Fission Yield Correction for Simplified Xe-135 and Sm-149 Decay Chains

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1. Introduction

The 'Two-Step' procedure which employs a detailed transport equation for fuel assembly analysis and a relatively cheap diffusion equation for whole core calculation serves as a cornerstone regarding modern reactor analyses. Whereupon, it is the retainment of the accuracy of transport solution whilst exploiting the numerical efficiency of diffusion calculation that determines the effectiveness of such a procedure.

The transport-based assembly analysis result, which is often referred to as lattice calculation, is homogenized into assembly-wise quantities. Various approaches have been proposed to preserve the reaction rate and (or) neutron currents through homogenized parameters [1,2]. Nevertheless, not much attention was allotted concerning the depletion aspect of the Two-Step procedure.

Two different approaches exist for depletion analyses, where detailed transmutation of each isotope is tracked or lumped variables with predetermined look-up tables are utilized. The former and the latter means are referred to as 'microscopic' and 'macroscopic' depletion methods respectively. Since Xe-135 and Sm-149 noteworthily affect the reactivity, simplified decay chains for such isotopes are often envisioned while implementing macroscopic depletion [3], however, equivalence for the acquired number densities cannot be guaranteed.

In this work, a concept of fictitious yield is introduced for the retainment of reference number densities concerning simplified decay chain models alongside macroscopic depletion. The details and numerical results of the proposed scheme are addressed.

2. Microscopic Depletion

The transmutation of nuclides subjected to nuclear reactions and radioactive decay can be expressed as

$$\frac{dN_i(t)}{dt} = \sum_{j=1}^{n}\left[\int_0^\infty dE f_{j\to i}(E)\sigma_j(E,t)\phi(E,t) + \lambda_{j\to i}\right]N_j(t) - \left[\int_0^\infty dE\sigma_i(E,t)\phi(E,t) + \sum_{j=1}^{n}\lambda_{i\to j}\right]N_j(t), \quad (1)$$

where $N_i(t)$ represents the number density for nuclide i, $\sigma_i(E,t)$ the transmutation cross-section, $f_{j\to i}$ the fraction of certain transmutation of interest, and all the other notations are that of the convention. A system of first-order ODEs for numerous isotopes can be casted into matrix expressions, which can be readily solved [4]. Nevertheless, direct utilization of such a method during diffusion analyses entails a noticeable computing burden.

3. Macroscopic Depletion

In place of considering each isotope as Eq. (1), homogenized assembly-wise parameters can be pre-tabulated at each burnup point which can be directly utilized to perform macroscopic depletion calculation.

As aforementioned, for an accurate appraisal of Xe-135 and Sm-149 densities, simplified decay chains are often considered alongside as shown in Fig.1. Note that tabulation of microscopic absorption cross-sections for Xe-135 and Sm-149 is necessary [3]. Nevertheless, one cannot guarantee the retainment of reference number densities, i.e., microscopic depletion result.

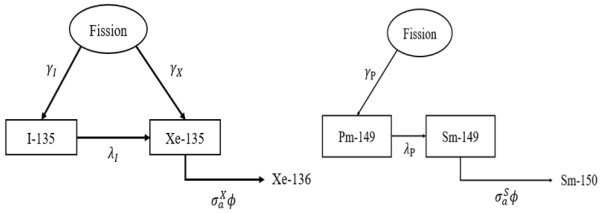

Fig. 1. Simplified decay chains for Xe-135 and Sm-149.

