The Study for Modeling of Spray Droplet Shape in 3-Dimensional Condition

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1. INTRODUCTION

The containment spray system removes fission products in atmosphere in containment. The shape of spray droplets has the pattern of behavior like ellipse objects or rain droplets. In this field, Lee's have carried out the shape modeling of spray droplets in 2012[1]. And also, fission products removal modeling study has carried out by Lee [1-2]. In this study, to promote Lee's model, the new advanced methodology is introduced. This study's object is focused to reduce the number of the modeling parameters. And that is based on the mathematical equation [1-4]. Here, the volume and surface area of spray droplet are calculated. Also, these are compared with experimental study in condition of non-spherical in falling mechanics [1-4]. The surface of spray droplets is efficiently calculated using new methodology. The mathematical technique in this work is based on ellipse equations and ellipse geometry.

3-dimensional surface area of water droplet and droplet size are simulated by Monte Carlo simulation. From these results, fission product's removal rate (removal efficiency) is generated using flow regime. Surface area and fission product's removal rate can be used to study the behavior of fission product's in containment of NPP in design basic accidents [1-8].

2. METHODOLOGY

2.1. Spray Droplet Size, Surface Area and Volume

Consider an ellipsoid centered at the coordinate origin, with rectangular Cartesian coordinate axes along the semi-axes a, b, c,

$$\frac{x^2}{a^2}+\frac{Y^2}{b^2}+\frac{z^2}{c^2}=1 \quad (1)$$

Spray droplet shape is similar to flat-ellipsoid and also its shape on the eccentricity δ.

The form and the surface strongly effect on the function of removing fission products in containments.

Generally, for the case in which two axes are equal to b=c, the surface is generated by rotation around the x-axis of the half-ellipse of equation (1) with Y>0.

On that half-ellipse, $dy/dx = -b^2x/(a^2y)$, and hence the surface area of the spheroid is written as below:

$$A = 2\int_0^a 2\pi y\sqrt{1+\frac{b^4x^2}{a^4y^2}}\,dx = 4\pi\int_0^a y\sqrt{y^2+\frac{b^4x^2}{a^4}}\,dx \quad (2)$$

$$A = 4\pi b\int_0^a \sqrt{1-\frac{x^2}{a^2}+\frac{b^2}{a^2}\frac{x^2}{a^2}}\,dx \quad (3)$$

$$A = 4\pi ab\int_0^1 \sqrt{1-\left(1-\frac{b^2}{a^2}\right)u^2}\,du \quad (4)$$

$$A = 4\pi ab\int_0^1 \sqrt{1-\delta u^2}\,du \quad (5)$$

Where, u=x/a and $\delta=1-\frac{b^2}{a^2}$, which is used to replace integral. $\delta=1-\frac{b^2}{a^2}$ is known as eccentricity for flatness of ellipsids.

Where, this equation can be selected by three options as below:

Option1: a>b

$$A = 2\pi b\left(a\times\frac{\arcsin\sqrt{\delta}}{\sqrt{\delta}}+b\right) \quad (6)$$

Option2: a=b

$$A = 2\pi b(a+b) = 4\pi a^2 \quad (7)$$

Option3: a<b

$$A = 2\pi b\left(a\times\frac{\operatorname{arcsinh}\sqrt{-\delta}}{\sqrt{-\delta}}+b\right) \quad (8)$$

Here, due to the falling spray droplet is crashed so the option1 is selected.

Continuously, option 1 is going on calculating the surface area of ellipse spray droplets.

Applying Power series into equation (6), it is changed as below:

$$A = \pi\left[2a^2+b^2\frac{1}{\sqrt{-\delta}}\log\left(\frac{1+\sqrt{-\delta}}{1-\sqrt{-\delta}}\right)\right] \quad (9)$$

$$A = 2\pi b\left(a\left[1+\frac{1}{6}\delta+\frac{3}{40}\delta^2+\frac{5}{112}\delta^3+\cdots\right]+b\right) \quad (10)$$

$$A = 4\pi ab\int_0^1 (1-\delta u^2)^{1/2}du \quad (11)$$

$$A = 4\pi ab\int_0^1\left(1-\frac{1}{2}\delta u^2+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\delta^2u^2-\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\delta^3u^6+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{4!}\delta^4u^8+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\left(-\frac{7}{2}\right)}{5!}\delta^5u^{10}+\cdots\right)du \quad (12)$$

