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Approximate Cone-Beam Filtered Backprojection for Limited Angle Tomography

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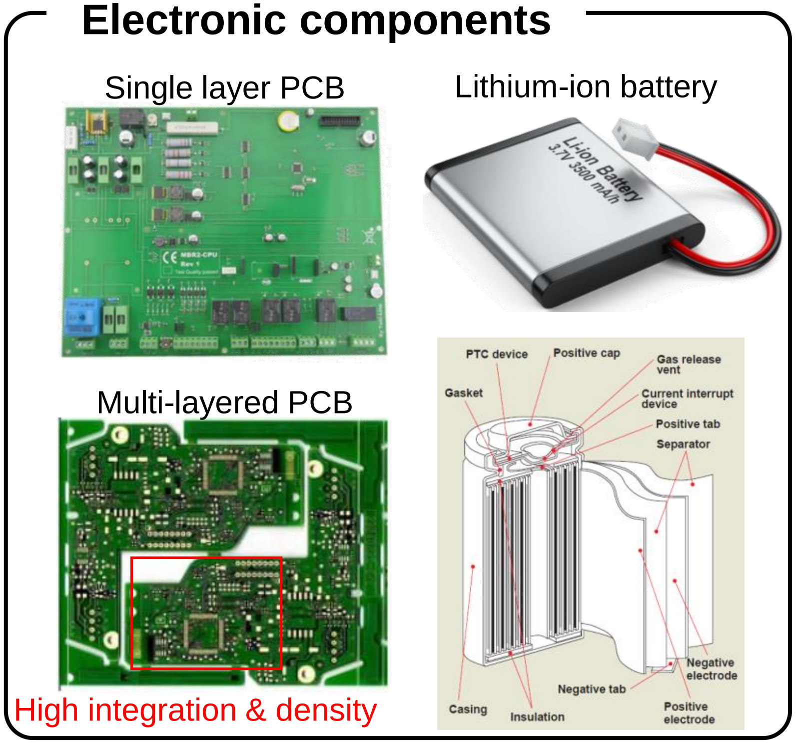
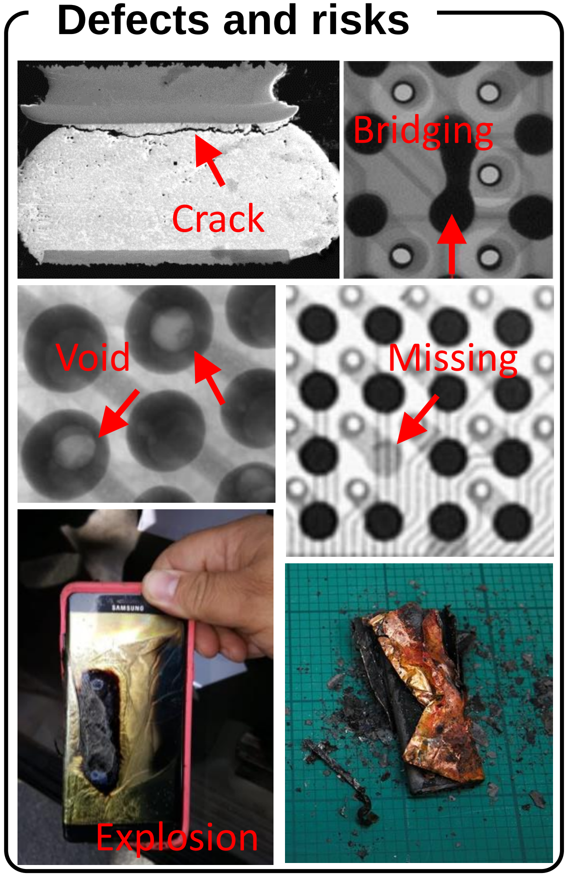
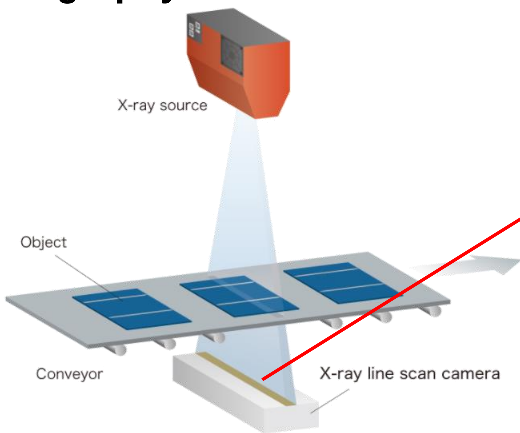
- Importance of non-destructive testing

