Improvement of *Phit-f k-ε* turbulence model for the prediction of heat transfer phenomena inside an air-cooled RCCS riser using OpenFOAM

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1. Introduction

Very high-temperature gas-cooled reactor (VHTR) is a GEN-IV reactor with a coolant outlet temperature of up to 950 ℃, which can be used for hydrogen production and industrial applications. Reactor Cavity Cooling System (RCCS) is a passive cooling system of the VHTR, which uses the natural circulation of atmospheric air to remove the decay heat emitted from the reactor vessel [1]. Korea Atomic Energy Research Institute (KAERI) has designed an air-cooled RCCS that incorporates rectangular riser channels, whose normal operation conditions are in turbulent forced convection [2].

In the RCCS risers, turbulent mixed convection with heat transfer deterioration can occur due to the decrease of chimney effect inducing lower flow rate of air circulation. Mixed convection has complex heat transfer characteristics that are different from those of forced and natural convections because of changes in the thermo-physical properties of the fluid. In addition, the RCCS riser is designed with narrow rectangular geometry, corner effect on the heat transfer needs to be examined. However, CFD analysis with existing RANS turbulence models shows poor predictions for the effects of the mixed convection and corner geometry.

In this study, heat transfer phenomena were investigated from the results of airflow visualization experiment, and the flow characteristics and heat transfer mechanism inside a heated rectangular riser were discussed. Based on the discussions, heat transfer deterioration mechanism and corner flow characteristics were reflected on RANS turbulence model, and the *Phit-f k-ε* turbulence model was improved using the open-source CFD tool, OpenFOAM v2012. In the end, by comparing with the experimental data, it was confirmed that the prediction of the heat transfer through the RCCS riser was improved with the modified turbulence model.

2. Flow characteristics in a heated rectangular riser

In Seoul National University (SNU), Four-Side Heating Riser Flow Visualization Experiment Facility (FROVE) was constructed to visualize the flow characteristics of the RCCS riser [3]. The heated test section has a rectangular geometry with a depth, width, and height of 0.02, 0.12, and 2 m, respectively, and it was fabricated with FTO-coated glass for the visual access with resistive heating. The facility also has a 1 m long unheated entry region for flow development. In this paper, the heat transfer phenomena inside the heated rectangular riser duct is examined through additional analyses and discussions with the experimental data.

2.1. Heat transfer deterioration in turbulent mixed convection

In turbulent mixed convection, as the buoyancy increases, the Reynolds shear stress distribution decreases, and it is known that the decrease of turbulence production results in the deterioration of heat transfer [4]. In this study, using quadrant analysis and Pearson's correlation coefficient, the mechanism of the Reynolds shear stress decrease by the wall heating effect would be explained.

Quadrant analysis is one of the approaches to investigate turbulent shear flows, and it examines the correlation between velocity fluctuations in the main flow direction and in the wall-normal direction [5]. Fig. 1 shows the normalized velocity fluctuation data from the FROVE experiment on the quadrants of the Reynolds shear stress plane, obtained at the Reynolds shear stress peak location along the mid-plane of the test section in turbulent forced and mixed convection conditions.

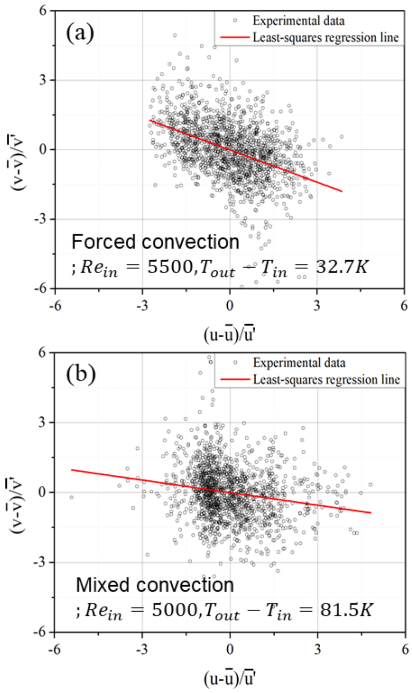

Fig. 1. Quadrant analysis results in the forced convection (a) and mixed convection (b) conditions.