Integrating equation (12), the result of integral term is written as equation (13).

$$I = \left(1-\frac{1}{2}\frac{\delta}{3}-\frac{1}{2}\frac{\delta^2}{3}-\frac{1}{16}\frac{\delta^3}{7}-\frac{5}{128}\frac{\delta^4}{9}-\frac{7}{256}\frac{\delta^5}{11}\cdots\cdots\right) \quad (13)$$

Here, equation (13) is resulted from Power series of $\frac{\arcsin\sqrt{\delta}}{\sqrt{\delta}}$. Finally, surface area A is written as below:

$$4\pi abI \quad (14)$$

And also, spray droplet's volume is written as equation (15).

$$Spheroids\ Volume = \frac{4}{3}\pi\ a\ b\ c \quad (15)$$

Because the initial condition of equation (1) is b=c, parameter b and parameter c of equation (15) have same value.

2.2. Modeling Strategy

a. Spray droplet' diameter is calculated from a and b of equation (1) by using Monte Carlo simulation.
b. Droplet diameter's range is assumed between 0 mm and 10 mm.
c. Spray droplet shape is assumed as spheroids and ellipsoid in 3-dimensional conditions.
d. Spray droplet distribution's skew value is ranged from 1.0 to 4.0(This value is derivative from raindrop falling experimental work).

2.3. Monte Carlo Modeling Parameters and Characteristics

In the previous chapters 2.2, spray droplet's surface area and volume is introduced about Monte Carlo modeling strategy.
The random parameters and the random number generation strategy are introduced as below.

Specially, spray droplet size distribution is famous as log-normal distribution. In Clift's experimental work, droplet distribution pattern is skewed log-normal distribution form.

a. Parameter a & b of equation (14) is same as spray droplet's diameter considering eccentricity. Because of that, they are used to calculated the surface area, volume and droplet size. It's random distribution type is shown below:
   Parameter a & b : log-normal distribution
   - Range: 0 mm ~ 10 mm
   - Skew: 1.0 ~ 4.0
b. Parameter δ of equation (13) is used to calculate the surface area from term I of equation(14). It's range and random distribution type as below:
   Parameter δ: homogeneous distribution
   - Random number range: 0.0 ~ 1.0
   - Parameter range: 0.0 ~ 1.4
c. Parameter a, b, c of equation (15) is used to calculate the volume of spheroids.
   It's range and random distribution type as below:
   Parameter a , b, c : log-normal distribution
   - Parameter range a: 0 mm ~ 10 mm
   - Parameter range b & c: 0 mm ~ 10 mm
   - Skew: 1.0 ~ 4.0

2.4. Fission Product's Removal Efficiency

Spray droplet's falling on containment ground is expressed by flow regime around a falling objects and then the removal efficiency is expressed as below:

$$Re = \frac{\gamma \times \rho_g \times E}{\mu_g} \quad (16)$$

$$Stk = \frac{{d_p}^2 \times \rho_g \times \gamma}{9 \times \mu_g \times D_d(e) \times E} \quad (17)$$

where
Re : Reynolds number
Stk : Stokes number
γ : Velocity of water droplet (spray drop)
E : Eccentricity (random exponential distribution)
μ_g : Viscosity of a spray drop
d_p : Diameter of aerosol particles (fission product)
$D_d(e)$: Diameter of spray droplets

From equation (16) and equation (17), the fission product's removal efficiency is calculated by equation (18). Equation (18) includes equation (19) and equation(20) as below:

$$\text{Fission product's removal efficiency} = \frac{(Vft + Re \bullet Pft/59)}{(1+Re/59)} \quad (18)$$

where

$$\text{Potential term:} Pft = 6.43\left[\frac{Stk}{Stk+\delta}\right]^2 (Stk - 0.06) \quad (19)$$

$$\text{Viscous term : } Vft = \left[1 + \frac{0.8\ln(3Stk)}{(Stk-1.3)}\right]^{-2} \quad (20)$$

From chapter 2.2 and 2.3, the calculated results is used to calculate the fission product's removal efficiency using equation (18). Finally, spray droplet's surface area and efficiency is calculated.