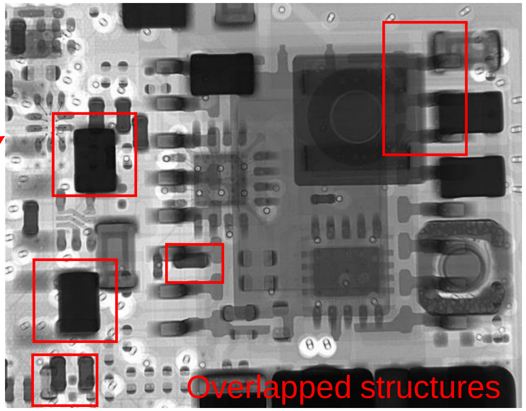




▪ Possible solutions: X-ray radiography & tomography

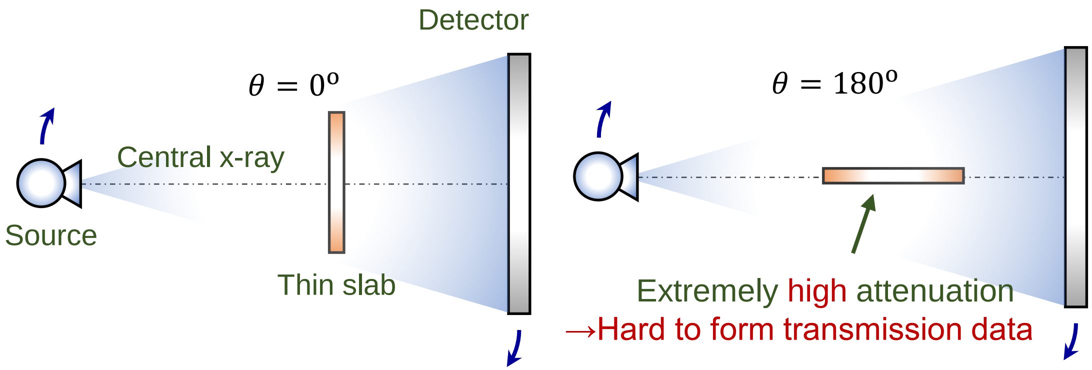
Radiography



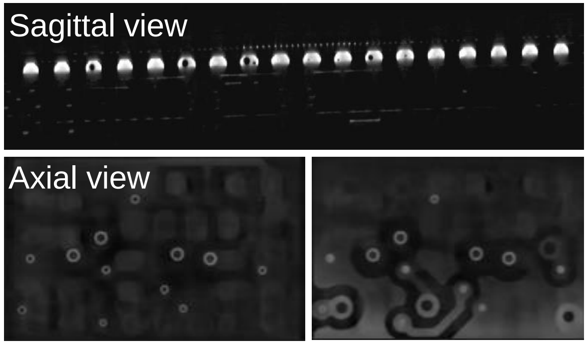


- Difficult to detect defects because of overlaps of structures

Computed tomography



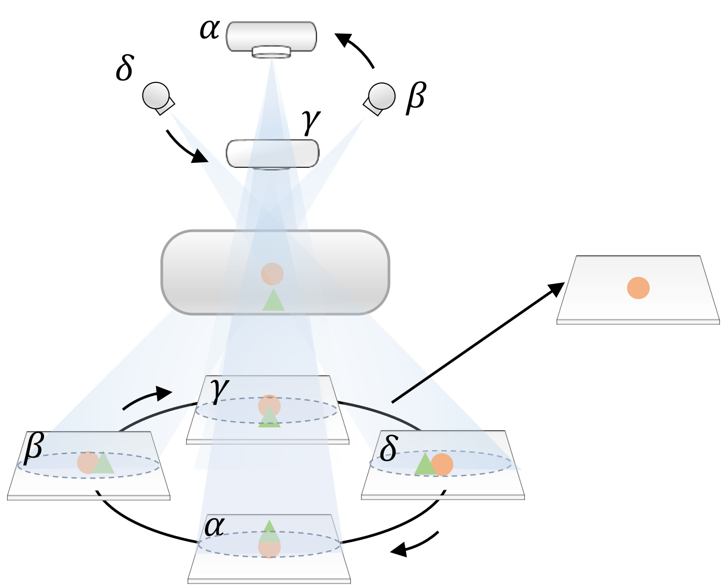
- Low resolution of reconstructed images



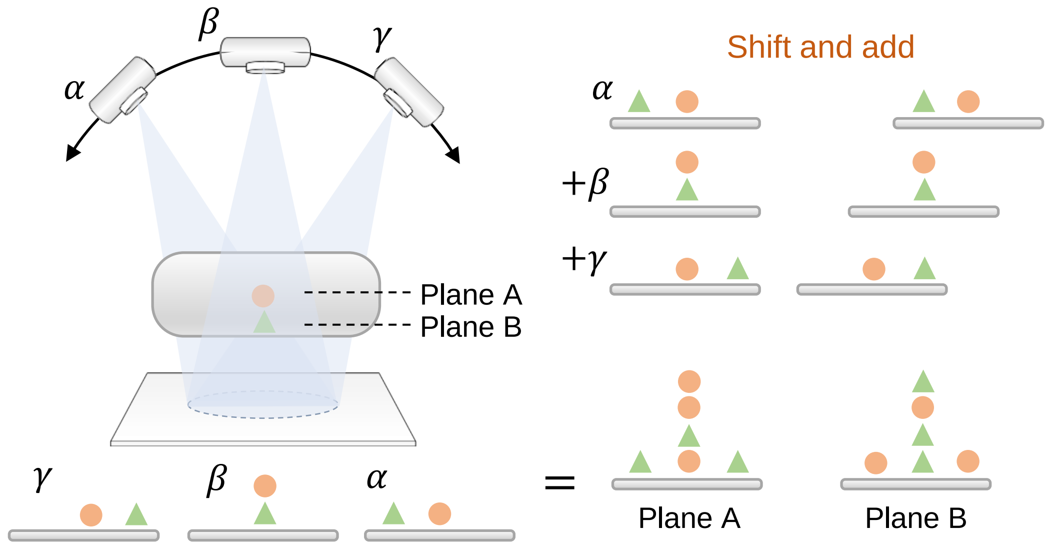
- Impractical for PCB having thin-slab geometry → Limited-angle tomography can be another solution

- Limited-angle tomography (LAT)

Laminography

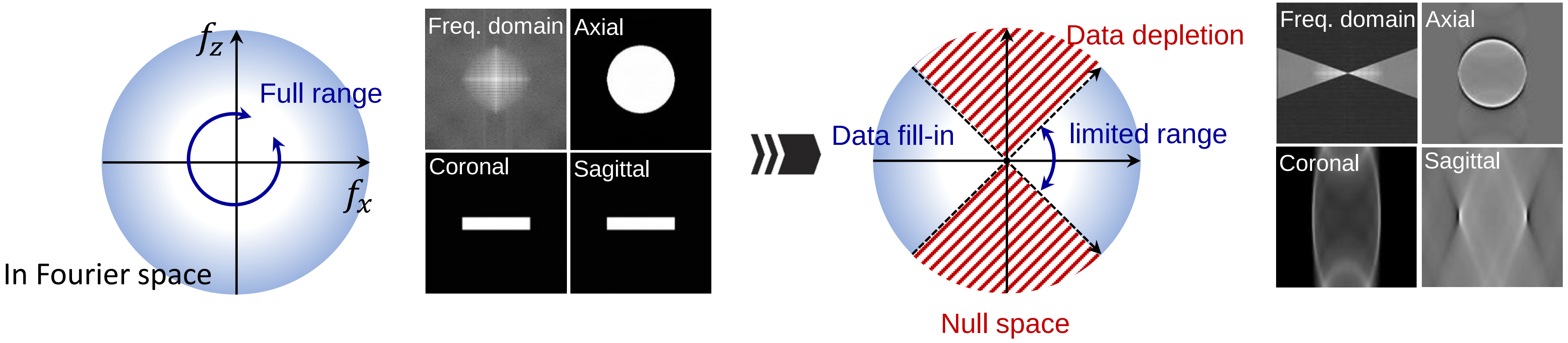


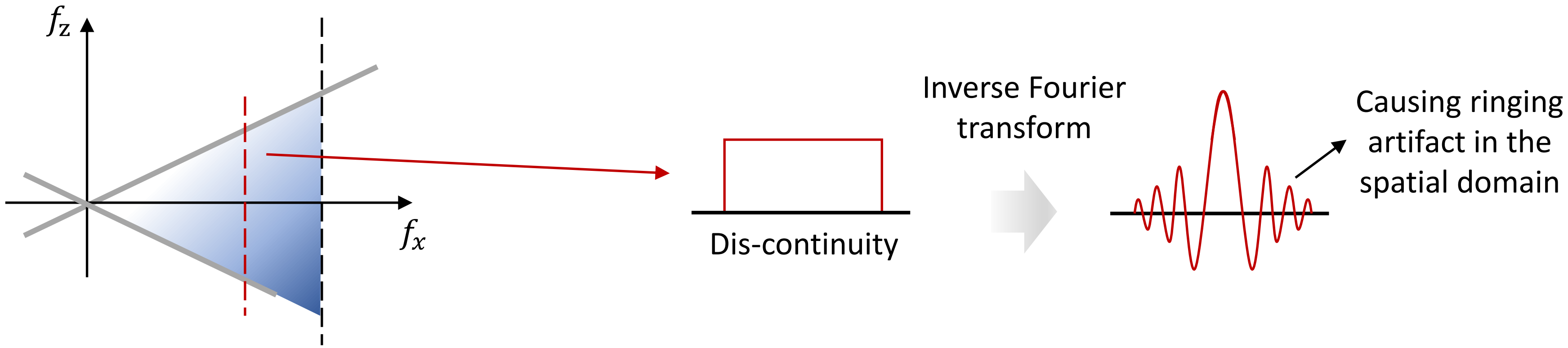
Digital tomosynthesis



▪ Fourier analysis of LAT

- Lack of Fourier data in limited-angle tomography [Fourier slice theorem]

