Prediction of the critical flow rate of subcooled water through the short length channel using analytical method

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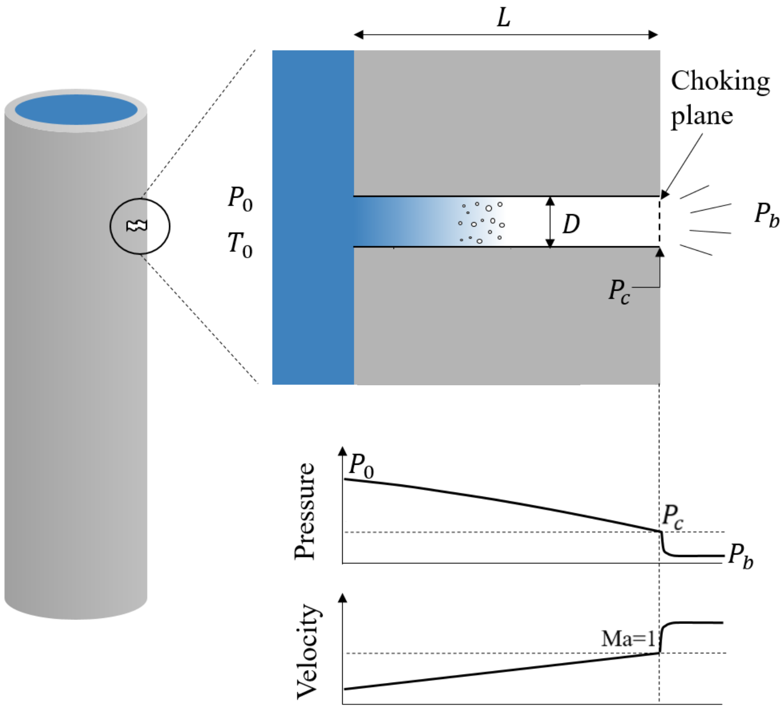
Introduction (1/2)

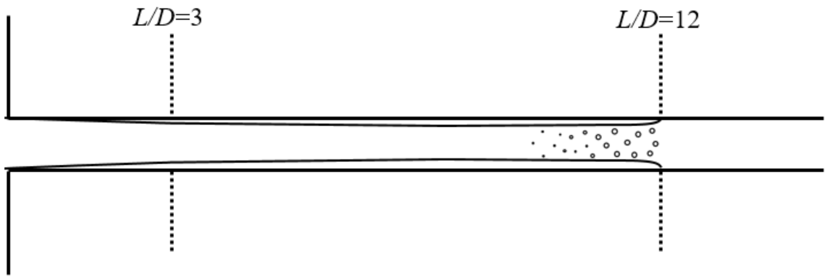
Two-phase critical flow

- The leakage of pressurized water through breaks is encountered in pipe line and heat transfer tube in nuclear power plants.
- When the velocity of leaking fluid reaches the sound speed, the leak rate cannot be increased more even though the downstream pressure decreases more (critical flow).
- The critical flow rate depends on the stagnation condition and channel geometry. Moreover, the two phase flow occurs due to the flashing of water.
- Most of studies of critical flow are related to long channel (L/D > 12), which are not suitable for the SG tubes (L/D ratios are generally 0.8~2.0) [1].



[Phenomenon of subcooled water critical flow through the penetrating crack on SG tube]

Flow patterns of critical flow [2]



[Flow patterns of initially subcooled liquid flowing through a sharp inlet]

L/D < 3 : Superheated liquid jet is surrounded by a vapor annulus

3 < L/D < 12 : Vapor bubbles are formed in the middle of the jet

12 < L/D : Dispersed two-phase region

Introduction (2/2)

Parameter range of two-phase critical flow experiments on short length channel

	Geometry	Channel length (mm)	Hydraulic diameter (mm)	L/D	P_0 (MPa)	ΔT_{sub} (ºC)	Remarks
Sozzi and Sutherland (1975) [3]	Tube	4.7-1778	12.7	0.37-140	~6.5	2-43	Henry-Fauske model was not comparable for ΔT_{sub}> 20ºC
Park et al. (2000) [4]	Tube	2.0-8.0	2.0-8.0	0.5-2.0	4	2-200	Empirical correlation was derived
Revankar et al. (2013) [1]	Slit	1.3	0.59-1.04	1.25-5.42	~6.8	14-51	RELAP5 H-F and R-T model: 30% and 15% error Burnell correlation was modified
Revankar et al. (2019) [5]	Crack	1.2-3.18	0.21-0.65	1.85-6.09	1.4~6.8	12-62	-

