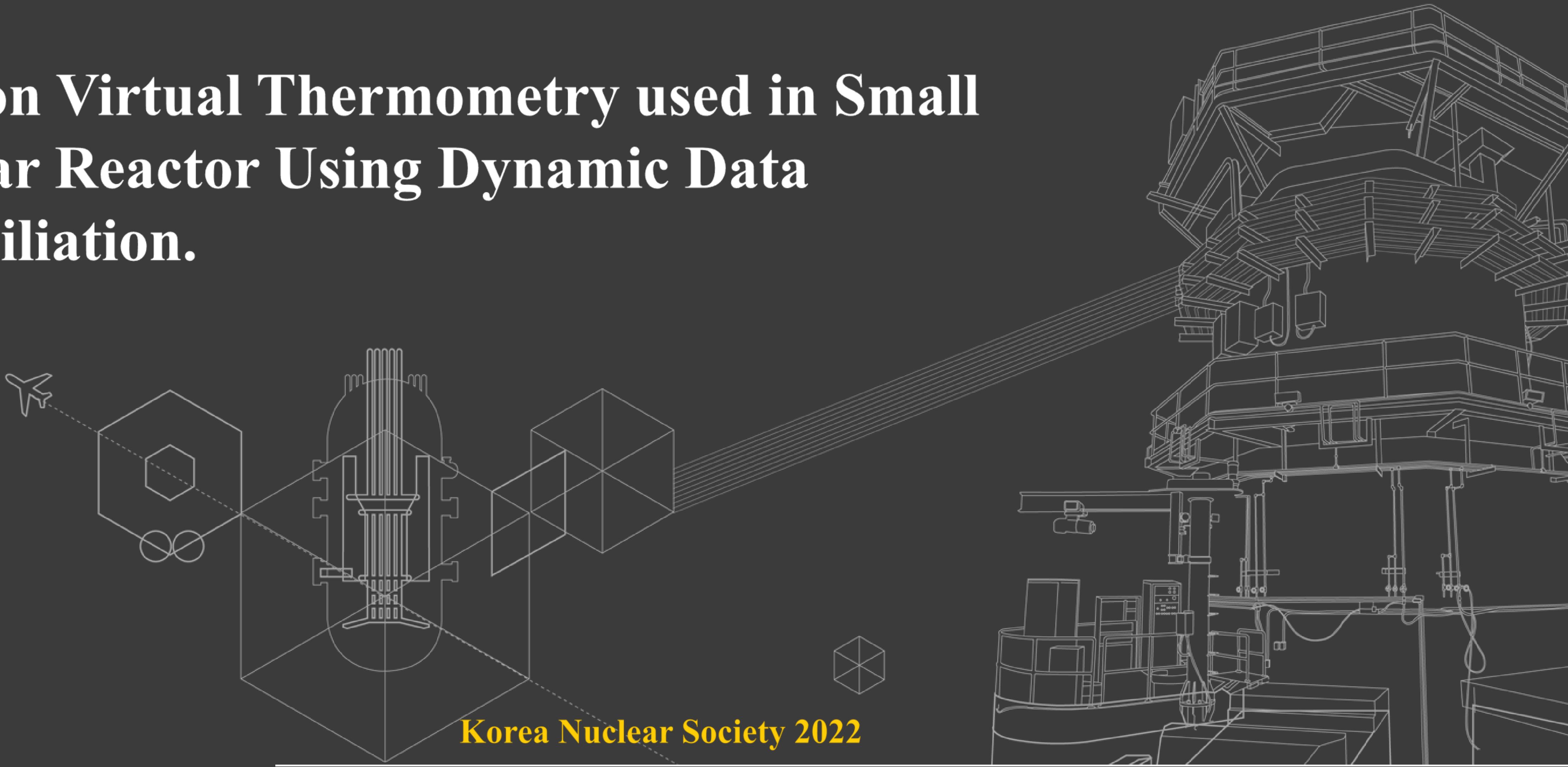
Study on Virtual Thermometry used in Small Modular Reactor Using Dynamic Data Reconciliation.



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Sangjun Park [a], Kyusik Oh [b], Gyunyoung Heo [b*]

[a]*Korea Atomic Energy Research Institute, Daedeok-daero 989 beon-gil,Yuseong-gu, Daejeon, South Korea, sangjun@kaeri.re.kr*
[b] *Kyung Hee University, Deok-young daero 1732, Yongin-si, Gyeonggi-do, 17104, South Korea,Corresponding author: gheo@khu.ac.kr*

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Study on Virtual Thermometry used in Small Modular Reactor Using Dynamic Data Reconciliation.

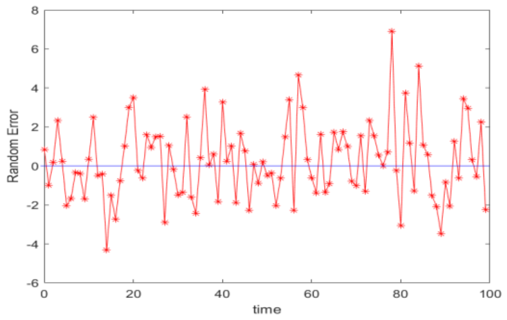
1

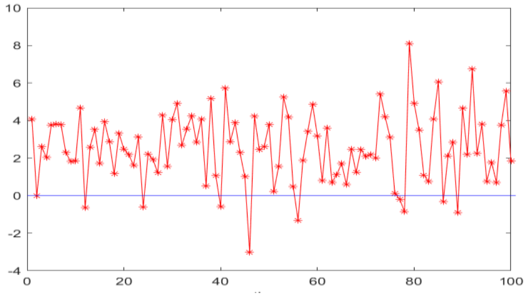
INTRODUCTION

- Process variables are important factor for representing the plant state.
  - ✓ ***Uncertainty of Instrument***
    - Imperfect instruments which have their own accuracy.
    - Signal transmission
    - Power fluctuation
    - Improper instrument installation
    - Miscalibration
    - Instruments malfunction

