Transactions of the Korean Nuclear Society Spring Meeting
Jeju, Korea, May 19-20, 2022

Development of Point Kinetics Equation Considering Delayed Photoneutrons

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1. Introduction

It is known that *photoneutrons* can be produced from photonuclear reactions of delayed photons emitted from decays of fission products with reactor core components. Deuterium and beryllium [1] are the major photonuclear isotopes because of their low threshold energy of 2.23 MeV and 1.67 MeV, respectively. The effect of delayed photoneutrons on the reactor dynamics has been studied [1-4] in research reactors that contain deuterium or beryllium in the coolant or reflector region.

One of the related topics is to develop a point kinetics model considering the delayed photoneutrons. In most of the previous studies [2-4], the delayed photoneutrons are treated as additional delayed neutron groups in the conventional form of the point kinetics equation (PKE). Jatuff et al. [5] derived neutron/photon-coupled PKE starting from an exact form of the neutron/photon-coupled transport equations. For the simplicity of its derivation, they applied the instantaneous photon transport approximation which assumes that the photon flux reaches its equilibrium almost immediately.

This paper aims to derive a PKE considering delayed photoneutrons without any approximations with clear definitions of adjoint weighting functions, starting from the exact neutron/photon-coupled transport equations. The proposed PKE can be used to accurately analyze the reactor dynamics for a reactor core where the photoneutron generation cannot be neglected.

2. Methodology

2.1 The exact neutron/photon-coupled Boltzmann transport equations

The neutron/photon-coupled transport equations and delayed neutron/photon precursor density equations can be expressed as

$$\frac{1}{v^n}\frac{\partial}{\partial t}\phi^n = -\mathbf{M}^n\phi^n + \mathbf{F}_p^n\phi^n + \sum_i \lambda_i c_i^n + \mathbf{\Gamma}\phi^\gamma \tag{1}$$

$$\frac{1}{v^\gamma}\frac{\partial}{\partial t}\phi^\gamma = -\mathbf{M}^\gamma\phi^\gamma + \mathbf{F}_p^\gamma\phi^\gamma + \mathbf{M}_{(n,\gamma)}^n\phi^n + \sum_j \lambda_j c_j^\gamma \tag{2}$$

$$\frac{\partial}{\partial t}c_i^n = \mathbf{F}_{di}^n\phi^n - \lambda_i c_i^n \tag{3}$$

$$\frac{\partial}{\partial t}c_j^\gamma = \mathbf{F}_{dj}^\gamma\phi^n - \lambda_j c_j^\gamma \tag{4}$$

$$\begin{aligned}\mathbf{M}^n\phi^n &= \Omega\cdot\nabla\phi^n(r,E,\Omega,t) + \Sigma_t(r,E,t)\phi^n(r,E,\Omega,t)\\ &-\int dE'\int d\Omega'\Sigma_s(r,E'\to E,\Omega'\to\Omega,t)\phi^n(r,E,\Omega,t)\end{aligned} \tag{5}$$

$$\mathbf{F}_p^n\phi^n = \frac{\chi_p^n(r,E,t)}{4\pi}\int dE'\int d\Omega'\,\nu_p^n(r,E',t)\Sigma_f(r,E',t)\phi^n(r,E',\Omega',t) \tag{6}$$

$$\mathbf{\Gamma}\phi^\gamma = \frac{\chi^{(\gamma,n)}(r,E,t)}{4\pi}\int dE'\int d\Omega'\Sigma_{(\gamma,n)}(r,E',t)\phi^\gamma(r,E',\Omega',t) \tag{7}$$

$$\begin{aligned}\mathbf{M}^\gamma\phi^\gamma &= \Omega\cdot\nabla\phi^\gamma(r,E,\Omega,t) + \mu(r,E,t)\phi^\gamma(r,E,\Omega,t)\\ &+\Sigma_{(\gamma,n)}(r,E,t)\phi^\gamma(r,E,\Omega,t)\\ &-\int dE'\int d\Omega'\mu_c(r,E'\to E,\Omega'\to\Omega,t)\phi^\gamma(r,E',\Omega',t)\end{aligned} \tag{8}$$

$$\mathbf{F}_p^\gamma\phi^\gamma = \frac{\chi_p^\gamma(r,E,t)}{4\pi}\int dE'\int d\Omega'\,\nu_p^\gamma(r,E',t)\Sigma_f(r,E',t)\phi^n(r,E',\Omega',t) \tag{9}$$

$$\mathbf{M}_{(n,\gamma)}^n\phi^n = \int dE'\int d\Omega'\Sigma_{(n,\gamma)}(r,E'\to E,\Omega'\to\Omega,t)\phi^n(r,E',\Omega',t) \tag{10}$$

$$\mathbf{F}_{di}^n\phi^n = \frac{\chi_{di}^n(r,E,t)}{4\pi}\int dE'\int d\Omega'\,\nu_{di}^n(r,E',t)\Sigma_f(r,E',t)\phi^n(r,E',\Omega',t) \tag{11}$$

$$\mathbf{F}_{dj}^\gamma\phi^n = \frac{\chi_{dj}^\gamma(r,E,t)}{4\pi}\int dE'\int d\Omega'\,\nu_{dj}^\gamma(r,E',t)\Sigma_f(r,E',t)\phi^n(r,E',\Omega',t) \tag{12}$$

