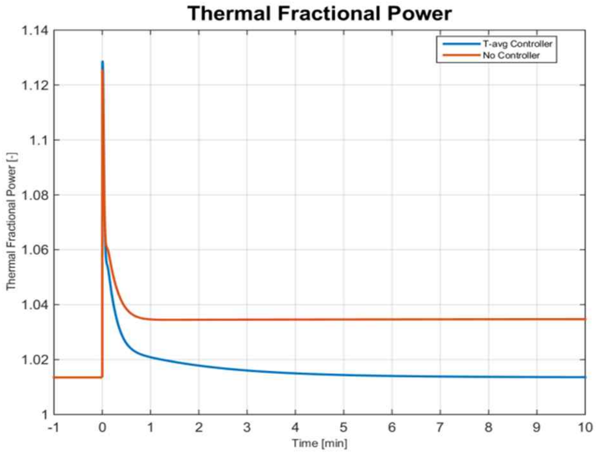

Fig. 5. Thermal Fractional Power in Ref. [1]

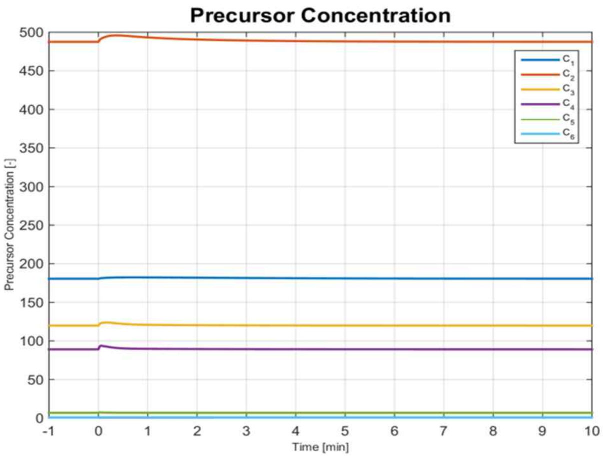

Fig. 6. Precursor Concentration in Ref. [1]

It can be observed that the simulation results demonstrate similarity to those presented in Ref. [1]. the above results indicate that the Padé approximant maintains accuracy.

4. Conclusion

In this study, we assessed the stability of a simulation model for the Babcock & Wilcox Generation mPower SMR using the Padé approximant. The simulation results presented here are based on a simulation that is only the reactor core model, rather than the entire model. Therefore, some errors have occurred, but they are generally similar. The simulation results demonstrated that the Padé approximant is an accurate and reliable method for simulating SMR responses.

In addition to the Padé method, simulations using the Euler method were also conducted. The results showed that the Euler method was able to similarly simulate the system with a time step of Δt=0.01 sec, but the values diverged when the time step was increased beyond this value. Overall, these results demonstrate that a simulation model based on Padé approximant can provide similar stability to the conventional Runge-Kutta method, while also being faster and simpler. This can be used to develop faster simulation models for the entire nuclear reactor system.

Based on these findings, efforts are underway to develop an SMR model that can provide faster simulation results for the entire SMR system. This model will allow for faster and more accurate simulations of the SMR's behavior under various operating conditions. The development of such a simulation model is expected to contribute to the development of safe and efficient SMR by enabling fast and diverse simulations of SMR.

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