Random Error (Imperfect instruments which have their own accuracy through Miscalibration)

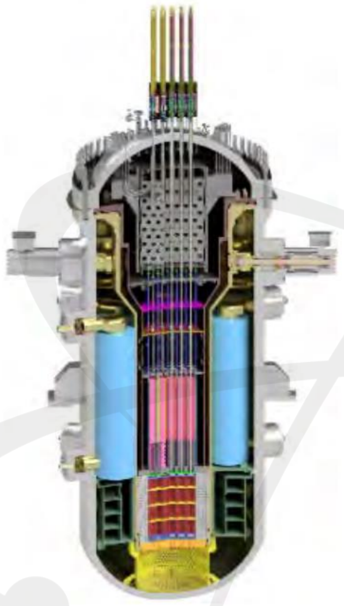
Gross error (Instruments malfunction)



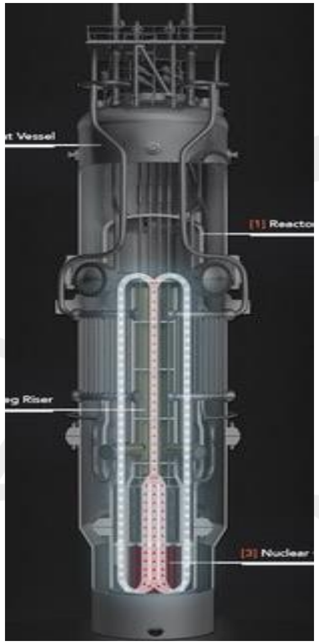


- ➢ **Due to the impoverished data quality → the process performance and control is deteriorated.**

- The uncertainty of process variables used for **Small Modular Reactors (SMRs)** is likely to be more increased
  - ➢ Due to the **compact system size**, the **geometry** by changed system, and the **harsh environment** by the SMRs nature.
  - ➢ Reduced the **diversity and redundancy** of instruments as well.
- The **estimation** of **accurate state** for SMR should be backup as a problem to be overcome in the future.



SMART
Ref. IAEA, SMR book, 2020



NuScale
Ref. nuscalepower.com

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INTRODUCTION

- In particular, it is not easy to estimate an accurate state in steady state, but the state estimation in dynamic state is even more difficult.
  - In other words, it is difficult for estimating accurate state in **transient state and load following operation in SMRs.**
    - There are limits to the efficient operation and control of SMRs.
    - It is affecting the safety of SMRs as well.
- In other to overcome this problem, **Dynamic Data Reconciliation(DDR)** is suitable method for estimating the accurate state in dynamic state by minimizing the uncertainty in physical model.
- Final goal of this study is for estimating the accurate state by minimizing the uncertainty of process variables applying the DDR technique.
- Prior to main study, an sensitivity analysis of measurement is performed by applying the DDR technique.
  - **The system state is continuously changed** due to **an inaccurate system model, measurement, and uncertainty of system parameters.**
  - Parameters from real-time acquisition system are accompanying **the uncertainty of the measurement** due to the **insufficient acquisition time.**

Data Reconciliation

❖ **Data Reconciliation(DR) is widely used the technique to minimize the uncertainty of process variables**

❖ **The DR is one of the *physics model based* solution to improve the impoverished quality data.**

- Estimate the true value → Using Physical Model Constraints

  Condition: spatial redundancy of instrument in constraint is satisfied

- Reconcile the Uncertainty → Using the Bayesian Update

} ***"Distinguishing Features of Data Reconciliation"***

x is Measured Value
$\hat{x}$ is True Value from model
V is covariance.

Data Reconciliation

Measured Value →

Least squared Function

$$min(\hat{x}, \hat{y}) = (x - \hat{x})^T V (x - \hat{x})$$

$f(\hat{x}, \hat{y}) = 0$ ➢ *First Principle*
$g(\hat{x}, \hat{y}) \geq 0$ ➢ *Empirical Eq*

→

" Minimize the **Random Errors**"

" Detect the **Gross Error** "

" Estimate the unmeasured Value"

❖ **DR is suitable for estimating the accurate performance parameter by optimizing the uncertainty and eliminating the gross error.**

❖ **DR can be applied by different methodologies depending on the usage environment.**

Data Reconciliation in Steady State

- Measurement is described by additive noise model

$$y = x + v$$

Where ***y*** is a vector of ***n*** × **1** and the ***v*** is a vector of random error.

- If the measurement are given by ***y*,** most likely estimate for ***x*** can be obtained by Maximum Likelihood(ML) function.

$$L\,(x_k|y_k) = \frac{1}{(2\pi)^{m/2}|\mathrm{V}|^{1/2}} \exp\left[-\frac{1}{2}(y_k - x_k)^T V^{-1}(y_k - x_k)^T\right]$$

- The ML estimation problem is equivalent to minimizing the function.

$$min(y) = (\mathrm{y} - \hat{y})^T\, V^{-1}(\mathrm{y} - \hat{y})$$

$$\text{subject to } f(\hat{y}) = 0,$$
$$g(\hat{y}) \geq 0$$

Where is the ***V*** is ***M*** x ***M*** covariance matrix. The matrix ***V*** is using the weight matrix for each measurements. The $\hat{\boldsymbol{y}}$ represent the measured and reconciled value having the ***m*** × **1**vector.