- Correlation of two-phase critical flow rate (L/D < 3)

Park et al. [4] $$G_c = C_{d,ref}\sqrt{2\rho_{ref}(P_0 - P_b)}\left\{1.04 - \frac{3.3}{1+\exp[(\Delta T^*_{sub} + 1.1)/0.49]}\right\}, \quad \Delta T^*_{sub} = \frac{T_{sat} - T_0}{T_{sat} - T_{ref}}$$

Revanakr et al. [1] $$G_c = \sqrt{2\rho_0(P_0 - kP_b)}, \qquad k = 1 + 11.6\left(\frac{\Delta T_{sub}}{T_{sat}}\right)^{1.7}$$

Limitations of existing models and correlations

- Analytical models such as Henry-Fauske and Trapp-Ranson model cannot be used for the critical flow through short length channel. Most of models were developed for the range of L/D > 12.

- Although the correlations were developed for L/D > 12, they are not evaluated for various geometries. The developed correlation might be applied only for the specific geometry.

Objectives and Scope

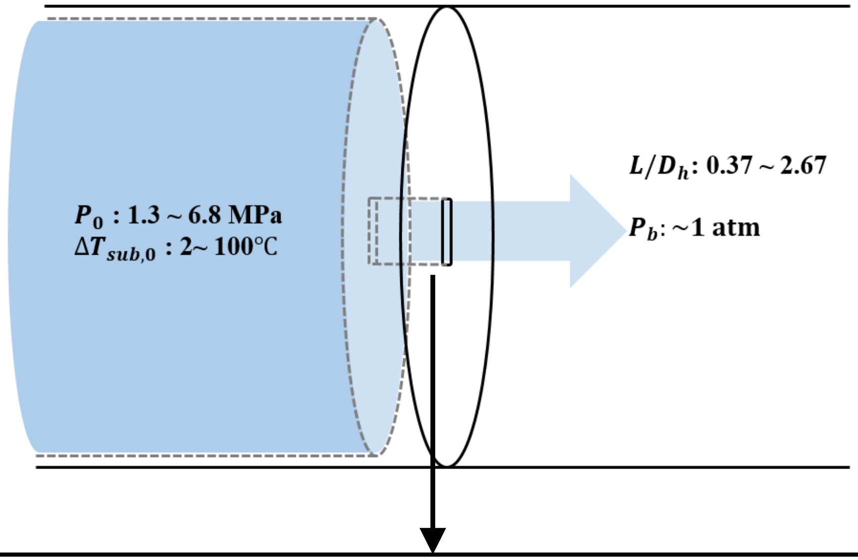
Objectives

- To investigate the characteristics of critical flow through short length channels
- To develop the analytical method to predict two-phase critical mass flux

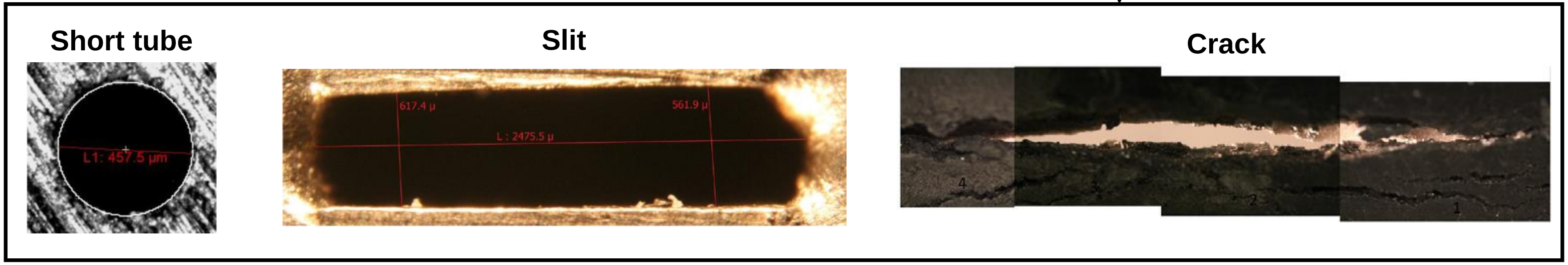
Scope

- Development of analytical method for critical mass flux
  - -Problems of Henry-Fauske Model
  - -Modification of existing analytical method
- Comparison with the previous
  - -Experiments that measured the critical mass flux through short tube, slit, and crack
  - -Correlations developed for L/D < 3