The definition for operators used in Eq. (1) to Eq. (4) is described in Eq. (5) to Eq. (12). ϕ denotes the angular flux, $\phi(r,E,\Omega,t)$, and subscripts n and γ mean that the corresponding variables are for neutron and photon, respectively. In the same manner, subscripts (n,γ) and (γ,n) mean the operator or variables are for that reaction. c_i^n and c_j^γ are i-th group neutron precursor density and j-th group photon precursor density. The operator $\mathbf{\Gamma}$ is for the photoneutron production from photon and $\mathbf{M}_{(n,\gamma)}^n$ is for the photons induced from neutron reaction except for fission. μ is the total photoatomic cross section which is the sum of the cross section of photoelectric reactions, pair productions, and Compton scattering. μ_c denotes Compton scattering.

2.2 Definition of photoneutron production operator

For the further derivations, Eq. (2) can be written as Eq. (13) by defining the total cross section of a photon as Eq. (14) and grouping the photon production term into Compton scattering term in Eq. (16) and neutron originated term in Eq. (17).

$$\frac{1}{v^\gamma}\frac{\partial}{\partial t}\phi^\gamma(r,E,\Omega,t)+\Omega\cdot\nabla\phi^\gamma(r,E,\Omega,t)$$
$$+\Sigma_t^\gamma(r,E,t)\phi^\gamma(r,E,\Omega,t)=S^\gamma(r,E,\Omega,t) \tag{13}$$

$$\Sigma_t^\gamma(r,E,t)=\mu(r,E,t)+\Sigma_{(\gamma,n)}(r,E,t) \tag{14}$$

$$S^\gamma(r,E,\Omega,t)=S_s^\gamma(r,E,\Omega,t)+Q^{n\to\gamma}(r,E,\Omega,t) \tag{15}$$

$$S_s^\gamma(r,E,\Omega,t)=\int dE'\int d\Omega'\mu_c(r,E'\to E,\Omega'\to\Omega,t)\phi^\gamma(r,E',\Omega',t) \tag{16}$$

$$Q^{n\to\gamma}(r,E,\Omega,t)=\frac{\chi_p^\gamma(r,E,t)}{4\pi}$$
$$\times\int dE'\int d\Omega'\nu_p^\gamma(r,E',t)\Sigma_f(r,E',t)\phi^n(r,E',\Omega',t)$$
$$+\int dE'\int d\Omega'\Sigma_{(n,\gamma)}(r,E'\to E,\Omega'\to\Omega,t)\phi^n(r,E',\Omega',t)$$
$$+\sum_j\lambda_j c_j^\gamma(r,E,\Omega,t) \tag{17}$$

By applying the method of characteristic, Eq. (13) can be written as

$$\frac{d}{ds}\phi^\gamma(r_0+s\Omega,E,\Omega,t_0+\frac{s}{v})+\Sigma_t^\gamma\phi^\gamma=S^\gamma(r_0+s\Omega,E,\Omega,t_0+\frac{s}{v}) \tag{18}$$

By introducing an integrating factor on both sides of the equation to solve this integral equation, we can yield the photon flux as the following expression.

$$\phi^\gamma(r,E,\Omega,t)=\int_0^\infty\exp\left(-\int_0^{s'}\Sigma_t^\gamma(r-s''\Omega,E)ds''\right)\cdot S^\gamma(r-s'\Omega,E,\Omega,t-\frac{s'}{v})ds' \tag{19}$$

We can now define photon collision density as $\psi^\gamma(r,E,\Omega,t)=\Sigma_t^\gamma(r,E,t)\phi^\gamma(r,E,\Omega,t)$ and follow the conventional step of getting Neumann series solution as we did while solving the neutron collision density equation. Finally, we yield Eq. (20) when *P=(r,E,Ω,t)*. Here, it can be noted that K_j^γ is a multiplication of T^γ, C_s^γ which are transition kernel and scattering collision kernel of the photon.

$$\psi^\gamma(r,E,\Omega,t)=\sum_{j=0}^\infty\int dP'K_j^\gamma(P'\to P)Q^{n\to\gamma}(P') \tag{20}$$

$$K_j^\gamma(r',E',\Omega'\to r,E,\Omega)=T^\gamma(E,\Omega;r'\to r)C_s^\gamma(r';E',\Omega'\to E,\Omega) \tag{21}$$