- Weight Factor for measurements.

$$\min(y) = \sum_{i=1}^{n}(y_i - x_i)^2/\sigma_i^2$$

$\boldsymbol{\sigma_i}$ is standard deviation of measurement ***i***.

- To solve the objective function under the constraint.

$$\mathcal{L}(\hat{y}, \lambda) = (\mathrm{y} - \hat{y})^T V^{-1}(\mathrm{y} - \hat{y}) - 2\lambda^T f(\hat{y})$$

"Lagrange Multipliers"

- To perform successive linearization by approximating a nonlinear constraint with a linear transformation.

$$\frac{\partial\mathcal{L}}{\partial y} = J(A_y)^T\lambda = 0 \qquad \frac{\partial\mathcal{L}}{\partial\lambda} = f(y) = 0$$

"Jacobian Matrix"

- Find the Reconciled Value.

$$\hat{y} = y - VA^T(AVA^T)^{-1}A_y$$

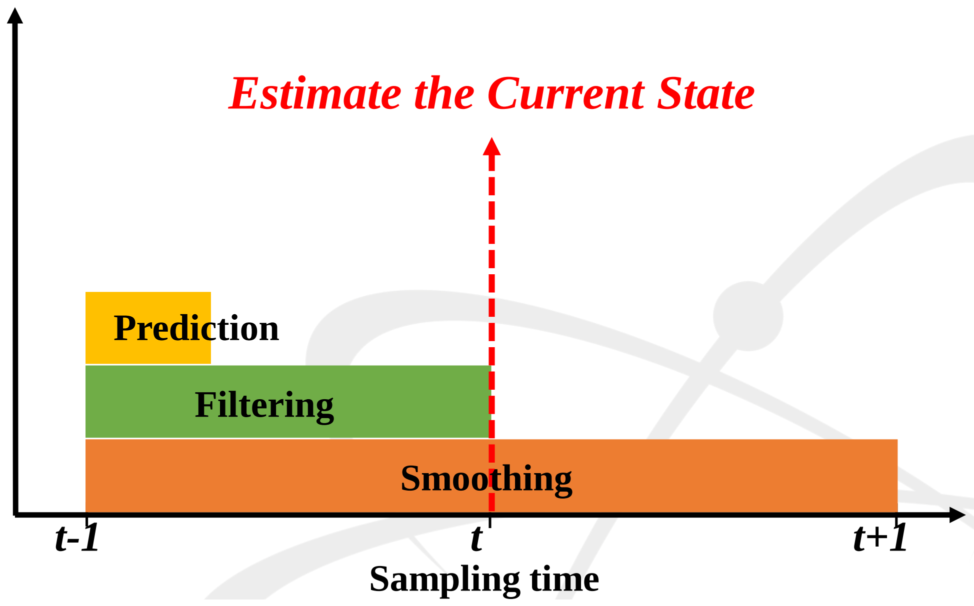
Only one set of data at current time is used and used spatial redundancy.

Dynamic Data Reconciliation

- The ***Dynamic Data Reconciliation(DDR)*** is suitable methodology for estimating the accurate state of time dependent parameters.
- DDR is a filtering method for estimating the current state using the data from measurement prior to ***t*** to measurement at time ***t***.

- $\boldsymbol{x}_k = \boldsymbol{A}_k\boldsymbol{x}_{k-1} + \boldsymbol{B}_k\boldsymbol{u}_{k-1} + \boldsymbol{w}_{k-1}$ ➢ $w \sim N(0, Q)$ →
  - Inaccurate model
  - Uncertainty of model parameters.
  - Inaccurate input value
- $\boldsymbol{y}_k = \boldsymbol{H}_k\boldsymbol{x}_k + \boldsymbol{v}_k$ ➢ $v \sim N(0, R)$
- $\min f(\hat{\boldsymbol{y}}_k) = \sum_{t=0}^{k}(\boldsymbol{y}_k - \hat{\boldsymbol{y}}_k)^T \boldsymbol{R}_k(\boldsymbol{y}_k - \hat{\boldsymbol{y}}_k)$ $\quad \hat{y}_k = x_k + w_k$
- $\min f(\hat{\boldsymbol{y}}_k) = (\boldsymbol{y}_k - \hat{\boldsymbol{y}}_k)^T \boldsymbol{R}_{M,k}^{-1}(\boldsymbol{y}_k - \hat{\boldsymbol{y}}_k) + (\boldsymbol{x}_k - \hat{\boldsymbol{y}}_k)^T \boldsymbol{Q}_{S,k}^{-1}(\boldsymbol{x}_k - \hat{\boldsymbol{y}}_k)$
- $\frac{\partial f}{\partial \hat{\boldsymbol{y}}_k} = 2\boldsymbol{R}_{M,k}^{-1}(\boldsymbol{y}_k - \hat{\boldsymbol{y}}_k) - 2\boldsymbol{Q}_{S,k}^{-1}(\boldsymbol{x}_k - \hat{\boldsymbol{y}}_k) = 0$
- $\hat{\boldsymbol{y}}_k = \left(\boldsymbol{R}_{M,k}^{-1} + \boldsymbol{Q}_{S,k}^{-1}\right)^{-1}\left(\boldsymbol{R}_{M,k}^{-1}\,\boldsymbol{y}_k + \boldsymbol{Q}_{S,k}^{-1}\,\boldsymbol{x}_k\right)$