The simplified decay chains of interest can be mathematically expressed as below:

$$\frac{dI}{dt} = \gamma_I\sum_{g=1}^{G}\Sigma_{fg}\phi_g(\boldsymbol{r},t) - \lambda_I I(\boldsymbol{r},t), \quad (2)$$

$$\frac{dX}{dt} = \gamma_X\sum_{g=1}^{G}\Sigma_{fg}\phi_g(\boldsymbol{r},t) + \lambda_I I(\boldsymbol{r},t) - \lambda_X X(\boldsymbol{r},t) - \sum_{g=1}^{G}\sigma_{a,g}^X\phi_g(\boldsymbol{r},t)X(\boldsymbol{r},t), \quad (3)$$

$$\frac{dP}{dt} = \gamma_P\sum_{g=1}^{G}\Sigma_{fg}\phi_g(\boldsymbol{r},t) - \lambda_P P(\boldsymbol{r},t), \quad (4)$$

$$\frac{dS}{dt} = \lambda_P P(\boldsymbol{r},t) - \sum_{g=1}^{G}\sigma_{a,g}^S\phi_g(\boldsymbol{r},t)S(\boldsymbol{r},t), \quad (5)$$

where γ_i and λ_i represent yield value and decay constant for isotope of interest respectively. The analytic solutions for Eqs. (2) to (5) can be easily acquired by assuming flux and yield values being constant, which is directly applied to predictor-corrector scheme based macroscopic depletion [5].

4. Fission Yield Correction

It is the retainment of reference number densities from simplified decay chains under macroscopic depletion that is interested in this work. One could envisage adjustment of fission yield values for Eqs. (2) to (5) that

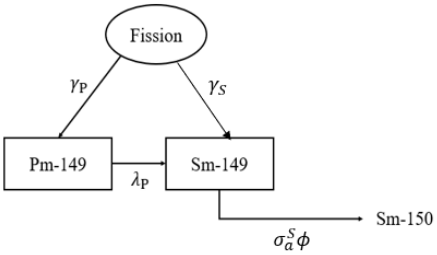

Fig. 2. Modification in the simplified decay chain for Sm-149 including fictitious Sm-149 yield.

forces number densities to coincide with the reference values if the solution is known. For instance, the I-135 density can be preserved by adjusting γ_I value in Eq. (2). For Xe-135, since all the other parameters excluding γ_X are fixed, preservation can be accomplished by correcting its fission yield value (γ_X).

However, only a single yield value, which can be freely adjusted, exists for Pm-149 and Sm-149 decay chain. Mathematically, an additional degree of freedom is imperative to preserve both Pm-149 and Sm-149 densities. Hence, a fictitious yield value for Sm-149 is proposed as a remedy in this study as depicted in Fig. 2, and its modified decay chain is expressed as

$$\frac{dS}{dt} = \gamma_S \sum_{g=1}^{G} \Sigma_{fg}\phi_g(\boldsymbol{r},t) + \lambda_P P(\boldsymbol{r},t) - \sum_{g=1}^{G} \sigma_{a,g}^S \phi_g(\boldsymbol{r},t) S(\boldsymbol{r},t). \quad (6)$$

5. Numerical Results

A fuel assembly for an APR1400 core without any burnable absorbers is selected to test the validity of the proposed yield correction scheme in this work. Figure 3 depicts the configuration and the reference microscopic depletion result obtained from Serpent 2.129 with an ENDF-VII.1 data library [6]. A more detailed description of the test problem is enumerated in Table 1.

Table 1. Description of APR1400 Fuel Assembly

Pin radius	4.7498 [mm]
P/D ratio	1.329 [-]
Assembly size	20.875 [cm]
UO2 Fuel enrichment	4.95 w/o
Power Density	38.0 W/gU

5.1. Reference Depletion

For calculating the reference microscopic depletion result, a sequence of fine burnup points was considered as shown in Fig. 3. As aforementioned, one could mathematically preserve the number densities of interest when such a reference solution is known. Figure 4 compares both the original yield values and adjusted yield values for the problem of interest, where salient difference exists for Pm-149 fission yield values and non

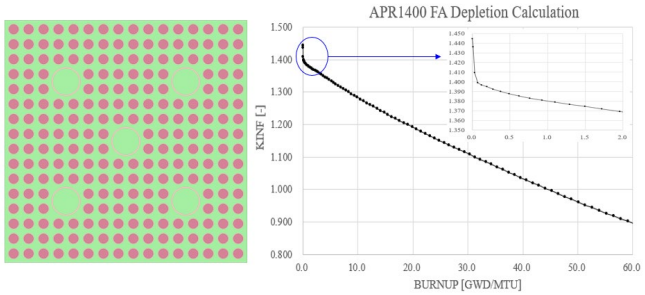

Fig. 3. Configuration of 16x16 APR1400 fuel assembly and its associated reference depletion calculation result.