[Reference experimental conditions]



Type of flow channel



Calculation method of Henry-Fauske Model

→ Ideal momentum equation

$$G = \sqrt{\frac{2(P_0 - P_c)}{v_{f,0}}} \quad ...(eq.1)$$

→ Henry-Fauske model for subcooled water [6]

$$G = \left[(v_{g,eq} - v_{f,0}) \frac{N}{(s_{g,eq} - s_{f,eq})} \frac{ds_{f,eq}}{dP_t} \right]^{-1/2} \quad ...(eq.2)$$

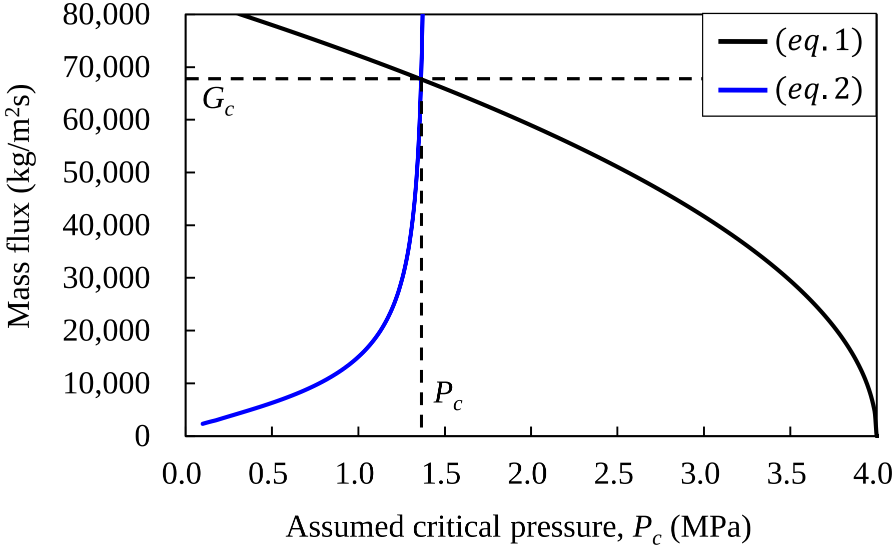
$$N = \begin{cases} x_{eq}/0.14 & x_{eq} \leq 0.14 \\ 1 & x_{eq} > 0.14 \end{cases}$$

- By assuming P_c, the critical mass flux and pressure are determined at the point where the mass flux calculated by eq. (1) and eq. (2) become equal.

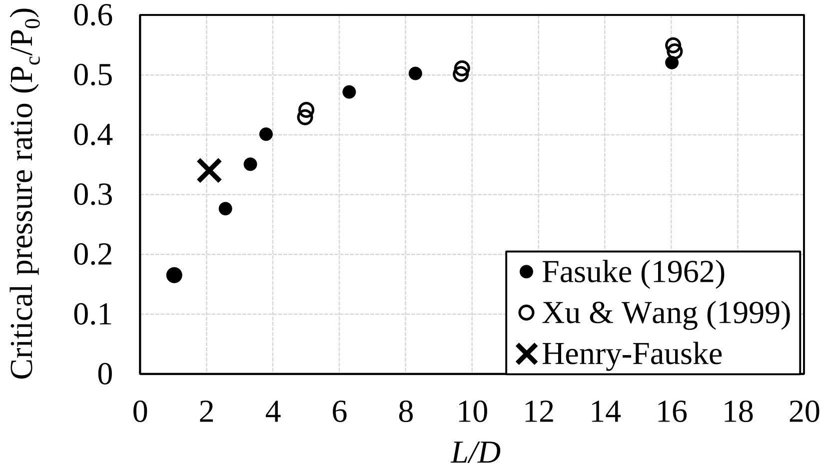
- Henry-Fauske model = 67,589 kg/m²s
  Experimental result = 54,400 kg/m²s

- Critical pressure ratio is higher than previous investigations

Henry-Fauske model	Fauske's experimental result
P_c/P_0 =0.34	P_c/P_0 =0.28



[Henry−Fauske model result, Calculation condition [4]
: P_0=4.0 MPa, $\Delta T_{sub,0}$=55.1 K, L= 8 mm, D= 4 mm, L/D = 2.0]



[Critical pressure ratio of initially saturated water flow through the nozzle [7]]