Now let us introduce Eq. (20) into the photoneutron production term $\mathbf{\Gamma}\phi^\gamma$ in Eq. (1) following the procedure described in Eq. (22), and define the photoneutron production operator **D** as Eq. (23). By revisiting the definition of $Q^{n\to\gamma}$ described in Eq. (17), Eq. (24) can be derived. It can be noted that the photon transport equation is merged with the neutron transport equation successfully by exploiting the Neumann series solution of the photon transport equation.

$$\mathbf{\Gamma}\phi^\gamma=\frac{\chi^{(\gamma,n)}(r,E,t)}{4\pi}\int dE'\int d\Omega'\Sigma_{(\gamma,n)}(r,E',t)\phi^\gamma(r,E',\Omega',t)$$
$$=\frac{\chi^{(\gamma,n)}(r,E,t)}{4\pi}\int dE'\int d\Omega'\frac{\Sigma_{(\gamma,n)}(r,E',t)}{\Sigma_t^\gamma(r,E',t)}\psi^\gamma(r,E',\Omega',t)$$
$$=\frac{\chi^{(\gamma,n)}(r,E,t)}{4\pi}\int dE'\int d\Omega'\frac{\Sigma_{(\gamma,n)}(r,E',t)}{\Sigma_t^\gamma(r,E',t)}$$
$$\times\sum_{j=0}^\infty\int dr'\int dE''\int d\Omega''K_j(r',E'',\Omega''\to r,E',\Omega')\int dr''T(E'',\Omega'';r''\to r')Q^{n\to\gamma}(r'',E'',\Omega'',t'')$$
$$=\mathbf{D}Q^{n\to\gamma} \tag{22}$$

$$\mathbf{D}Q^{n\to\gamma}\equiv\frac{\chi^{(\gamma,n)}(r,E,t)}{4\pi}\int dE'\int d\Omega'\frac{\Sigma_{(\gamma,n)}(r,E',t)}{\Sigma_t^\gamma(r,E',t)}$$
$$\times\sum_{j=0}^\infty\int dr'\int dE''\int d\Omega''K_j(r',E'',\Omega''\to r,E',\Omega')$$
$$\times\int dr''T(E'',\Omega'';r''\to r')Q^{n\to\gamma}(r'',E'',\Omega'',t'') \tag{23}$$

$$\mathbf{\Gamma}\phi^\gamma=\mathbf{D}Q^{n\to\gamma}=\mathbf{D}(\mathbf{F}_p^\gamma\phi_n+\mathbf{M}_{(n,\gamma)}^n\phi_n+S_d^\gamma)$$
$$=\mathbf{D}(\mathbf{F}_p^\gamma\phi_n+\sum_j\mathbf{F}_{dj}^\gamma\phi_n)-\mathbf{D}\sum_j\mathbf{F}_{dj}^\gamma\phi_n+\mathbf{D}\mathbf{M}_{(n,\gamma)}^n\phi_n+\mathbf{D}S_d^\gamma$$
$$=\mathbf{D}\mathbf{F}^\gamma\phi_n-\mathbf{D}\sum_j\mathbf{F}_{dj}^\gamma\phi_n+\mathbf{D}\mathbf{M}_{(n,\gamma)}^n\phi_n+\mathbf{D}S_d^\gamma \tag{24}$$

2.3 Derivation of point kinetics equations considering photoneutrons

Introducing the relation in Eq. (24), Eq. (1) can be expressed as

$$\frac{1}{v^n}\frac{\partial\phi_n}{\partial t}=-\mathbf{M}^n\phi_n+\mathbf{F}_p^n\phi_n+S_d^n+\mathbf{\Gamma}\phi_n$$
$$=-\mathbf{M}^n\phi_n+(\mathbf{F}_p^n\phi_n+\sum_i\mathbf{F}_{di}^n\phi_n)-\sum_i\mathbf{F}_{di}^n\phi_n+S_d^n+\mathbf{\Gamma}\phi_n$$
$$=-\mathbf{M}^n\phi_n+\mathbf{F}^n\phi_n-\sum_i\mathbf{F}_{di}^n\phi_n+S_d^n+\mathbf{\Gamma}\phi_n \tag{25}$$
$$=\mathbf{F}^n\phi_n+\mathbf{D}\mathbf{F}^\gamma\phi_n+\mathbf{D}\mathbf{M}_{(n,\gamma)}^n\phi_n-\mathbf{M}^n\phi_n$$
$$-(\sum_i\mathbf{F}_{di}^n\phi_n+\mathbf{D}\sum_j\mathbf{F}_{dj}^\gamma\phi_n)+S_d^n+\mathbf{D}S_d^\gamma$$