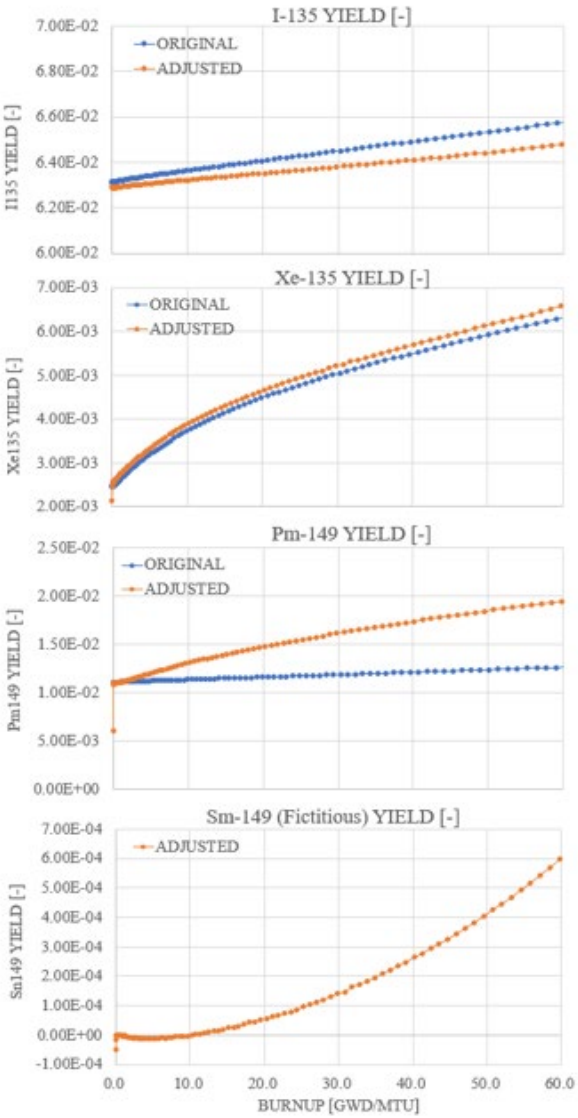

Fig. 4. Reference corrected yield values for simplified decay chains utilized in the macroscopic depletion.

negligible magnitude of fictitious Sm-149 fission yield value can be observed. Such observations insinuate that reference number densities cannot be retained without proper adjustment in the fission yield values for simplified decay chains that would incur inaccurate reactivity estimation.

Nevertheless, the proposed adjustment scheme based on a fine burnup step is impractical for realistic implementation solely due to its associated computing burden, i.e., coarser burnup step needs to be considered. Figure 5 shows the acquired adjusted yield values from coarser burnup steps, where an equilibrium state was intentionally postulated for I-135 and Xe-135 decay chains. Even with the reduced number of burnup steps, the macroscopic depletion results can retain enough accuracy as illustrated in Fig. 6.