From these quadrant analysis results, the change of the correlation of the two velocity fluctuations can be examined between the forced and mixed convection conditions. A linear correlation between the two velocity components is evident under the forced convection, but that was distorted in the mixed convection. In this study, to examine the correlation between the two velocity fluctuations, Pearson's correlation coefficient (ρ) was introduced, which can quantify the direction and strength of the linear correlation between two datasets as follows:

$$\rho_{X,Y} = \frac{\mathrm{cov}(X,Y)}{\sigma_X \cdot \sigma_Y} \qquad (1)$$

where cov(X, Y) is the covariance of the two datasets and σ is the standard deviation of a variable. If the correlation coefficient is obtained for the two velocity fluctuations used in the above quadrant analysis, it can be represented by the Reynolds shear stress and the related two Reynolds normal stresses as follows:

$$\rho_{u\prime,v\prime} = \frac{\mathrm{cov}(u\prime,v\prime)}{\sigma_{u\prime} \cdot \sigma_{v\prime}} = \frac{\overline{u'v'}}{\sqrt{\overline{u\prime^2}}\sqrt{\overline{v\prime^2}}} = \frac{R_{xy}}{\sqrt{R_{xx}R_{yy}}} \qquad (2)$$

which is the same as the slope of the regression lines in Fig. 1. The linear correlation coefficients of the experiments are $\rho_{FC} = -0.6731$ and $\rho_{MC} = -0.2791$, under the forced and mixed convection conditions, respectively. It indicates that the magnitude of the correlation between the two velocity fluctuation components was greatly reduced by the heating effect from the forced convection to the mixed convection.

Relatively more reduction of Reynolds shear stress compared to Reynolds normal stress means that the correlation induced by near-wall eddy motions is greatly reduced rather than the magnitude of the velocity fluctuation itself. Based on this assumption, it was estimated that additional vortex motions are generated by the wall heating effect, independent of the vortex motions formed by the near-wall shear.

Fig. 2 describes the rotational direction of the vortex motion by the wall shear and that of the of the vortex motion induced by the density gradient near the heated wall. The magnitude of the correlation by the vortex motion from the heating effect would increase as the density gradient increases near the heated wall along the wall-normal direction. Therefore, each vortex induced in opposite rotational directions cancels each other out, which results in the reduction of Reynolds shear stress and the heat transfer deterioration under mixed convection conditions.

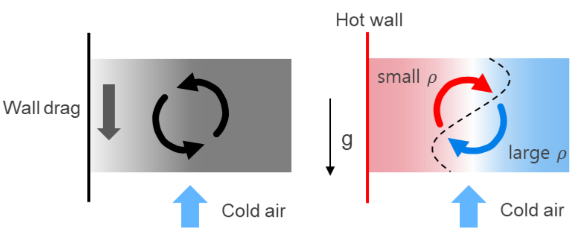

Fig. 2. Formation of wall-bounded and horizontal density (ρ) gradient-induced vortexes near the heated wall.

2.2. Corner symmetry along corner bisectors

From the FROVE experimental results, it was confirmed that the Reynolds shear stress distribution showed a more complex and low distribution near the corners than at the center of the test section, as described in Fig. 3. Based on the experimental results, it was estimated that the flow behavior at the corner bisector was similar to that on the symmetry line between two parallel plates. This estimation was also confirmed from the results of previous studies that investigated the flow characteristics inside a rectangular duct [6].

Based on this estimation, it can be considered that the Reynolds shear stress is canceled out along the corner bisector and line of symmetry. Considering that the Reynolds shear stress distribution directly affects the production of turbulence kinetic energy, it can be seen that this distribution is consistent with the existing DNS analysis results, in which the production of turbulence near corners is reduced, as shown in Fig. 4 [7].