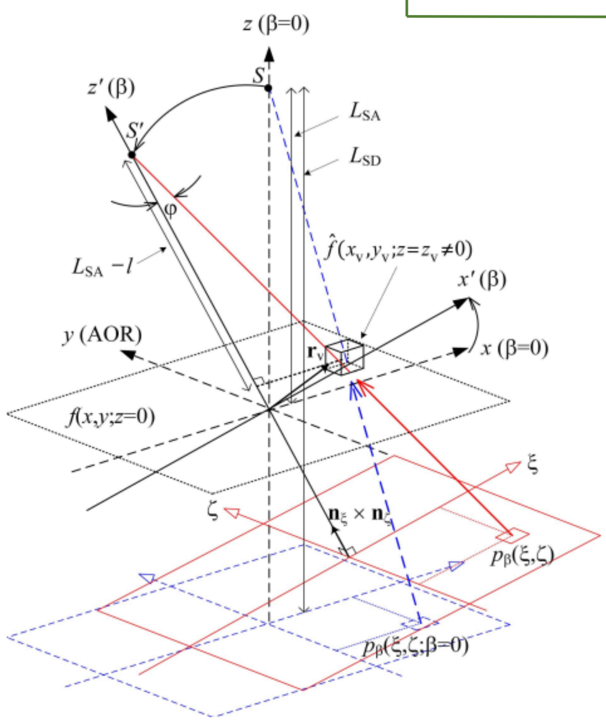




- The null space (also known as missing cone) causes artifacts in the reconstructed images
  - Linear filtering: not possible to recover the missing Fourier data
  - Interpolation and extrapolation (Bayesian): sensitive to noise and inaccurate reconstructed images
  - Iterative algorithms: good at noise performance but timely expensive
- Strategies for reducing artifacts in FBP
  - Limiting the relative volume fraction of null space by using additional apodization
    - Uniform apodization along the depth direction [H. E. Knutsson, *IEEE T. Bio-Med. Eng.* (1980)]
    - Adaptive apodization along the depth direction [W. Haerer. (2002). U.S. Patent No. 6,442,288)]
- Investigation of the strategies with respect to various scan parameters.
  - Scan angle
  - Number of projections

- FBP-LAT
  - Slice image $f(x,y)$ at a depth z can be given by
    - $f(x,y;z) = k \int_{\beta_{\text{scan}}} \frac{L_{SD}^2}{(L_{SA}-l)^2} \int_{-\infty}^{\infty} [\frac{L_{SD}}{\sqrt{L_{SD}^2+\xi^2+\zeta^2}} p_\beta(\xi,\zeta) h(\xi)] d\xi d\beta$

Backprojection



- Preweighting (cosine)

$$\frac{L_{SD}}{\sqrt{L_{SD}^2+\xi^2+\zeta^2}}$$

- Projection at β

$$p_\beta(\xi,\zeta)$$

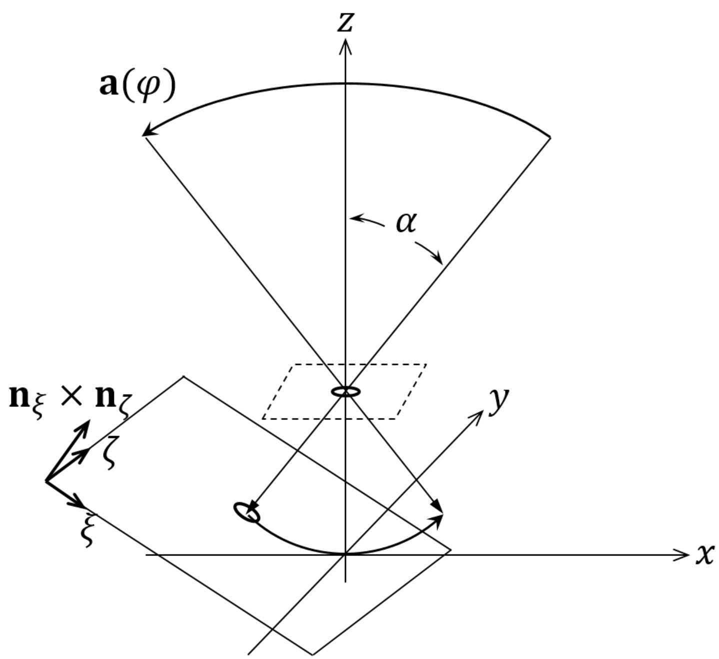
- Filter

$$h(\xi)$$

- Weighting considering beam divergence

$$\frac{L_{SD}^2}{(L_{SA}-l)^2}$$

- FBP-LAT
  - Filter function



- Measurement = Transfer function $*$ Objection
  - $g(\mathbf{x}) = h(\mathbf{x}) * f(\mathbf{x})$
  - $f(\mathbf{x}) = \mathcal{F}^{-1}\{H^{-1}(\mathbf{f})G(\mathbf{f})\}$

- Transfer function (parallel beam)
  - $H(\mathbf{f}) = \begin{cases} \frac{1}{2\tan\alpha\sqrt{(f_x)^2+(f_z)^2}} & \text{for } |f_z| \le |f_x|\tan\alpha \\ 0 & \text{otherwise} \end{cases}$
  - $f_x = f_u \cos\theta,\ f_z = f_u \sin\theta$

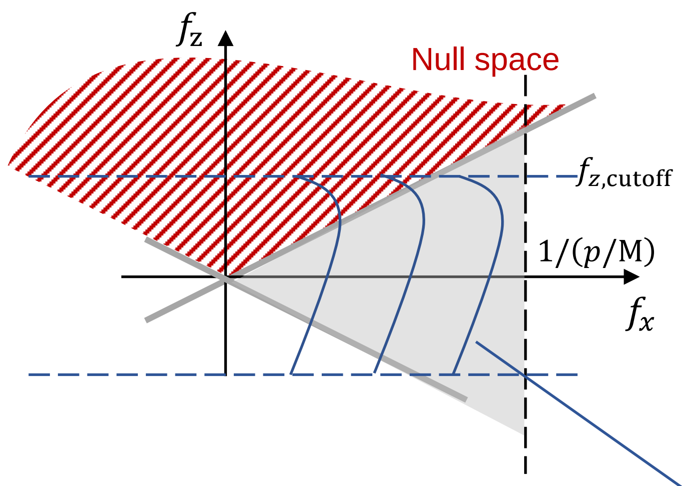
- Reconstructed voxel size

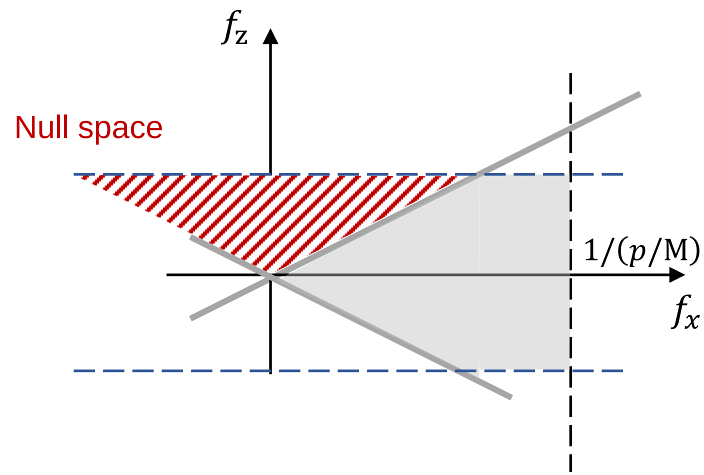
$$\Delta x \times \Delta y \times \Delta z = \left(\frac{p}{M}\right)^2 \times \left(\frac{p}{M} \times \tan(\beta_{\mathrm{scan}}/2)\right)$$

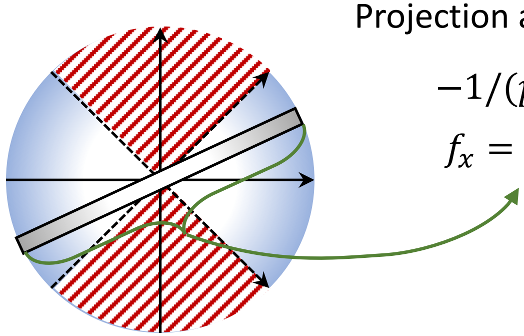
  - $H^{-1}(\mathbf{f}) = 2\tan\alpha\,|f_u| = H_{\mathrm{RA}}(\mathbf{f})$
  - $H_{\mathrm{RA}}(\mathbf{f}) \approx 2\alpha|f_u|$ when $\alpha \ll 1$