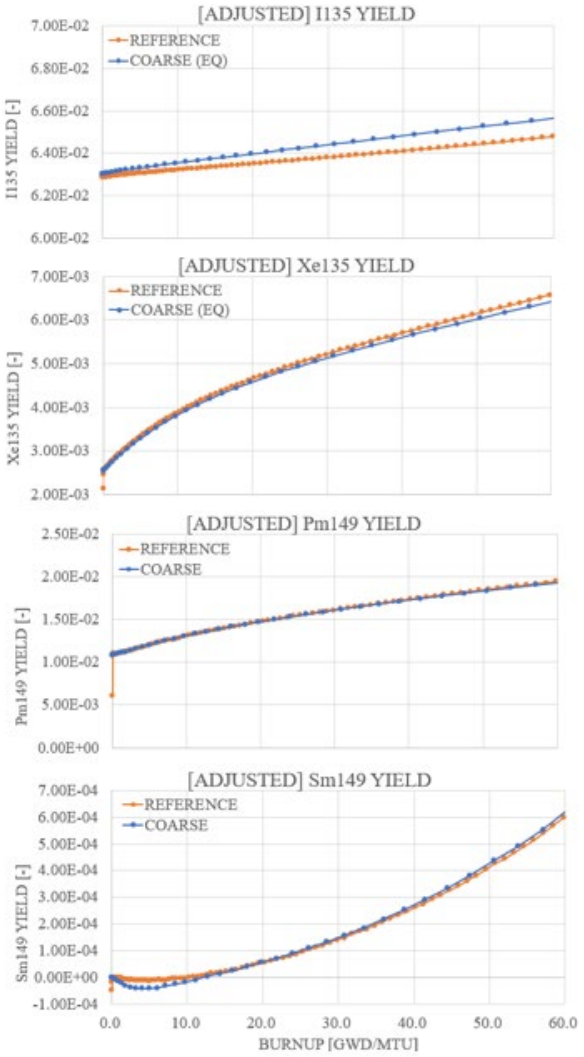

Fig. 5. Corrected fission yield values based on coarse burnup steps.

5.2. Load Follow Operation

A depletion scenario which mimics a load follow operation (LFO) was devised in this study to test the applicability of the proposed scheme as depicted in Fig. 7. Three different initial burnup values were considered to resemble BOL, MOL, and EOL conditions: 0.00 GWD/MTU, 20.65 GWD/MTU, and 42.96 GWD/MTU.

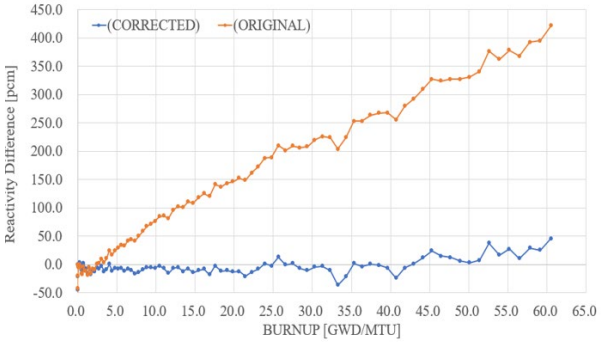

Fig. 6. Reactivity difference for macroscopic depletion calculation result.