$\boldsymbol{x}_k$: true value of state variables at time t.	$\boldsymbol{B}$: equation of optional control input
$\boldsymbol{u}_k$: manipulated input	$\boldsymbol{H}$: state equation related measurement y_k
$\boldsymbol{w}_k$: system model disturbance	Subscript $\boldsymbol{k}$: time
$\boldsymbol{y}_k$: measured values	$\boldsymbol{R}_k$: covariance matrix related measurement
$\boldsymbol{v}_k$: measurement error	$\boldsymbol{Q}_k$: covariance matrix related system model.
$\boldsymbol{A}$: equation of system model	



➢ ***A*** **is representing the system state**

➢ ***H*** **is representing the measurement state.**

➢ ***v and w*** **are representing the uncertainty.**

Kalman Filter for Dynamic Data Reconciliation.

Predict (Time Update)

- Prediction from system model

$$\hat{x}_k^- = A\hat{x}_{k-1}^- + Bu_k$$

- The covariance matrix of system model predictor

$$P_k^- = AP_{k-1}^- A^T + Q$$

Correct (Measurement Update)

- Calculate the Kalman Gain(difference with measurement and predictor of system model).

$$K_k = P_k^- H^T \left(HP_k^- H^T + R\right)^{-1}$$

$$\hat{x}_k = \hat{x}_k^- + K_k(Z_k - H\hat{x}_k^-)$$

- **Update the estimation error of system model.**

$$P_k = (I - K_k H)P_k^-$$

$\hat{x}_k^-$: a priori estimate.	$\hat{x}_k$: a posteriori state estimate.
A : equation of system model.	B: equation of optional control input .
u_k: manipulated input .	Z_k: measurement.
H: state equation related measurement	R: measurement error covariance
Q: system model error covariance	P_k: a posteriori estimate error covariance
P_k^-: a priori estimate error covariance	Subscript k: time

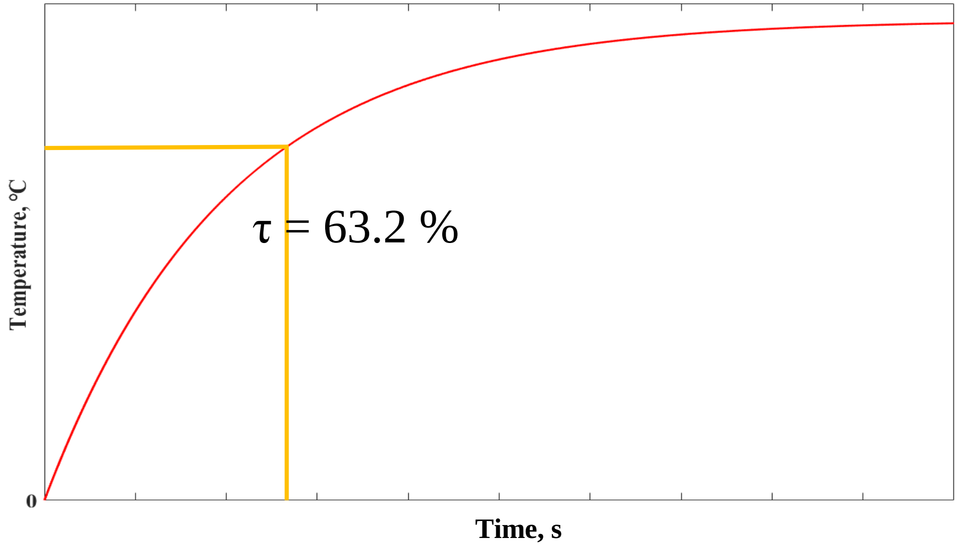
- DDR is estimated by only system model.
- The system model should be calculated by the numerical solution for estimating the accurate state at every time step.
- But, kalman filter is updated by reflecting the changed weighting factor between measurement and covariance of system model.
- Kalman filter is more suitable method for estimating accurate state in dynamic state.

Case Study: Thermometry on SMRs

- ❖ Temperature is one of the importance parameter for representing the plant state..
- ❖ Thermometry is a most restricted instrument such as an environment and location of SMRs.
- ❖ The Resistance Temperature Detector (RTD) and Thermocouple (TC)
  - ➢ Each of thermometry have opposite characteristics
- ❖ According to International Electrotechnical Commission (IEC) 60751 and ASTM E644 standards, the specifications of RTD and TC are below table.

Type	Response time(τ)	Tolerance
RTD	6 sec	Class A: ± (0.15 + 0.002*t)°C
		Class B: ± (0.3 + 0.005*t)°C
TC	2 sec	Class A: ± 1.5°C (or ±0.4%)
		Class B: ± 2.5°C (or ±0.75%)

The Graph of Time Constant.



- ❖ Which thermometry is more suitable for dynamic state

Case Study: System Model

- In this study, the one-dimensional heat conduction equation was used as the physical model to demonstrate the DDR.

$$\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2}$$

Where ***T*** is temperature, ***t*** is time, ***x*** is length and ***k*** is thermal conductivity.