Near the corner, the flow characteristics are simultaneously affected by two perpendicular walls. A wall-bounded vortex is formed in the direction perpendicular to the wall, and the wall-bounded vortexes in the two vertical walls occur independently of each other. Then, the correlations between u' and v' and between u' and w' decrease depending on the distance from the two walls. Consequently, the Reynolds shear stresses from each perpendicular wall cancel each other near the corner.

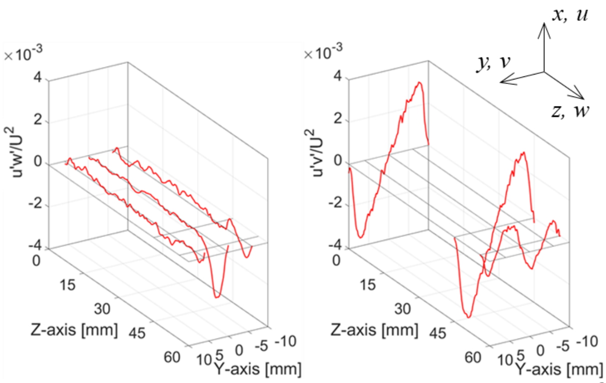

Fig. 3. Normalized Reynolds shear stresses for $u'w'/U^2$ and $u'w'/U^2$ in the forced convection condition ($Re_{in} = 5500$, $T_{out} - T_{in} = 32.7K$)

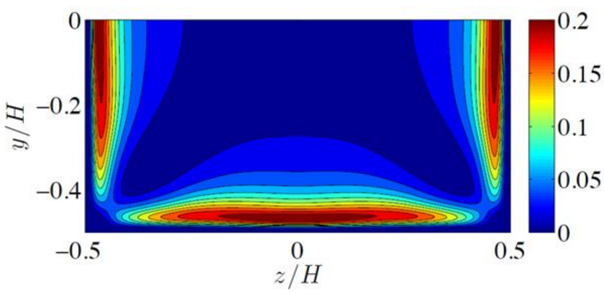

Fig. 4. DNS analysis result for the production of turbulence kinetic energy scaled with $u_{\tau,c}^4/\nu$ inside a square duct ($Re_{\tau,c} \approx 180$, Marin et al., 2016)

3. Improvement of the *Phit-f k-ε* turbulence model

Based on the two estimated flow characteristics in a heated rectangular duct, which are the horizontal density gradient-induced vortex and the reduced turbulence production near the corner bisectors, the predictability of RANS turbulence model was improved. In this study, transport equations of the *Phit-f k-ε* turbulence model, which is one of the variations of the V2F *k-ε* turbulence model, were modified using OpenFOAM.

In the actual phenomenon, the heated fluid near the wall and cold fluid in the center of the test section mix with the wall-bounded vortex described in Fig. 2. In this process, fluids of different densities are arranged in the vertical direction (or gravity parallel direction), and turbulent mixing occurs according to the velocity distribution and buoyancy force. This phenomenon is naturally reflected in LES and DNS analyses, which simulate the behavior of large eddies. However, in RANS turbulence modeling, which predicts the average temperature field, the turbulent mixing effect due to buoyancy production is negligible in the vertical direction owing to the small vertical average temperature gradient in the developed flow region. Therefore, this phenomenon is inherently excluded in RANS turbulence modeling.

Therefore, in this study, based on the concept of passive scalar transport analysis adopted in turbulence modeling, an additional vortex motion was inferred by proposing new turbulence production term. The vortex motion is repeatedly induced by the density gradient in the radial direction or the wall-normal direction, and affects the Reynolds shear stress distribution near the wall. This vortex motion is formed regardless of the flow direction or wall-bounded vortex. Therefore, the Reynolds shear stress is determined by the interaction between the wall-bounded and horizontal density gradient-induced vortexes.