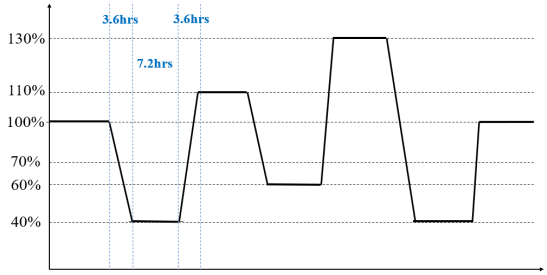

Fig. 7. Load follow operation (LFO) scenario

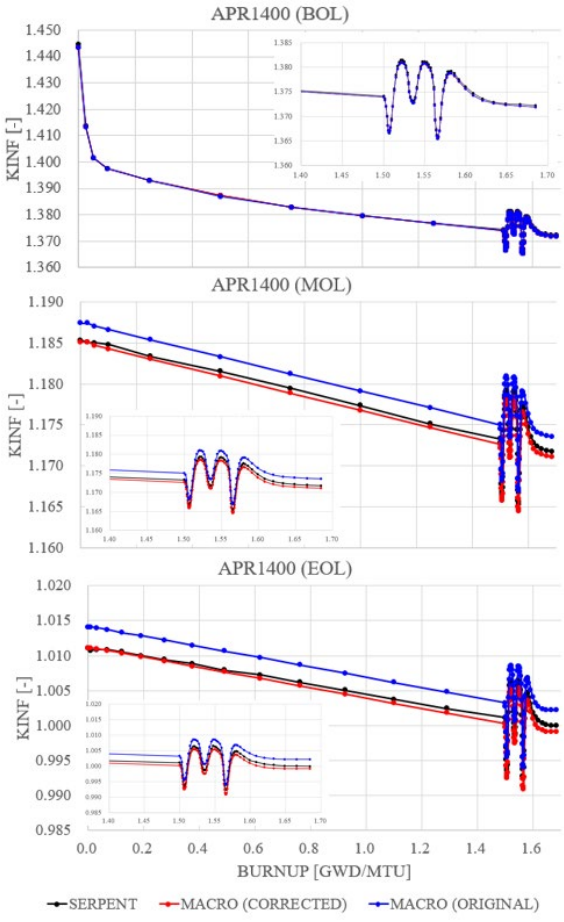

Fig. 8. LFO depletion calculation

Figure 8 enumerates the macroscopic depletion result for each initial burnup condition, where usage of the adjusted yield values preserves the multiplication factor at the onset of calculation regardless of the initial burnup value. The deviation in the reactivity during the LFO calculation is illustrated in Fig. 9. It is noteworthy to mention that single assembly calculation under reflective boundary conditions does not require any spatial homogenization, i.e., nodal method independent. It could be seen that the exploitation of corrected yield values noteworthily stifles the deviation in the reactivity for MOL and EOL conditions. Such an observation attests to the importance of equivalence in isotopes of interest while utilizing macroscopic depletion.

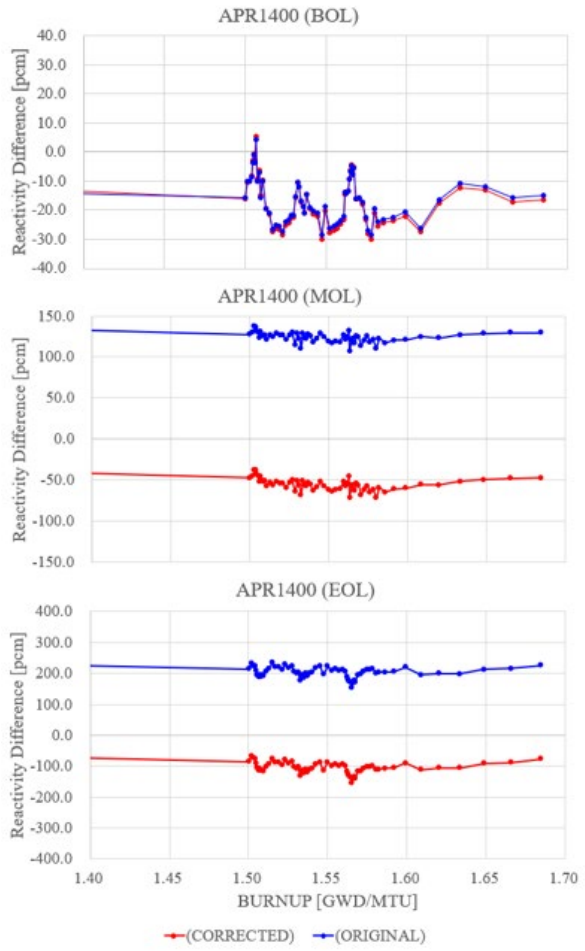

Fig. 9. Reactivity difference during LFO calculation

6. Conclusions

The presented study highlights the importance of retaining number densities for isotopes of interest including Xe-135 and Sm-149 whilst implementing macroscopic depletion calculation alongside simplified decay chains. Under the predictor-corrector scheme for macroscopic depletion, which postulates constant yield values between two consecutive burnup points, an equivalence can be met via an adjustment in the yield values. Particularly, a concept of fictitious fission yield regarding Sm-149 decay chain was proposed for the same purpose.

The acquired adjusted yield values from coarse burnup step well resemble the reference solution-based values, and the associated macroscopic depletion calculation exhibits reasonable retainment of reactivity. Especially, the depletion calculation that mimics a load follow operation plainly demonstrates the effectiveness and importance of the proposed correction scheme.

ACKNOWLEDGEMENTS

This work was supported by the National Research Foundation of Korea (NRF) Grant funded by the Korean Government (MSIP) (NRF-2016R1A5A1013919)

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