- The finite-difference method was applied to calculate the node-wise temperature distribution

$$T_i^{k+1} = k\frac{\Delta t}{\Delta x^2}T_{i-1}^k\left(1-2k\frac{\Delta t}{\Delta x^2}\right)T_i^k + k\frac{\Delta t}{\Delta x^2}T_{i+1}^k$$

$$A = \begin{bmatrix} \left(1-2k\frac{\Delta t}{\Delta x^2}\right) & k\frac{\Delta t}{\Delta x^2} & & \\ k\frac{\Delta t}{\Delta x^2} & \left(1-2k\frac{\Delta t}{\Delta x^2}\right) & k\frac{\Delta t}{\Delta x^2} & \\ & k\frac{\Delta t}{\Delta x^2} & \left(1-2k\frac{\Delta t}{\Delta x^2}\right) & k\frac{\Delta t}{\Delta x^2} \\ & & k\frac{\Delta t}{\Delta x^2} & \left(1-2k\frac{\Delta t}{\Delta x^2}\right) \end{bmatrix} \begin{bmatrix} T_1^k \\ T_2^k \\ \vdots \\ T_N^k \end{bmatrix}$$

- **Reference Data Set**

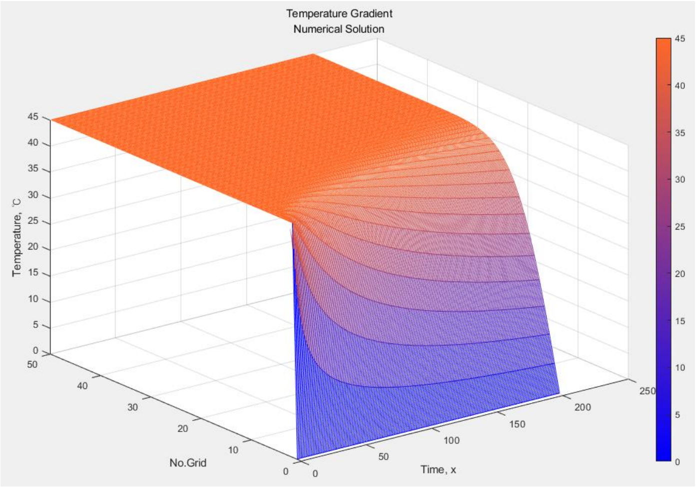
T=0 at x=0, for t>0

T = 45℃ for t=0, x>0

*k = 4.7*E-7 m^2/s*

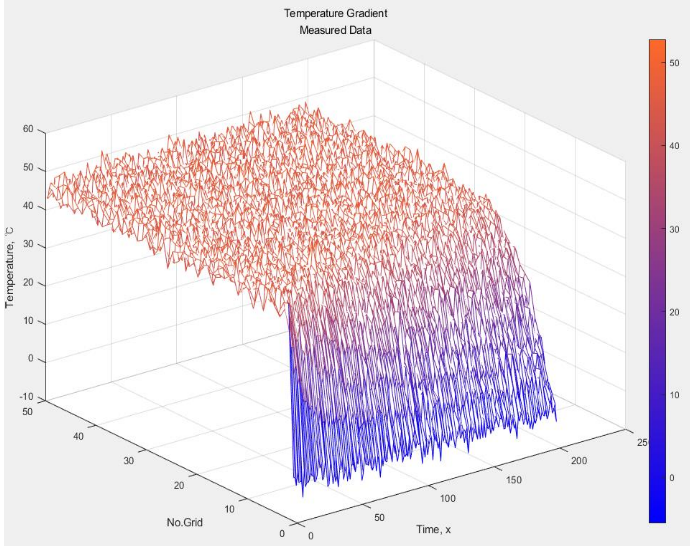
$r = k\Delta t/\Delta x^2$

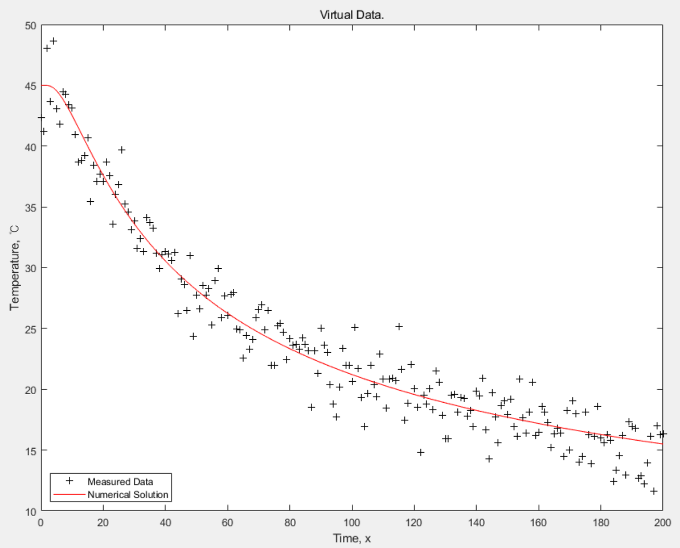
Final time is 200sec



- **Virtual Data Set**
  - *Reference data + σ_{vk} * randn. (σ_{vk} = 2℃)*

Where ***randn*** is the normally distributed random numbers

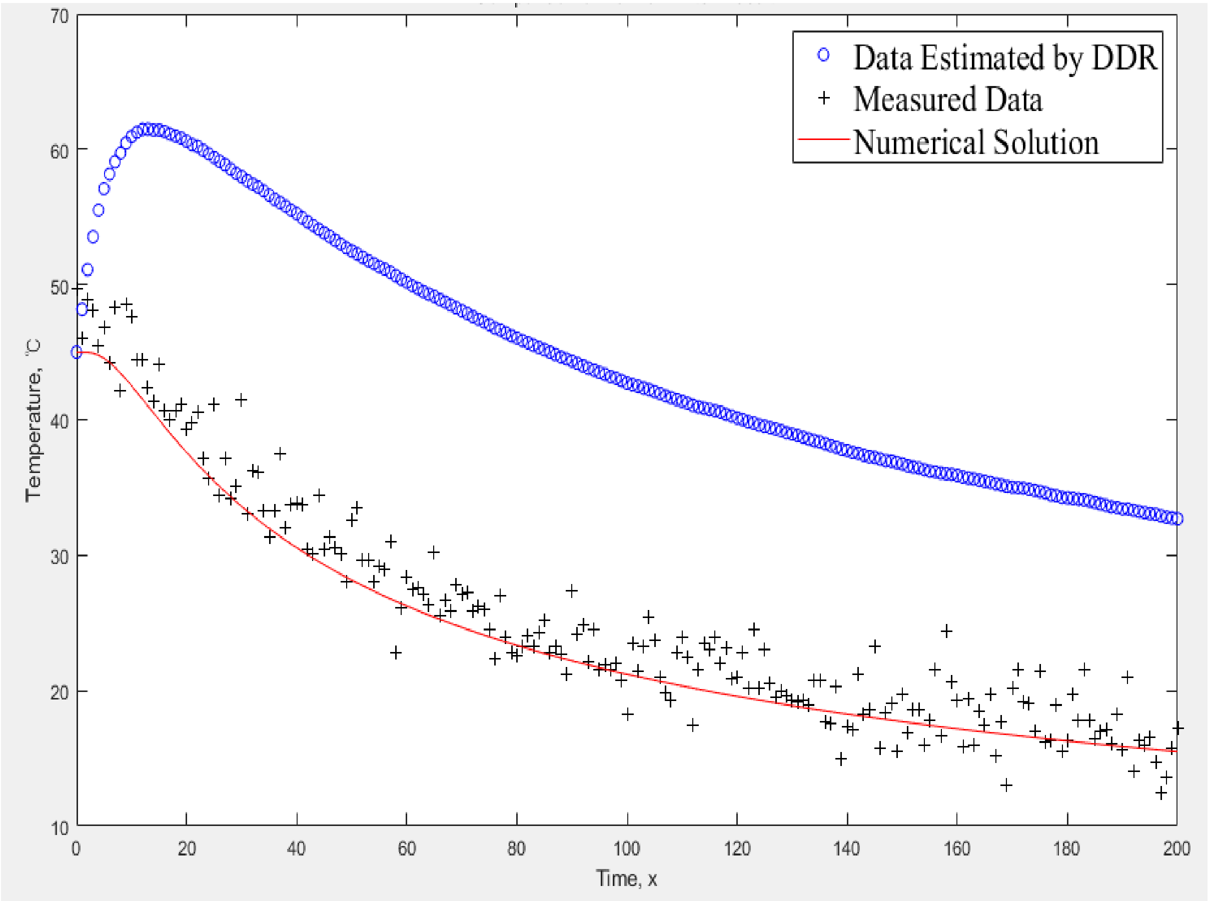




Results(1)

❖ **Result of Case 1.** ($\sigma_s = 0.1, \sigma_y = 2, \sigma_i = 0.8 + rand, H = 0.5$)

where σ_s is Standard Deviation of System Model, σ_y is Standard Deviation of Measured Value, σ_i is Standard deviation with random value following the Gaussian distribution, and H is the state of measurement.