- Apodization
  - Hann window $W_{\mathrm{HN}}(f_u) = \frac{1}{2}\left[1 + 1\cos\left(\frac{\pi f_u}{A}\right)\right]$
  - A= f_{u_NY} (Cut-off frequency)

Uniform apodization







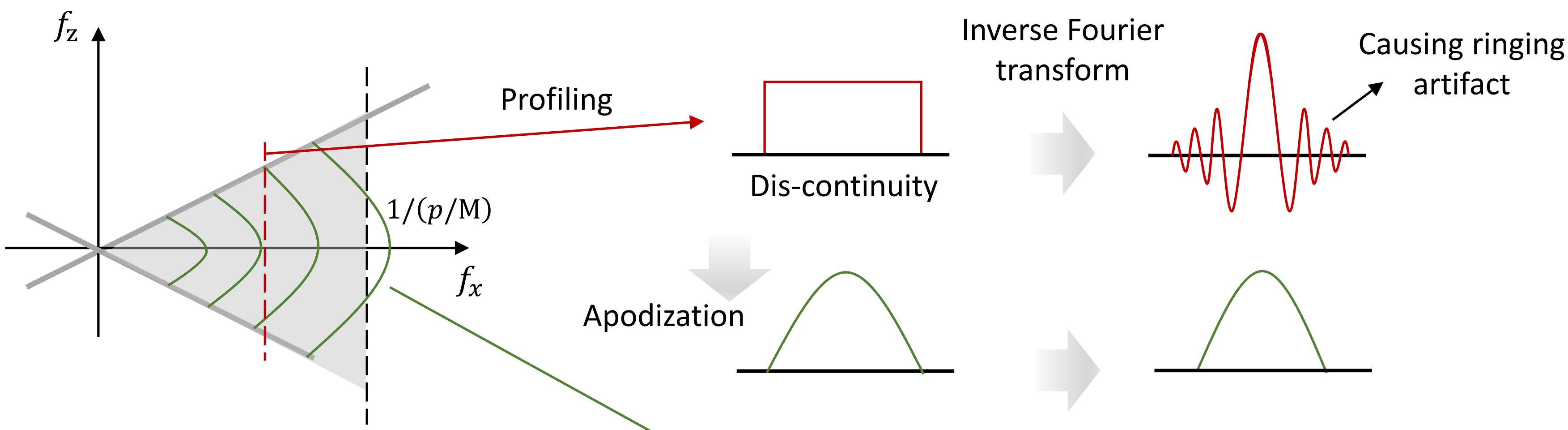
Projection at θ

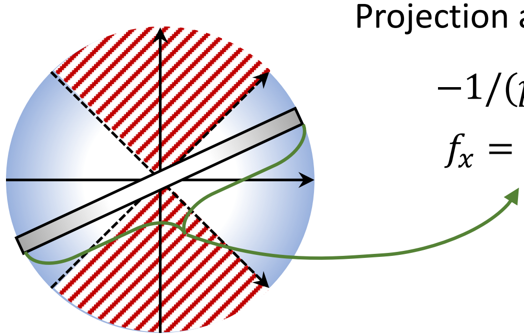
$$-1/(p/\mathrm{m}) \le f_u \le 1/(p/\mathrm{m})$$

$$f_x = f_u \cos\theta,\ f_z = f_u \sin\theta$$

- Apodization
  - $H_{\mathrm{ST}}(f_z) = \begin{cases} \frac{1}{2}\left[1 + \cos\left(\frac{\pi f_z}{f_{z,\mathrm{cutoff}}}\right)\right] & \text{for } |f_z| \le f_{z,\mathrm{cutoff}} \\ 0 & \text{otherwise} \end{cases}$
  - $f_{z,\mathrm{cutoff}}$ = cut-off frequency
- Filtered projection
  - $p_\beta{}^* = \mathcal{F}^{-1}\{\mathcal{F}(p_\beta) H_{\mathrm{RA}}(\mathbf{f})\ W_{\mathrm{HN}}(\mathbf{f})\ H_{\mathrm{ST}}(\mathbf{f})\}$

▪ Adaptive apodization





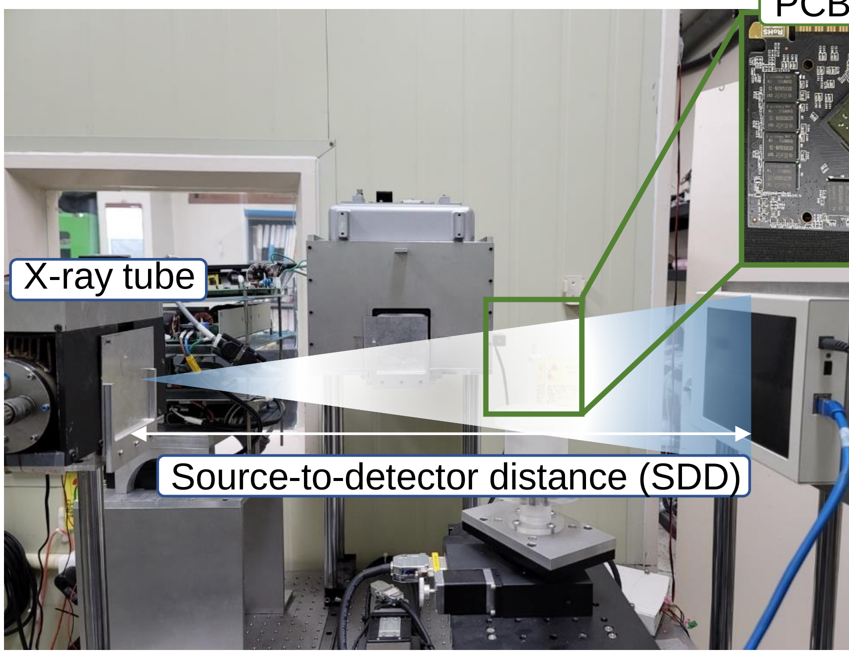
Projection at θ

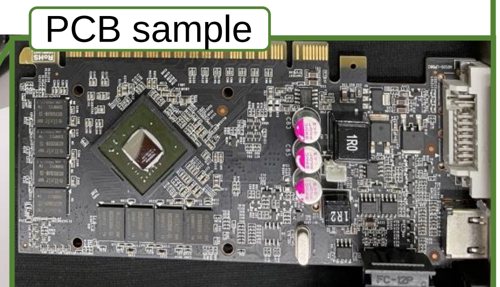
$$-1/(p/\mathrm{m}) \le f_u \le 1/(p/\mathrm{m})$$

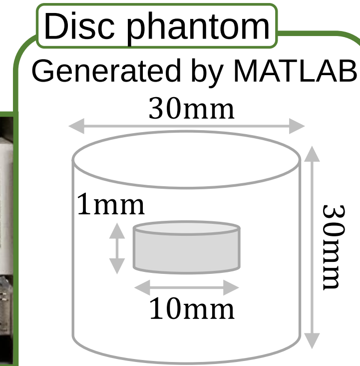
$$f_x = f_u \cos\theta,\ f_z = f_u \sin\theta$$

