Depletion Analysis of Spatial Self-shielding Effect for Double Heterogeneous Region Near Burnable Poison

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1. Introduction

Recently, an effective homogenized cross section method with explicit TRISO model was proposed to treat the spatial self-shielding factor for a non-resonant double heterogeneous (DH) region such as VHTR fuel compact in the thermal energy range [1]. It is known that the spatial self-shielding effect is not negligible in a fuel compact near burnable poison. [1, 2]

The method was developed based on a spherical unit cell model with explicit coated layers and a matrix layer, which is consistent MOC calculation region with PSM-DH [3] for the DH resonance treatment. It also adopted the conventional collision probability solution method which has been well defined for general spherical cell model.

In this paper, the depletion analysis of the effective homogenized cross section method is examined using VHTR mini block problems with burnable poison and the change of the spatial self-shielding effect with the depletion is investigated.

2. Review of the Effective Homogenized Cross Section Method

The effective homogenized cross section [4] for a compact model can be derived by preserving the total reaction as follows:

$$\tilde{\Sigma}\bar{\phi}V = \sum_i \Sigma_i \phi_i V_i , \qquad (1)$$

where

i : sub-region index for matrix and particle layers,

$\tilde{\Sigma}$: effective homogenized total macroscopic cross section,

$\bar{\phi}$: homogenized flux for a compact,

V : volume of a compact,

Σ_i : total macroscopic cross section at sub-region *i*,

ϕ_i : flux at sub-region *i*,

V_i : volume of sub-region *i*.

Eq. (1) can be rewritten with self-shielding factor and volume fraction as follows:

$$\tilde{\Sigma} = \sum_i \Sigma_i \frac{\phi_i}{\bar{\phi}} \frac{V_i}{V} = \sum_i \Sigma_i f_i p_i \ , \qquad (2)$$

where

$f_i = \dfrac{\phi_i}{\bar{\phi}}$: self-shielding factor for sub-region *i*,

$p_i = \dfrac{V_i}{V}$: volume fraction for sub-region *i*.

Considering the relation between the collision probability and the reaction rate, the ratio of the collision probability in a sub-region and the collision probability in all sub-region should be equal to the ratio of the reactions in the same regions. This relation can be expressed as follows:

$$\frac{P_i}{\sum_i P_i} = \frac{\Sigma_i \phi_i V_i}{\sum_i \Sigma_i \phi_i V_i} = \frac{\Sigma_i \phi_i V_i}{\tilde{\Sigma}\bar{\phi}V} = \frac{\Sigma_i p_i}{\tilde{\Sigma}} f_i \ , \qquad (3)$$

where P_i is first collision probability at sub-region *i* for the particles entering boundary surface.

Then, the self-shielding factor can be derived as follows:

$$f_i = \frac{\tilde{\Sigma}}{p_i \Sigma_i} \frac{P_i}{\sum_i P_i} , \qquad (4)$$

If $\tilde{\Sigma}$ for the compact model and P_i at all sub-regions are known, the self-shielding factors for all sub-regions can be calculated.

In order to consider the explicit particle layers without the mathematical complexity, the spherical unit cell model as shown in Figure 1 was proposed in our previous work [1]. The collision probabilities for the multi-layered spherical geometry can be obtained using well-defined collision probability solution methods.

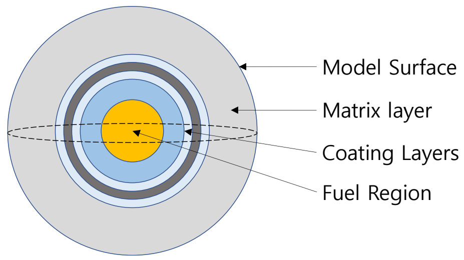


Fig. 1. Spherical unit cell model with a graphite matrix layer and a TRISO particle for a fuel compact

After obtaining the collision probability, P_{ij}, which is defined as the probability that a neutron born in sub-region i has its first collision at sub-region j, the P_i can be obtained using the following relations [5]:

$$P_i = \bar{l} p_i \Sigma_i \left(1 - \sum_j P_{ij}\right), \quad (5)$$

where $\bar{l}$ is the average chord length through the model surface.

For obtaining the effective homogenized total macroscopic cross section, $\tilde{\Sigma}$, in the Eq. (4), the transmission probability for the homogenized spherical compact as shown in Figure 2 can be derived.

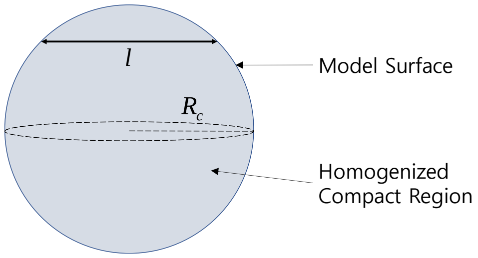


Fig. 2. Homogenized spherical compact model

The transmission probability for the whole spherical compact can be expressed using as in the following:

$$T = \int_0^{2R_c} t(l) f(l) dl = \int_0^{2R_c} e^{-\tilde{\Sigma} l} \frac{l}{2R_c^2} dl , \quad (6)$$

where $t(l)$ is the transmission probability without any collisions in a chord, l, and $f(l)$ is the probability density function for the chord length.