The buoyancy production term (G_b) is calculated using the dot product of the temperature gradient and gravitational acceleration vector as follows:

$$G_b = \beta \frac{\mu_t}{\mathrm{Pr}_t}(\nabla \bar{T} \cdot \vec{g}) = -\frac{\mu_t}{\mathrm{Pr}_t}(\nabla \bar{\rho} \cdot \vec{g})/\bar{\rho} \qquad (3)$$

In this study, the magnitude of the production term by the horizontal (or gravity-perpendicular) density gradient-induced vortex is defined in a form similar to that of the buoyancy production term. The sign of this production term is based on its relationship with the wall-bounded vortex. As a result, the production term by the horizontal density gradient-induced vortex is given by:

$$G_{gperp} = -\frac{\frac{\mu_t}{\mathrm{Pr}_t}|\vec{g}|\left(\nabla\bar{\rho}\cdot\frac{-\nabla(U\cdot\hat{g})}{|\nabla(U\cdot\hat{g})|}\right)}{\bar{\rho}} \qquad (4)$$

$\hat{g}$ is the unit vector parallel to the gravitational acceleration vector. This newly proposed term is added to the turbulence production term of all transport equations of the *Phit-f k-ε* turbulence model.

RANS turbulence models adopting the linear eddy viscosity assumption cannot predict the anisotropy of Reynolds stresses, which are important for predicting flow characteristics near corners. However, according to the estimation in Section 2.2, in the vicinity of corners, turbulence production decreases along the corner bisectors because it is more affected by the symmetry of the two adjacent walls than the turbulence anisotropy near the wall or the increase of turbulence production by the wall shear. Therefore, to predict the symmetric flow characteristics along the corner bisectors, the elliptic relaxation equation of the *Phit-f k-ε* turbulence model was modified in the present study.

Durbin introduced an elliptic relaxation function as an additional transport equation to predict the near-wall flow behavior [8]. This approach can simulate the symmetrical behavior along the symmetry line between the two parallel plates, therefore, the elliptic relaxation equation was modified to predict the symmetrical flow characteristics along the corner bisector in this study.

Durbin proposed an elliptic relaxation equation between two-dimensional infinite plates as follows:

$$L^2 \frac{\partial^2}{\partial y^2} f_{22} - f_{22} = -\frac{\Pi_{22}}{k} - \frac{[\overline{v^2}/k - \frac{2}{3}]}{T} \qquad (5)$$

To simulate the symmetrical behavior along the corner bisector, as shown in Fig. 5 with the $y' - z'$ coordinate system, a formula was proposed based on the above elliptic relaxation equation as follows:

$$L^2 \frac{\partial^2}{\partial y'^2} f(y', z') - f(y', z') = g(k, \nu_t) \qquad (6)$$

If the chain rule is used to rotate the $y' - z'$ coordinate system to the existing $y - z$ coordinate system, a mixed derivative term is required in addition to two second derivative terms as follows:

$$\frac{\partial^2}{\partial y'^2} = \cos^2\left(\frac{\pi}{4}\right)\frac{\partial^2}{\partial y^2} - 2\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)\frac{\partial^2}{\partial y \partial z} + \sin^2\left(\frac{\pi}{4}\right)\frac{\partial^2}{\partial z^2} = \frac{1}{2}\left(\frac{\partial^2}{\partial y^2} - 2\frac{\partial^2}{\partial y \partial z} + \frac{\partial^2}{\partial z^2}\right) \qquad (7)$$

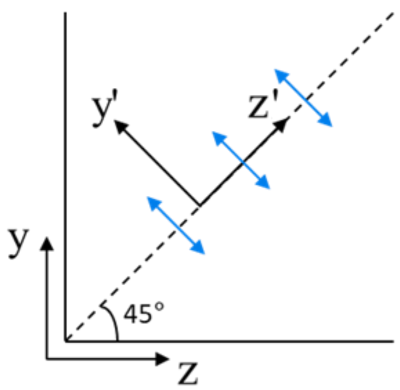

Fig. 5. Line of symmetry between two perpendicular walls with different coordinate systems