→ Henry-Fauske model for subcooled water [6]

$$G = \left[\left(v_{g,eq} - v_{l,0} \right) \frac{N}{\left(s_{g,eq} - s_{l,eq} \right)} \frac{ds_{l,eq}}{dP} \right]^{-0.5}$$

- Wang et al. [8] suggested the following non-equilibrium factor

$$N = 0.0376 \frac{L}{D} - 0.163 \quad \text{(Saturated)}$$

$$N = \left(0.0376 \frac{L}{D} - 0.163 \right) \exp(-0.322 \Delta T_{sub}) \quad \text{(Subcooled)}$$

They were compared with experiments (L/D = 16.1, 25.6)
but, cannot be used for L/D < 4.34

- Ghosh et al. [9] suggested that the critical pressure should be calculated by considering various pressure losses.

$$\Delta P_{tot} = P_0 - P_c$$

$$\Delta P_{tot} = \Delta P_e + \Delta P_f + \Delta P_{aph} + \Delta P_{aa} + \Delta P_K$$

ΔP_e = Entry loss

ΔP_f = Friction pressure drop

ΔP_{aph} = Acceleration pressure drop after flashing

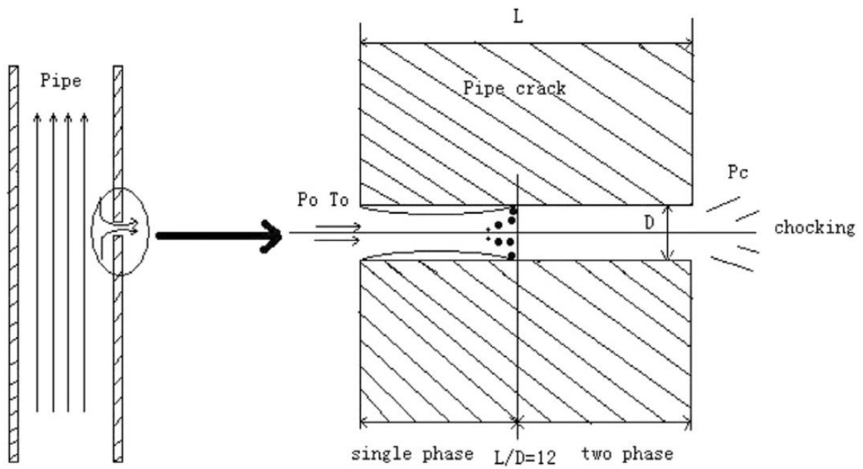
ΔP_{aa} = Pressure loss by area change

ΔP_K = Pressure loss by protrusion in actural crack

Some loss terms are not significant for L/D < 3
Some loss terms are difficult to be calculated

→ Ideal momentum equation

$$G = \sqrt{\frac{2(P_0 - P_c)}{v_{f,0}}}$$



[Diagram of pipe leakage [9]]

- **Previous methods for the modification of Henry-Fauske Model are not suitable for the condition of L/D < 3**

New prediction of critical mass flux (1/3)

Characteristics of two-phase flow in short length channel [10]

There is no enough time to become a equilibrium condition at the choking location.

Fluid passing will not have sufficient time to completely nucleate before leaving the pipe or tube.

Non-equilibrium factor and steam quality

Reference	Non-equilibrium factor (N)	Steam quality (x)	Remark
Henry [2]	$N = 20x_{eq}\ (x_{eq} \le 0.05)$ $1\quad (x_{eq} > 0.05)$	$x = Nx_{eq}\{1 - \exp[-0.0523(L/D - 12)]\}$	Applicable for L/D > 12
Henry and Fauske [6]	$N = x_{eq}/0.14\ \ (x_{eq} \le 0.14)$ $1\quad (x_{eq} > 0.14)$	$x = Nx_{eq}$	Applicable for low quality region
Xu and Wang [7]	$N = (0.037 \cdot L/D - 0.164)\exp\left(-20.7\frac{\Delta T_{sub}}{T_c}\right)$	$x = Nx_{eq}$	Applicable for L/D > ~4.34

Homogeneous Frozen model [11]

- The equation of sound speed of two-phase flow can be derived by combining the mass and momentum conservation of two-phase flow in one-dimension

$$a_{tp}^2 = \left\{\left[\alpha^2 + \alpha(1-\alpha)\frac{\rho_f}{\rho_g}\right]\frac{d\rho_g}{dP} + \left[(1-\alpha)^2 + \alpha(1-\alpha)\frac{\rho_g}{\rho_f}\right]\frac{d\rho_f}{dP} + (\rho_g - \rho_f)\frac{\alpha(1-\alpha)}{x(1-x)}\frac{dx}{dP} - \alpha(1-\alpha)(\rho_g - \rho_f)\frac{dk}{dP}\right\}^{-1}$$