3. RESULTS AND DISCUSSIONS

3.1. Simulation Results and Experimental Comparison

The Monte Carlo simulation results based on equation (13), (14) and (15). And the comparison results between MC simulation and experimental work (LiLuo et al and Clift's experimental work) are shown in Figure 1.
From Figure 1, the skewed 4.0 distribution is not matched with the experimental work but the skewed 2.8 distribution is in good agreement with the experimental work.
From this results, skewed 2.8 log-normal distribution of spray droplet's size is generated by Monte Carlo simulation as like Figure 2.
Figure 3 shows the sorting results of the averaged droplet size.
Smaller average diameter is larger than larger average diameter in skew value.

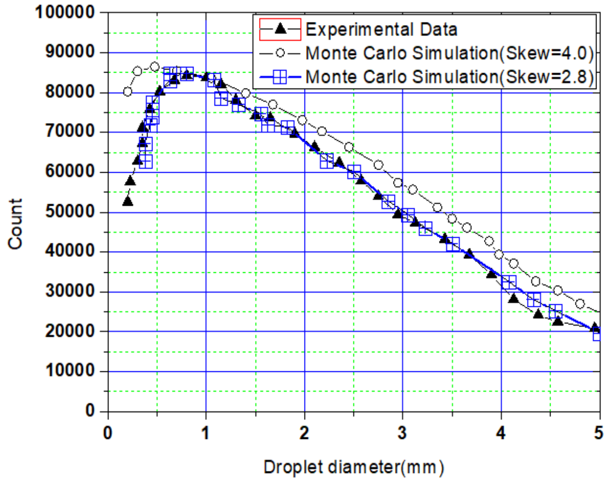

Figure 1. Monte Carlo simulation comparing with experimental data in water droplet size distribution.

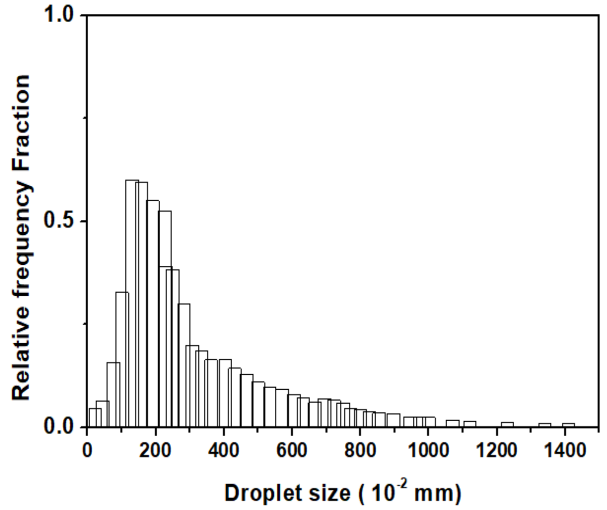

Figure 2. Monte Carlo simulation results of normal raindrop (in the condition of 300000 count generations).

From the simulation equation (13), (14) and (15), Figure 1 and Figure2 are generated.

From Figure 1 and Figure2, the equation (18) is used. The equation (18) can calculate the fission product's removal efficiency (Removal Rate) of the surface area of spray droplet size.
Additionally, the relation between droplet size and surface area is exactly shown in Figure 4.
Using Figure 4 and equation (18), the fission product's removal efficiency of surface area can be calculated as shown in Figure5.

Table1 shows the fission products removal rates in this study using the Power's model.
Power's model includes some parameters such as surface area of droplet and droplet diameters.
The results of Table 1 are calculated by each summation of the 50% cut off range and 90% cut off range in Figure2.
In spray removal rate, 50% cut off range is 11.97 ~ 12.11 and 90% cut off range is 27.99 ~ 28.12(Table 1).

The spray removal rates of Table 1 are calculated by the 50% cut off summation and the 90% cut off summation using the frequency weighted value considering the log-normal distribution of Figure 2.