To predict the symmetrical behavior along the bisectors at the four corners in a rectangular duct, the absolute value of the mixed derivative term is added to the existing elliptic relaxation equation as follows:

$$L^2\left(\nabla^2 f - 2C_{corner}\left|\frac{\partial^2}{\partial y \partial z} f\right|\right) - f = -\frac{\Pi_{ij}}{k} - \frac{\left[\overline{u_i u_j}/k - \frac{2}{3}\right]}{T} \quad (8)$$

The corner coefficient C_{corner} is a constant for empirically evaluating the effect of the mixed derivative term on the existing elliptic relaxation equation. The coefficient was determined by comparing the flow characteristics from the FROVE experiment and reviewing the heat transfer phenomena. The value of the coefficient was determined to be 1.4.

4. Results of the calculation

As discussed in Section 3, both the effect of heating and the distribution of turbulence quantities near the corners can reproduce a more reduced Reynolds shear stress and turbulence production compared to the prediction by the conventional RANS turbulence modeling. The predictability of the modified turbulence model was examined comparing with existing experimental data under mixed convection conditions. For validation, the experimental conditions and results of the Riser Heat Transfer Experiment Facility (RHEF) experiment performed at SNU were used [9]. The dimensions of the heated test section in the RHEF experiment were 0.04 × 0.24 × 4 m, whose aspect ratio is the same as the FROVE experiment.

Fig. 6 shows a comparison between the experimental results and analysis results of the Re8921HF1070 case along the height of RHEF. The heat transfer coefficient prediction was improved by modifying the turbulence model under the heat transfer deterioration condition with increased buoyancy. The modified turbulence model predicted the wall temperature distribution well.

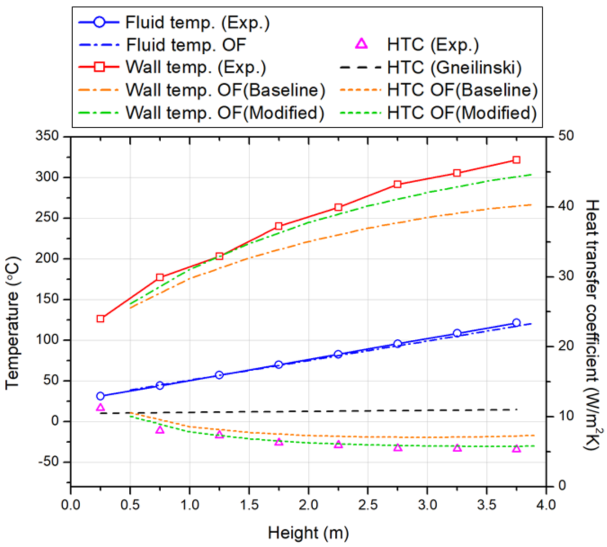

Fig. 6. Experimental data and CFD analysis results with and without the modification of the turbulence model (RHEF experiment, Re_{in} = 8921, heat flux = 1070 W/m^2).

The difference in the prediction of the wall temperature is about 37 K near the outlet with and without the modification. Therefore, it was verified that the modified turbulence model performs an improved prediction of the existing experimental data, particularly in predicting the wall temperature distribution with intense wall heating.

5. Conclusions

In this study, the prediction of heat transfer phenomena in a heated rectangular riser was improved by modifying the RANS turbulence model. The reasons for the less predictive ability of RANS turbulence models regarding the decrease in Reynolds shear stress and heat transfer in the heated rectangular riser were discussed. Based on the discussion, the *Phit-f k-ε* turbulence model of OpenFOAM was improved. The horizontal density gradient was added to consider the turbulence production term in the transport equations, and a mixed derivative term was added to the elliptic relaxation equation to consider symmetrical flow characteristics along the corner bisector. Through these modifications, the prediction of the CFD analysis was improved in the mixed convection regime.

ACKNOWLEDGEMENT

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