- Apodization
  - $H_{\mathrm{ST}}(f_z) = \begin{cases} \frac{1}{2}\left[1+\cos\left(\frac{\pi f_z}{f_{z,\mathrm{cutoff}}}\right)\right] & \text{for } |f_z| \le f_{z,\mathrm{cutoff}} \\ 0 & \text{otherwise} \end{cases}$
  - $f_{z,\mathrm{cutoff}} = f_{z\_\mathrm{max}}$ at each f_x
- Filtered projection
  - $p_\beta{}^* = \mathcal{F}^{-1}\{\mathcal{F}(p_\beta) H_{\mathrm{RA}}(\mathbf{f})\ W_{\mathrm{HN}}(\mathbf{f})\ H_{\mathrm{ST}}(\mathbf{f})\}$

- Experimental set-up

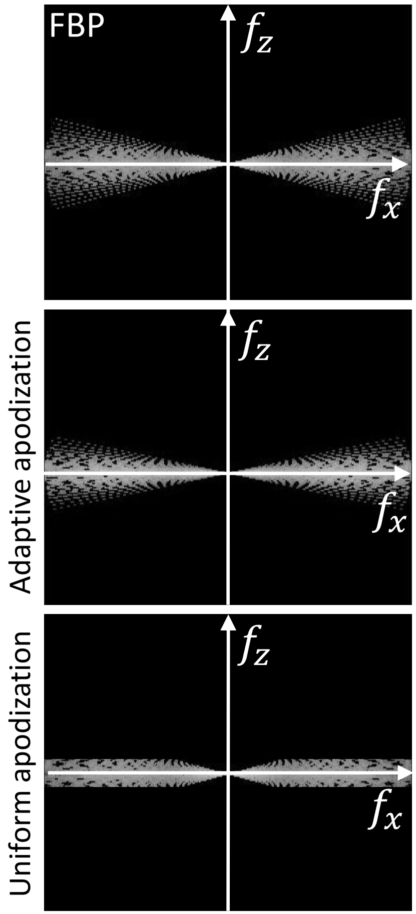


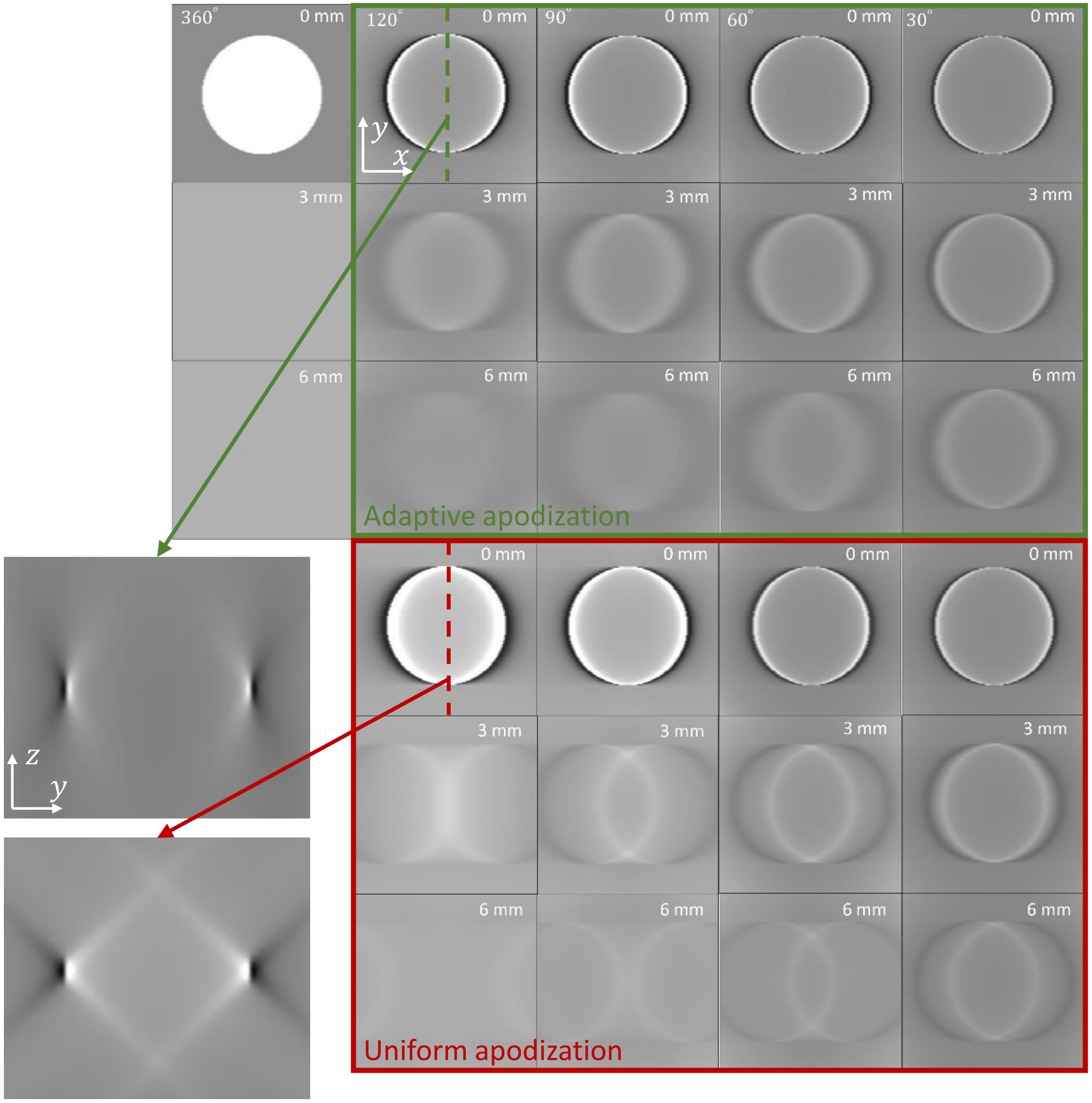




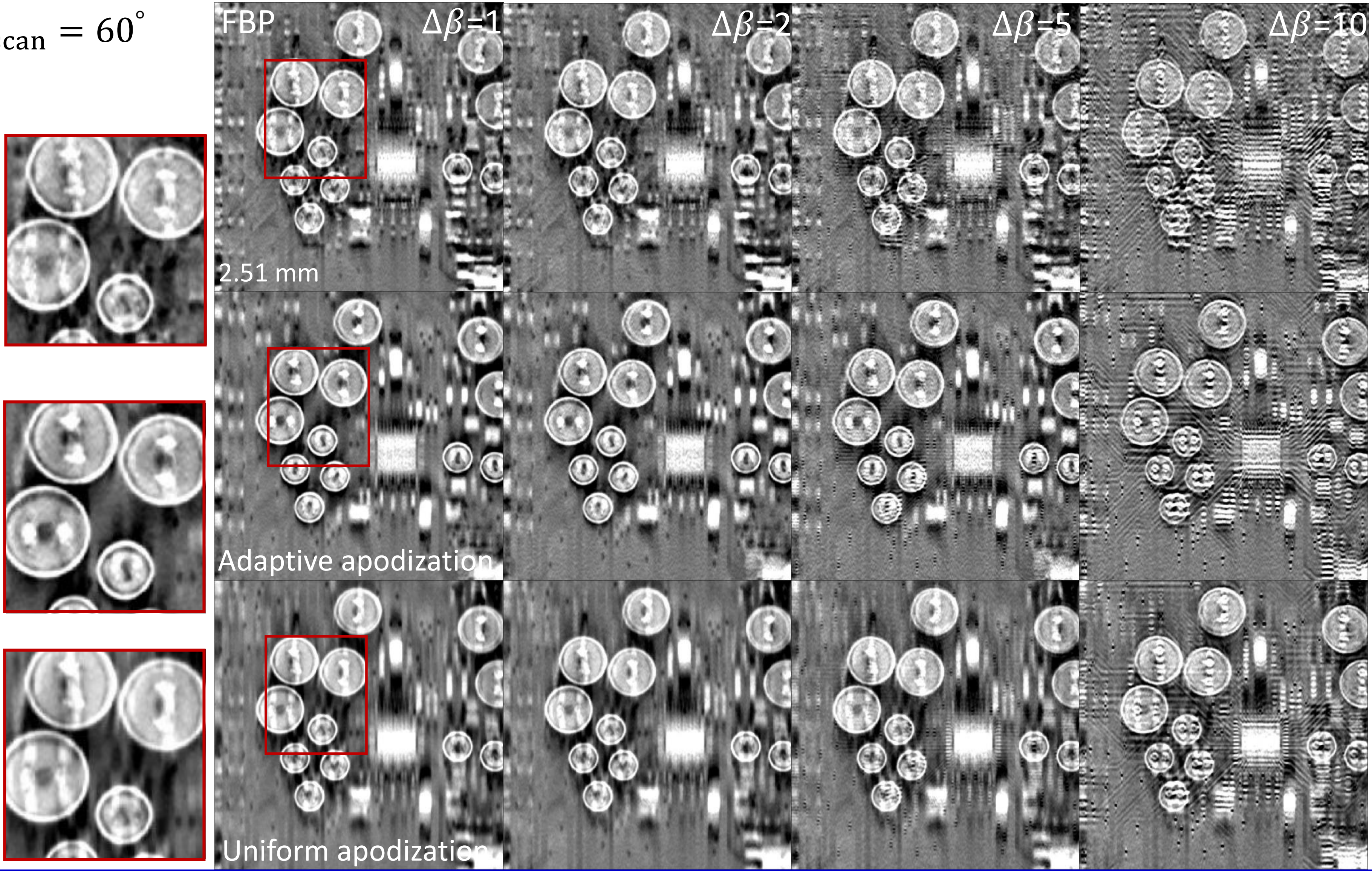
Parameters	Specification
Detector	Shad-o-Box 1548 HS / $p = 99\mu m$
	Pixels: 1032 x 1548 (10.2 x 15.3 cm)
SDD (mm)	665.86, Magnification (M) = 1.5
Energy (kVp)	45
Current (mA)	0.9