Polytropic, $\frac{d\rho_g}{dP} = \frac{\rho_g}{nP}$ Isentropic flow, $\frac{d\rho_f}{dP} = \frac{1}{a_f^2}$ **Adiabatic, $\frac{dx}{dP} = 0$** Homogeneous, $\frac{dk}{dP} = 0$

$$a_{HFM}^2 = \left\{\frac{1}{\left[\alpha^2 + \alpha(1-\alpha)\frac{\rho_f}{\rho_g}\right] + \left[(1-\alpha)^2 + \alpha(1-\alpha)\frac{\rho_g}{\rho_f}\right]\frac{nP}{\rho_g a_f^2}}\right\}\frac{nP}{\rho_g}$$

New prediction of critical mass flux (2/3)

→ Momentum equation

$$G = \boldsymbol{C_d}\sqrt{\frac{2(P_0 - P_c)}{v_{f,0}}} \quad \ldots (eq.3)$$

Discharge coefficient, $\boldsymbol{C_d} = \frac{actual\ flow\ rate}{ideal\ flow\ rate}$

→ Sound speed by **Homogeneous Frozen Model**

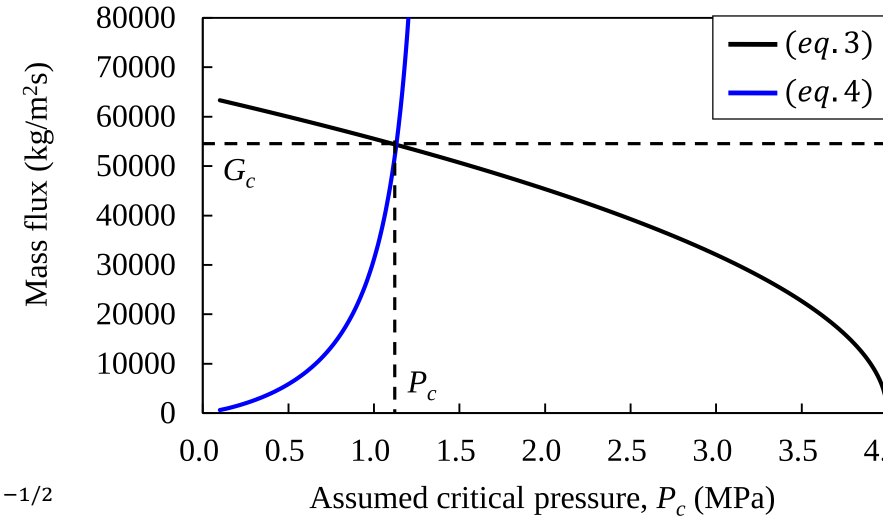
$$\boldsymbol{a_{HFM}} = \left\{\left[\alpha^2 + \alpha(1-\alpha)\frac{\rho_f}{\rho_g}\right] + \left[(1-\alpha)^2 + \alpha(1-\alpha)\frac{\rho_g}{\rho_f}\right]\frac{nP_c}{\rho_g a_f^2}\right\}\frac{nP_c}{\rho_g}^{-1/2}$$

Void fraction, $\alpha = \dfrac{1}{1 + \dfrac{(1-x)}{x}\dfrac{\rho_g}{\rho_f}}$

Quality, $x = N x_{eq},$

Mixture density, $\rho_{mix} = \alpha\rho_g + (1-\alpha)\rho_f$

Mass flux, $G = \rho_{mix} a_{HFM} \ldots (eq.4)$



Calculation condition [4]

[: P_0=4.0 MPa, $\Delta T_{sub,0}$=55.1 K, L= 8 mm, D= 4 mm, L/D= 2.0]

→ Correlation of discharge coefficient, Idelchik [12]

$C_d = \xi^{-0.5}$ ξ : total loss coefficient through sharp-edged orifice (0.015 < L/D < 2.5)