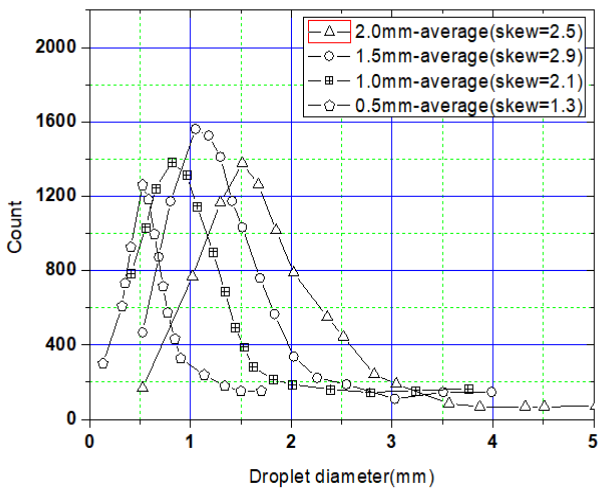

Figure 3. Droplet size distribution of various averaged cases.

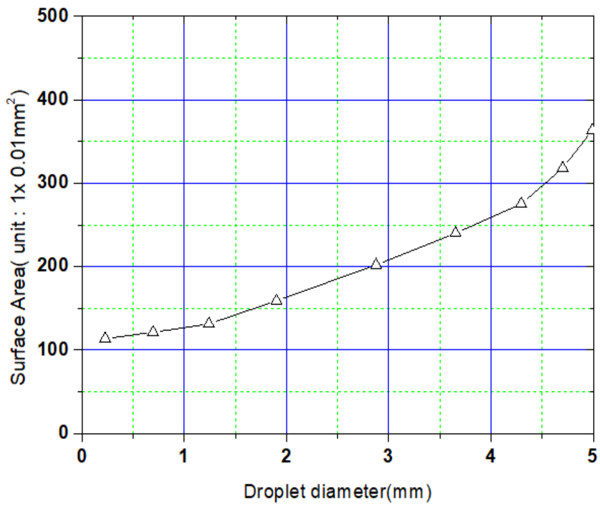

Figure 4. The relations in droplet radius vs surface area.

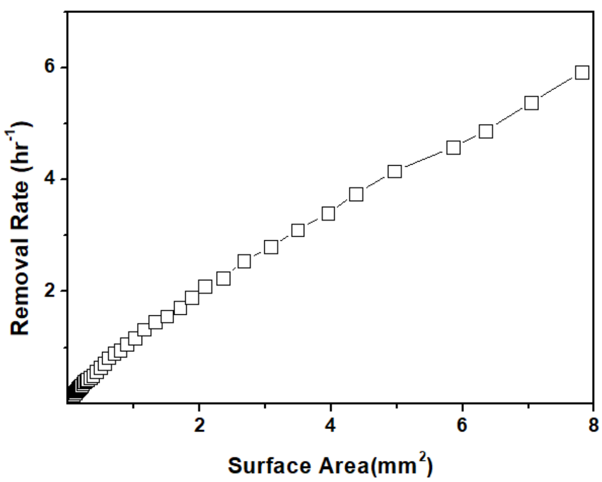

Figure 5. Fission product's removal efficiency in surface area of spray droplet.

Table 1. Calculation results of spray removal rate

Spray information	Spray Removal Rate (hr^{-1})
Header 1	50% : 12.11 90% : 28.12
Header 2	50% : 11.97 90% : 27.99
Minimum Value	50% : 11.97 90% : 27.99

3.2. Some Discussions from Calculation Results

According to the domestic nuclear power plant's final safety reports, Westinghouse type's spray removal rate is known as the range between 20 and 28.
These known values are very similar to this study results. From Table 1, 90 % cutoff value is in good agreement with Westinghouse type's known value.

4. CONCLUSIONS

In this study, 3-dimensional simple modeling of spray droplet is developed by Monte Carlo simulation and mathematical modeling. And fission product's removal rates are calculated. From this study, some conclusions are below:

a. Spray droplet size distribution between Monte Carlo simulation is in good agreement with and experimental data (Figure 1).
b. Spray removal rate is range 27.99 ~ 28.12 in the condition of 90% cutoff. And the range of 50% cutoff is between 11.97 and 12.11.
c. In each averaged droplet size, the frequency and distribution pattern are larger skew value in in smaller averaged droplet size(Figure3).
d. Removal rate of surface area is larger in the condition of the larger surface area.

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