As in the conventional derivation of PKE, we can multiply Eq. (25) by adjoint neutron flux ϕ_0^*, separate the neutron flux into an amplitude function *P(t)* and shape function $\psi(r,E,\Omega,t)$ with a normalization condition, which yields Eq. (28).

$$\phi^n(r,E,\Omega,t) = P(t)\cdot\psi(r,E,\Omega,t) \tag{26}$$

$$\frac{\partial}{\partial t}\left\langle \phi_0^*, \frac{\psi}{v^n}\right\rangle = 0 \tag{27}$$

$$\begin{aligned}\left\langle \phi_0^*, \frac{\psi}{v^n}\right\rangle \frac{dP(t)}{dt} &= \left\langle \phi_0^*, (\mathbf{F}^n + \mathbf{DF}^\gamma + \mathbf{DM}^n_{(n,\gamma)} - \mathbf{M}^n)\psi\right\rangle P \\ &- \left\langle \phi_0^*, (\mathbf{F}^n_d + \mathbf{DF}^\gamma_d)\psi\right\rangle P + \left\langle \phi_0^*, S^n_d\right\rangle + \left\langle \phi_0^*, \mathbf{D}S^\gamma_d\right\rangle\end{aligned} \tag{28}$$

Let us define the normalization constant I as Eq. (29) and divide both sides of Eq. (28) with it.

$$I = \left\langle \phi_0^*, \frac{\psi}{v^n}\right\rangle \tag{29}$$

Finally, the PKE considering delayed photoneutrons can be obtained as

$$\frac{dP(t)}{dt} = \frac{\rho(t)-\beta(t)}{\Lambda(t)}P(t) + \sum_i \lambda_i C_i^n(t) + \sum_j \lambda_j C_j^\gamma(t) \tag{30}$$

$$F(t) = \left\langle \phi_0^*, (\mathbf{F}^n + \mathbf{DF}^\gamma)\psi\right\rangle \tag{31}$$

$$\rho(t) = \left\langle \phi_0^*, (\mathbf{F}^n + \mathbf{DF}^\gamma + \mathbf{DM}^n_{(n,\gamma)} - \mathbf{M}^n)\psi\right\rangle \Big/ F(t) \tag{32}$$

$$\beta_i^n(t) = \left\langle \phi_0^*, \mathbf{F}^n_{di}\psi\right\rangle \Big/ F(t) \tag{33}$$

$$\beta_j^\gamma(t) = \left\langle \phi_0^*, \mathbf{DF}^\gamma_{dj}\psi\right\rangle \Big/ F(t) \tag{34}$$

$$\begin{aligned}\beta(t) &= \left\langle \phi_0^*, (\mathbf{F}^n_d + \mathbf{DF}^\gamma_d)\psi\right\rangle \Big/ F(t) \\ &= \sum_i \beta_i^n(t) + \sum_j \beta_j^\gamma(t)\end{aligned} \tag{35}$$

$$\Lambda(t) = \frac{I}{F(t)} \tag{36}$$

$$C_i^n(t) = \left\langle \phi_0^*, c_i^n\right\rangle \Big/ I \tag{37}$$

$$C_j^\gamma(t) = \left\langle \phi_0^*, \mathbf{D}c_j^\gamma\right\rangle \Big/ I \tag{38}$$

In the same manner used to derive Eq. (28), the PKE for delayed neutron and delayed photon precursor can be obtained as

$$\frac{dC_i^n(t)}{dt} = \frac{\beta_i^n(t)}{\Lambda(t)}P(t) - \lambda_i C_i^n(t) \tag{39}$$

$$\frac{dC_j^\gamma(t)}{dt} = \frac{\beta_j^\gamma(t)}{\Lambda(t)}P(t) - \lambda_j C_j^\gamma(t) \tag{40}$$

It can be noted from Eq. (31) that the neutron production term from fission, *F(t),* includes the contribution of photoneutron production from fission gammas. Eq. (34) gives a clear physical meaning of effective delayed photoneutron fraction as a delayed photoneutron production weighted with the neutron adjoint flux among the total neutron production from fission.

3. Conclusions and Future Work

The PKE considering photoneutrons is derived from neutron/photon-coupled transport equations without any approximations. The photon transport equation is successfully merged with a neutron transport equation by deriving its Neumann series solution and introducing the delayed photoneutron production operator.

The MC algorithm for the estimation of kinetics parameters can be easily obtained from the developed PKE in this paper [6]. As photonuclear physics is recently implemented in McCARD [7], delayed neutron fraction and delayed photoneutron fraction can be calculated during a forward k-eigenvalue mode simulation. After implementing a function to handle delayed photon production in every fission event, we plan to apply it to reactor dynamics analysis in research reactors.

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