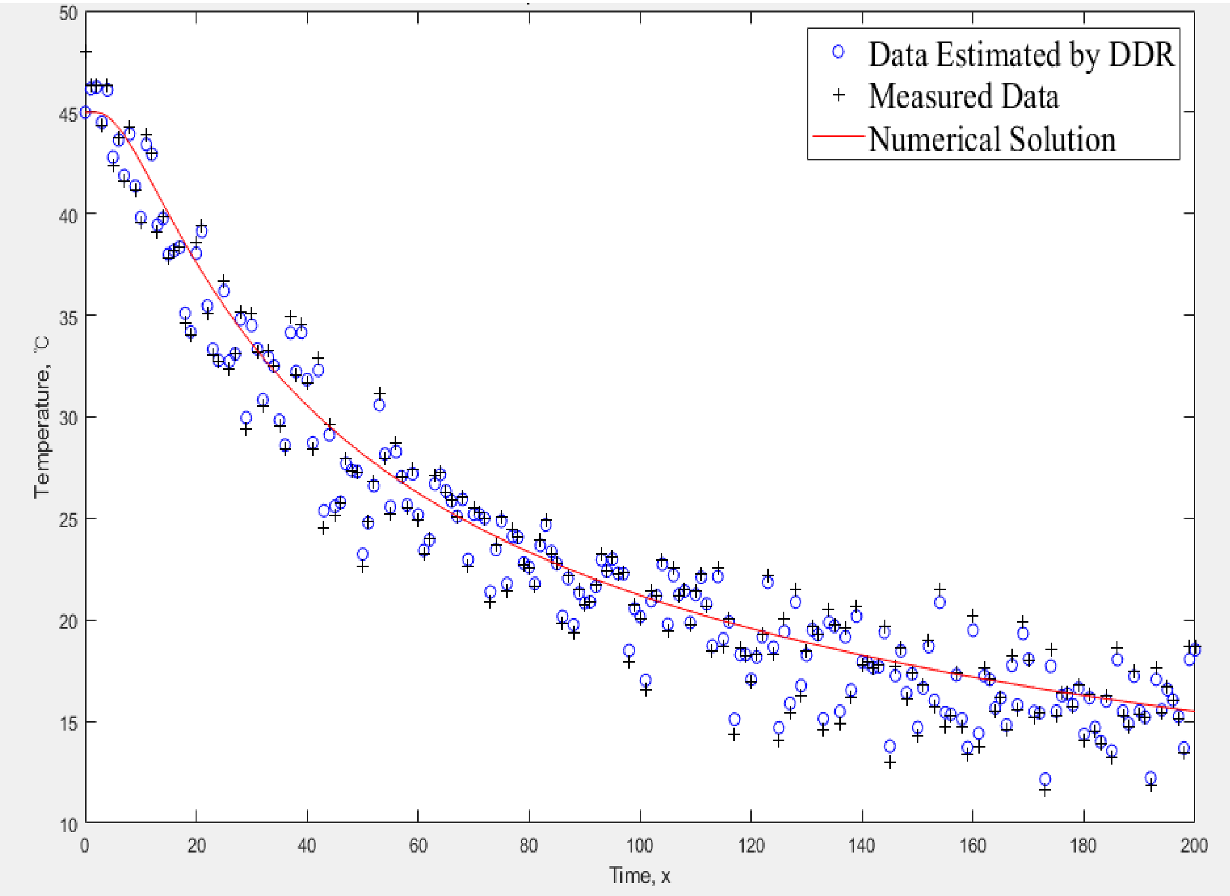


- This case is representing the state of measurement having a high uncertainty
- $y_k = H_k x_k + v_k$
- Case 1 is significantly getting out of the numerical solution(true value).
- It can not represent an accurate state due to the information of inaccurate measurement.

Results(2)

❖ **Result of Case 2. ($\sigma_s = 5, \sigma_y = 2, \sigma_i = 0.8 + rand, H = 1$)**

where σ_s is Standard Deviation of System Model, σ_y is Standard Deviation of Measured Value, σ_i is Standard deviation with random value following the Gaussian distribution, and H is the state of measurement.

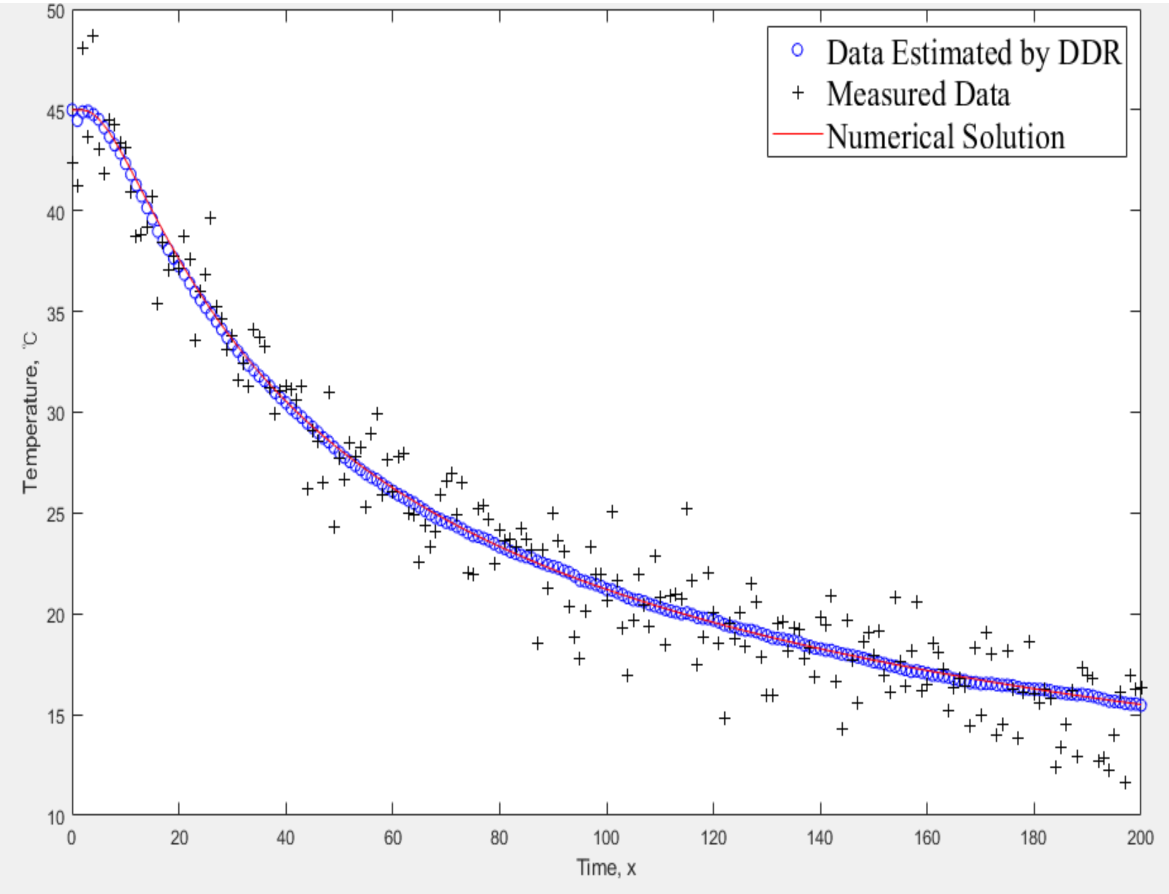


- Case 2 is representing the inaccurate system state.
- $x_k = A_k x_{k-1} + B_k u_{k-1} + w_{k-1}$
- If the accuracy of the system state is decreased, it is difficult to estimate the accurate state.
- when σ_s becomes larger than σ_y, the w_y (weight of the measurement) is increased and the estimation is following the measure.