After integrating Eq. (6), the solution can be obtained as follows:

$$T = \frac{1}{2R_c^2 \tilde{\Sigma}^2}\left(1 - e^{-2R_c\tilde{\Sigma}}(1 + 2R_c\tilde{\Sigma})\right) . \quad (7)$$

If applying Taylor expansion with fourth order for the exponential function, the effective cross section can be approximated as follows:

$$\tilde{\Sigma} \cong \frac{1}{2R_c}\left(1 - \frac{5}{\sqrt[3]{3\hat{T}}} + \frac{\hat{T}}{\sqrt[3]{9}}\right), \quad (8)$$

where

$$\hat{T} = \sqrt[3]{27 - 54T + 2\sqrt{3}\sqrt{92 - 243T + 243T^2}} .$$

Because the transmission probability for the heterogeneous sphere model is identical to the transmission probability for the homogeneous sphere model, T in Eq.(8) is calculated as follows:

$$T = 1 - \sum_i P_i . \quad (9)$$

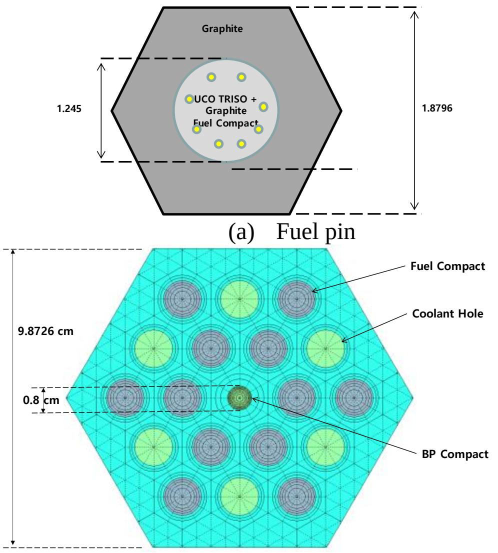
Finally, the self-shielding factor, Eq. (4), can be calculated from Eq. (5), (8), and (9).

3. Numerical Results

For depletion analysis of the spatial self-shielding effect, OECD/NEA MHTGR-350 benchmark [6] were applied in this work. Figure 3 shows the configuration of the small size single fuel block which consists of the MHTGR-350 fuel pins and burnable poison pin with homogeneous mixture in the center.

The calculation was performed using DeCART with ENDF/B-VII.1. The DH effect for the resonance nuclide and resonance energy range was calculated using PSM-DH module [3] of DeCART. On the contrary, the self-shielding factors for the fuel compact in the thermal energy range were obtained using the above described effective homogenization method. It is noted that the DH effect for the resonance energy range is outside the scope of this work.

The references for accuracy comparison were obtained using the McCARD [7] based on the Monte Carlo method.



(a) Fuel pin

(b) Single Block

Fig. 3. Configuration of the MHTGR single block problem with DH region

Table 1 compares the k_{inf} and DH effect for the fuel block problem. It is observed that the DH effect in the thermal range are not small (about 370 pcm) in the initial burnup step and decreases with depletion. Also, it can be seen that the DH effect is negligible over 500 days considering the DH effect in the resonance range (about 4500 pcm) [3], because the BP effect almost disappear over the step. It is known that the DH effect of fuel compact near BP is originated from the change of the thermal utilization caused by the self-shielding effect in the thermal range [2].

Thus, it is clear that the method can predict the DH effect in the thermal energy range for the depletion problem and the effect for fuel block with BP must be taken into consideration.

4. Conclusions

In this work, the depletion analysis of the recently proposed effective homogenized cross section method was examined using VHTR mini block problems with burnable poison and the change of the spatial self-shielding effect with the depletion was investigated

The calculations revealed that the DH effect in the thermal range are not small in the initial burnup step and decreases with depletion. Also, it shows that the effect is negligible when the BP effect disappear.

ACKNOWLEDGEMENT

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Table 1. DH effect change with depletion in the thermal energy range

Burnup (Days)	TEMP=1200K, PF=35%					
	McCARD (M) (σ≈14 pcm)	DeCART with Hom.[1](H)	DeCART with Ehom.[2](E)	Diff. (H-M) (pcm)	Diff. (E-M) (pcm)	DH Effect[3]
0	1.14800	1.15250	1.14755	450	-45	-374
25	1.13260	1.13659	1.13179	399	-81	-374
50	1.14043	1.14377	1.13922	333	-121	-349
100	1.15520	1.15808	1.15399	288	-121	-306
150	1.16715	1.16997	1.16630	282	-85	-269
200	1.17722	1.17953	1.17627	231	-95	-235
250	1.18503	1.18714	1.18430	211	-73	-202
300	1.19145	1.19310	1.19069	165	-76	-170
350	1.19610	1.19763	1.19565	153	-45	-138
400	1.20010	1.20089	1.19933	79	-77	-109
450	1.20250	1.20299	1.20183	49	-67	-80
500	1.20387	1.20402	1.20324	15	-63	-54
625	1.20272	1.20244	1.20245	-28	-27	0
750	1.19603	1.19571	1.19626	-32	23	38
875	1.18548	1.18484	1.18569	-64	21	60
1000	1.17100	1.17077	1.17170	-23	70	68
1125	1.15419	1.15433	1.15517	14	98	63
1250	1.13581	1.13620	1.13678	39	97	45
1375	1.11604	1.11685	1.11704	81	100	15
1500	1.09544	1.09662	1.09627	118	83	-29

[1] Volume weighted homogenization for thermal energy range

[2] Effective homogenization method for thermal energy range

[3] $\frac{1}{k_H} - \frac{1}{k_E}$