Disc phantom

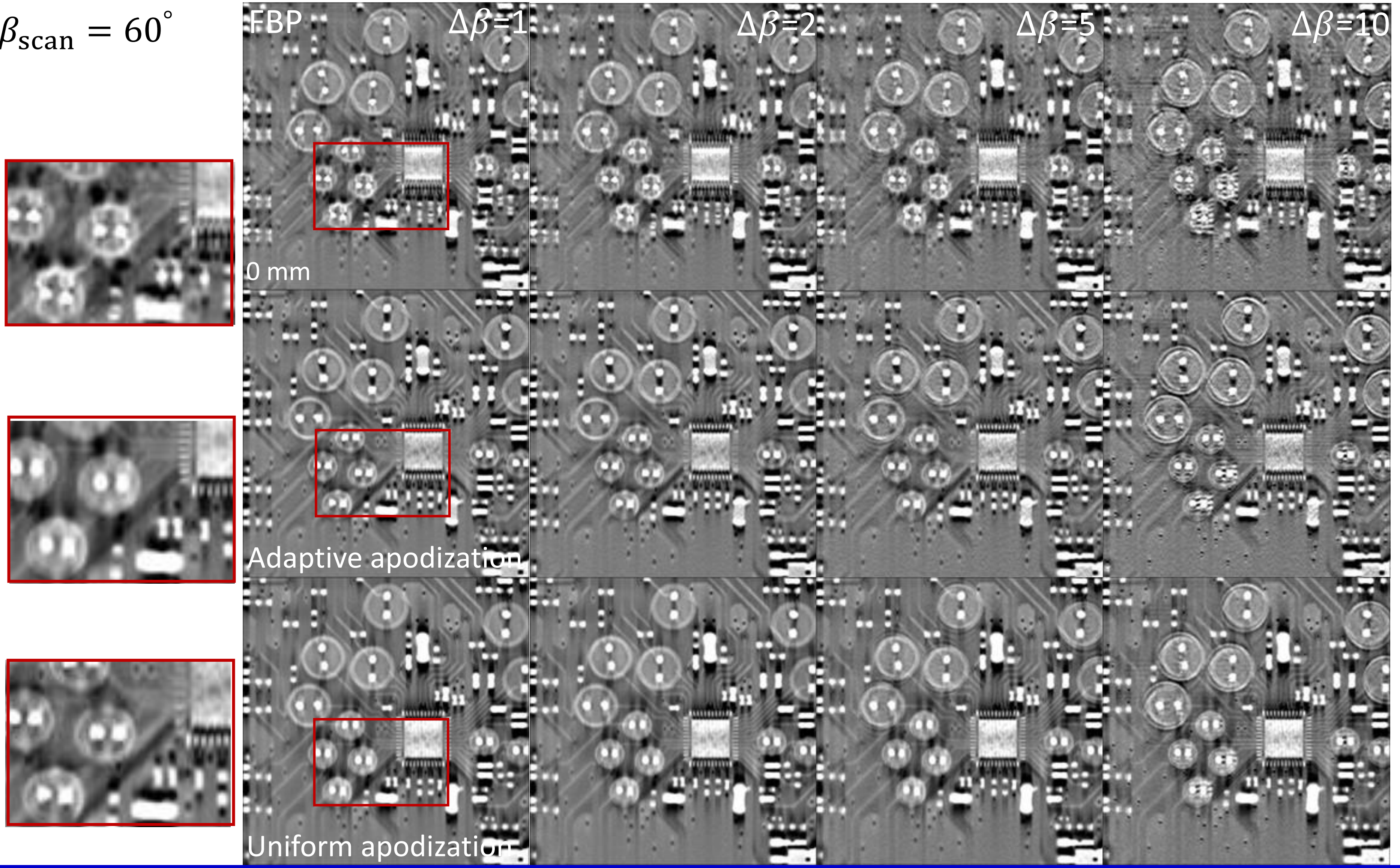




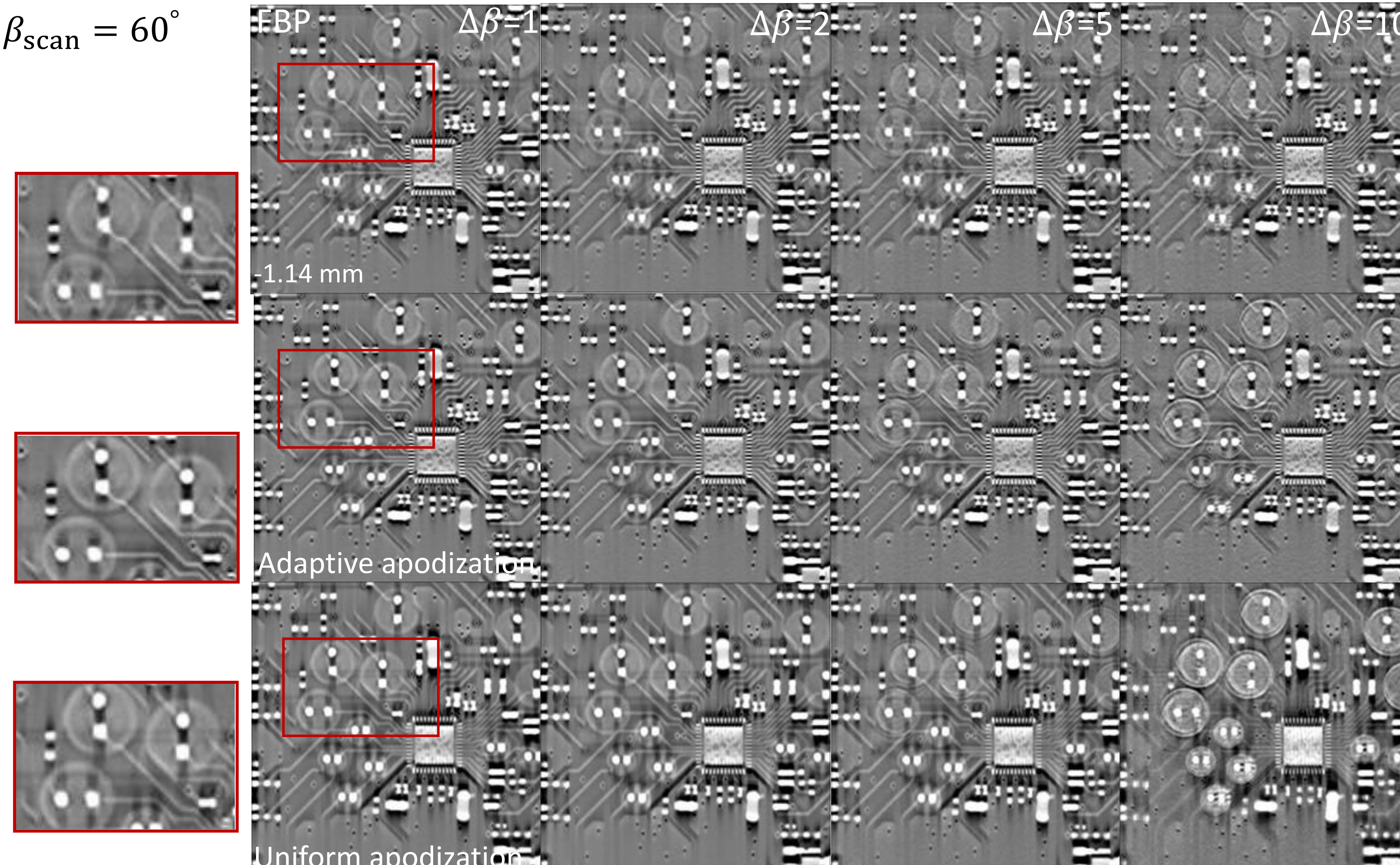
$\beta_{\text{scan}} = 60^{\circ}$



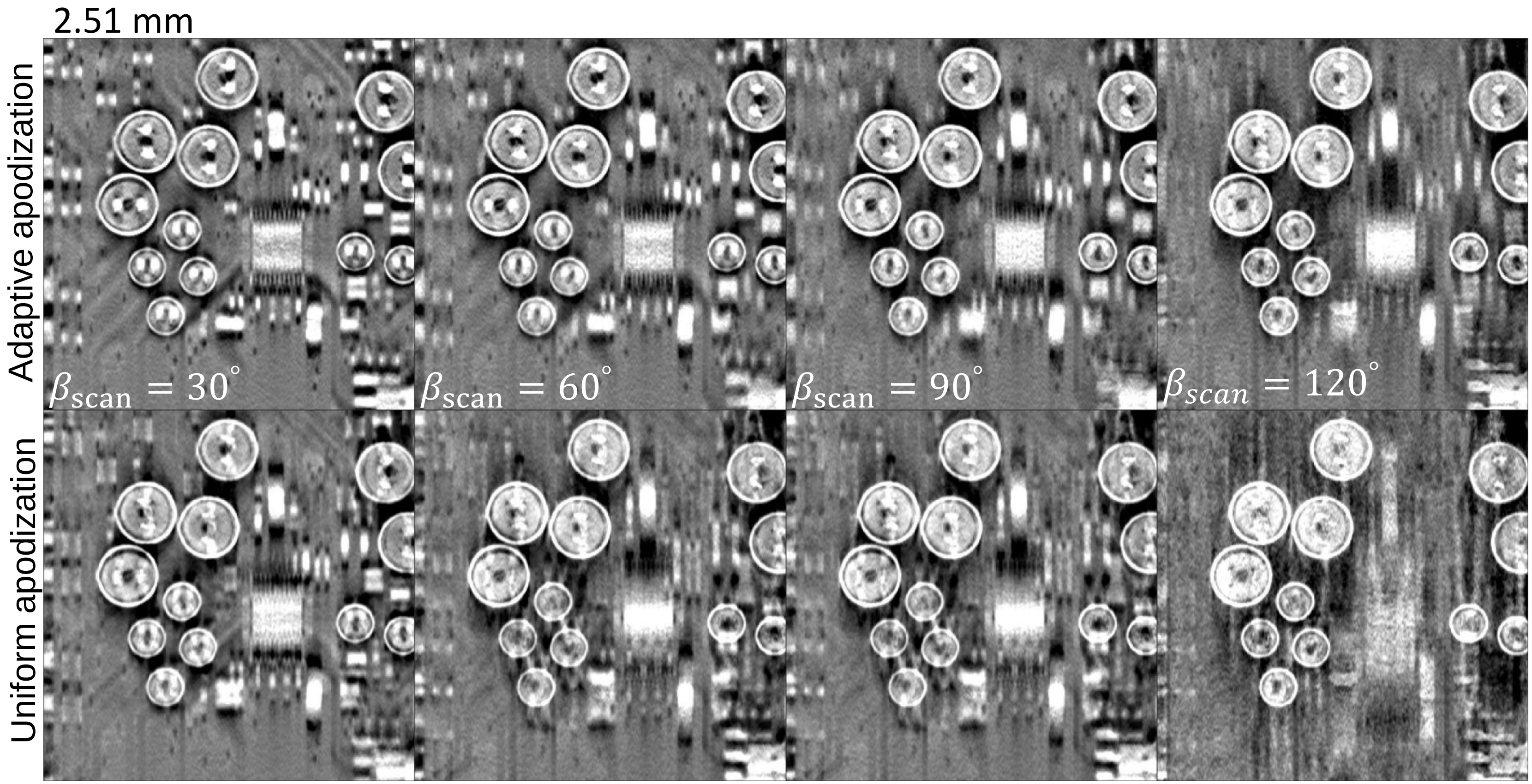
$\beta_{\text{scan}} = 60^{\circ}$



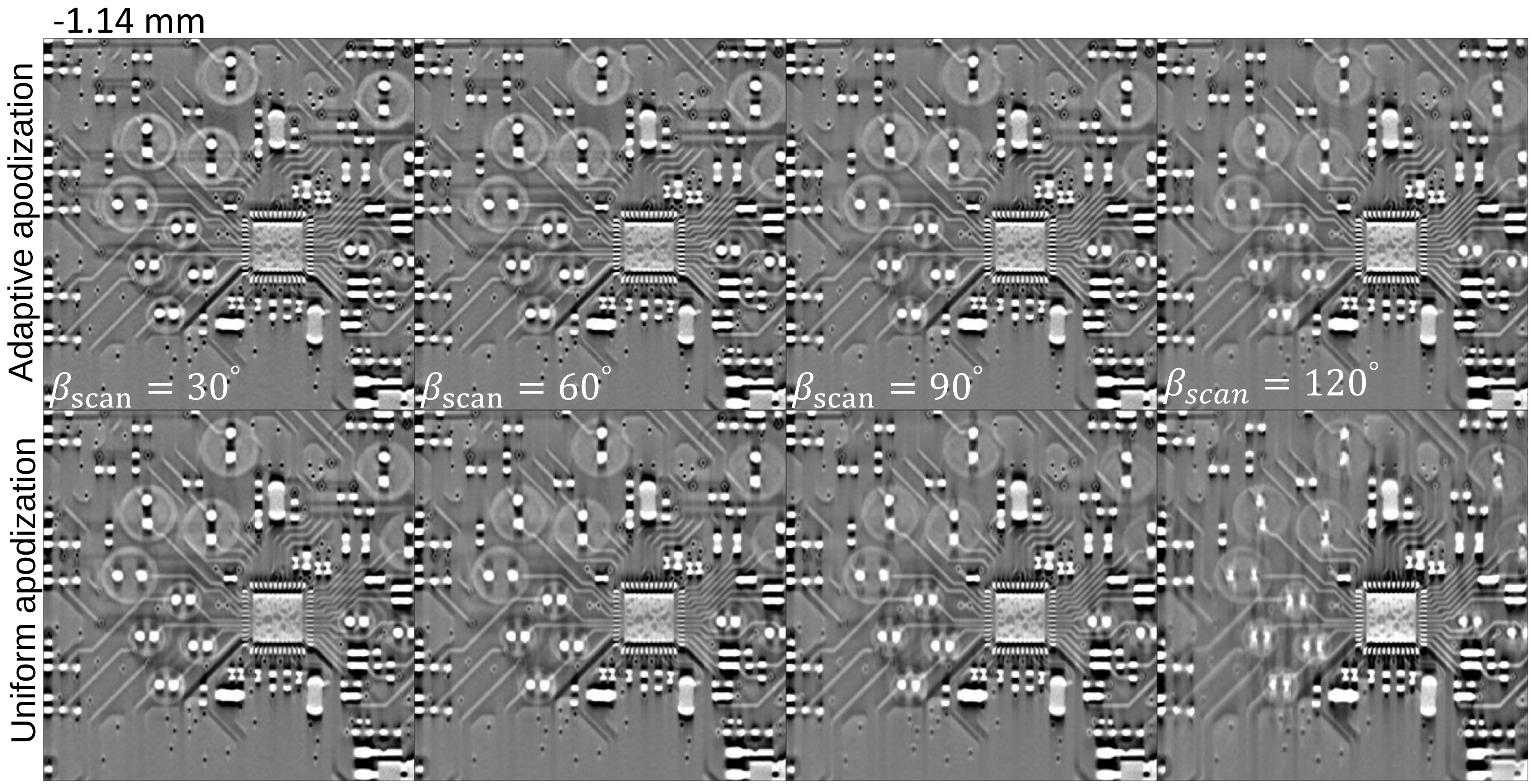
$\beta_{\mathrm{scan}} = 60^{\circ}$



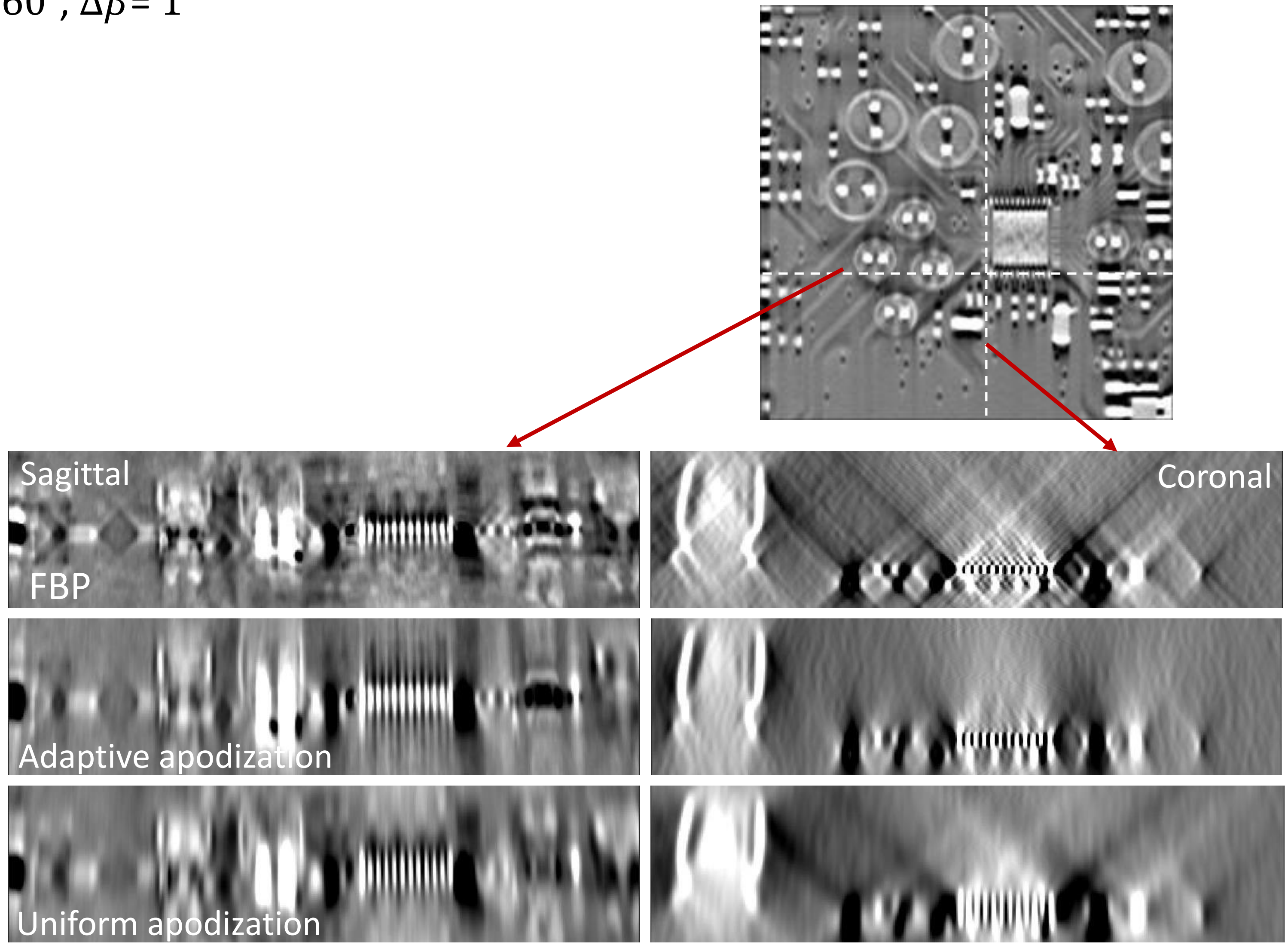
$\Delta\beta = 2$


2.51 mm
Adaptive apodization
Uniform apodization
$\beta_{\text{scan}} = 30^{\circ}$
$\beta_{\text{scan}} = 60^{\circ}$
$\beta_{\text{scan}} = 90^{\circ}$
$\beta_{scan} = 120^{\circ}$

$\Delta\beta = 2$


-1.14 mm
Adaptive apodization
Uniform apodization
$\beta_{scan} = 30^{\circ}$
$\beta_{scan} = 60^{\circ}$
$\beta_{scan} = 90^{\circ}$
$\beta_{scan} = 120^{\circ}$

$\beta_{\text{scan}} = 60^{\circ}, \Delta\beta = 1^{\circ}$



- The two types of apodization in the industrial x-ray NDT images were introduced and its feasibility was investigated using disc phantom and printed circuit board

- The two apodizations enhance image contrast of axial plane at near axis-of-rotation
  - As the depth increase, artifacts decrease in the adaptive apodization
  - But uniform apodization emphasized artifacts. So, appropriate $f_{z,\mathrm{cutoff}}$ should be determined for image performance

- As a result, they show the possibility of PCB having thin slab geometry, and appropriate parameters should be set according to quantitative analysis for image performance

- Our next study will include a more quantitative evaluation of apodizations in terms of contrast, noise, and depth resolution

Thanks for your attention