Results(3)

❖ **Result of Case 3. ($\sigma_s = 0.1, \sigma_y = 2, \sigma_i = 0.8 + rand, H = 1$)**

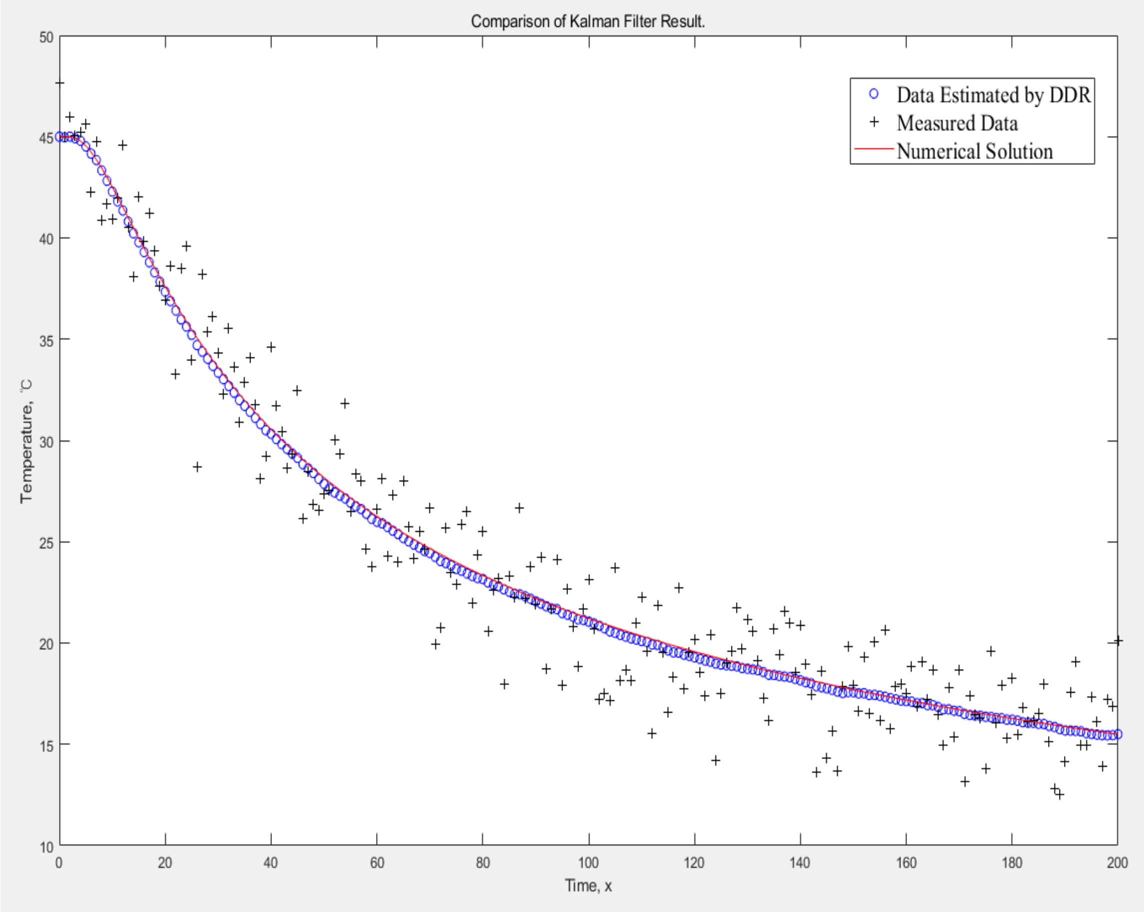
where $\boldsymbol{\sigma_s}$ is Standard Deviation of System Model, $\boldsymbol{\sigma_y}$ is Standard Deviation of Measured Value, $\boldsymbol{\sigma_i}$ is Standard deviation with random value following the Gaussian distribution, and H is the state of measurement.



- Estimated measurements by DDR are close to the true value
  - The uncertainty of measurements is minimized.
- If the system state is accurate, the estimating state is more and more close to the true state.
- We should consider what is more accurate.?
  - Inaccurate measurement by system state or Uncertain measurement by response time.?
  - In other to estimate an accurate state in dynamic state, the dynamic compensation is necessary at every seconds.

Results(4)

❖ Result of Regression Analysis on Case 1



- Of course, the dynamic compensation is used for estimating a state in NPPs.
  - ➢ But, It is used a historical data set in certain period.
- In order to estimate a more accurate state in dynamic, the data closed to the current state should be used rather than historical data
- Performed regression analysis using the sampling data within 1 second.
- In case of thermometry, since there is time constant equation, the value of accurate state can be estimated by regression analysis.

Conclusion

- ❖ This study is tried to an sensitivity analysis of the thermometry used in SMR by applying a DDR technique.
  - ➢ Inaccurate system state.
  - ➢ Uncertain measurement.
- ❖ According to results, the inaccurate measurement is more reliable than uncertain measurement.
- ❖ However, if an uncertain measurement is compensated by regression method, it is more accurate than an inaccurate measurement.
- ❖ In result 3, it is possible to estimate an accurate state through DDR.
- ❖ Further study
  - ▪ DDR will be performed to estimate the accurate state applying the actual system of the SMR.
    - ➢ In order to overcome various problems occurred by uncertainty of process variables in SMR.

THANK YOU

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