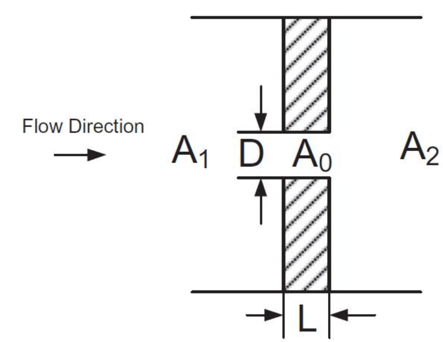
$$\xi = 0.5(1 - A_0/A_1)^{0.75} + (1 - A_0/A_2)^2 + \tau(1 - A_0/A_1)^{0.375}(1 - A_0/A_2) + fL/D$$

$$\tau = (2.4 - L/D) \times 10^{-\varphi},$$

$$\varphi = 0.25 + 0.535(L/D)^8/(0.05 + (L/D)^7)$$

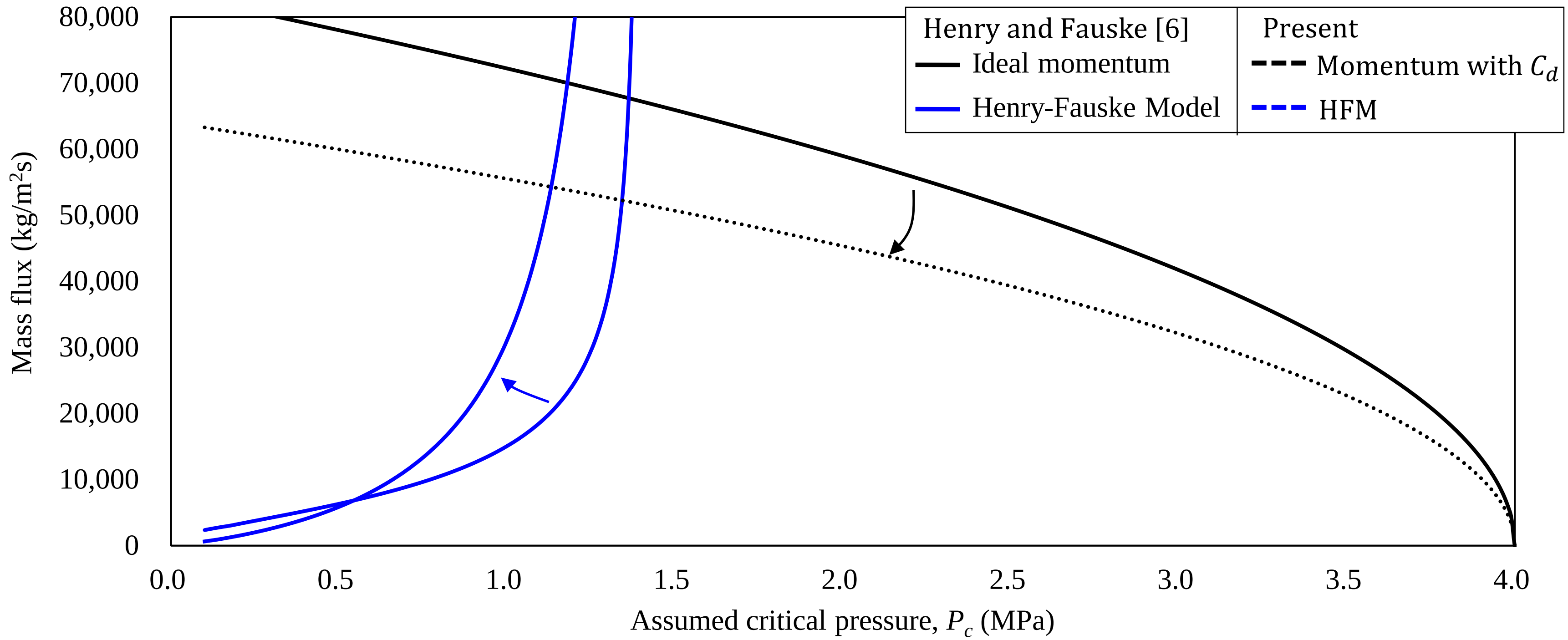
$$f = 1/(1.8\ln(Re) - 1.64)^2$$

Filonenko and Altshul formula:
Friction factor for turbulent flow through smooth circular tube



New prediction of critical mass flux (3/3)

Comparison of present method and Henry-Fauske Model



[Calculation condition [4]: P_0 =4.0 MPa, $\Delta T_{sub,0}$=55.1 K, L= 8 mm, D= 4 mm, L/D = 2.0]

	Experiment [4]	Henry-Fauske [6]	Present
Critical mass flux (kg/m²s)	54,400	67,589	53,955
Critical pressure ratio	-	0.34	0.28
Discharge coefficient	0.77	-	0.81

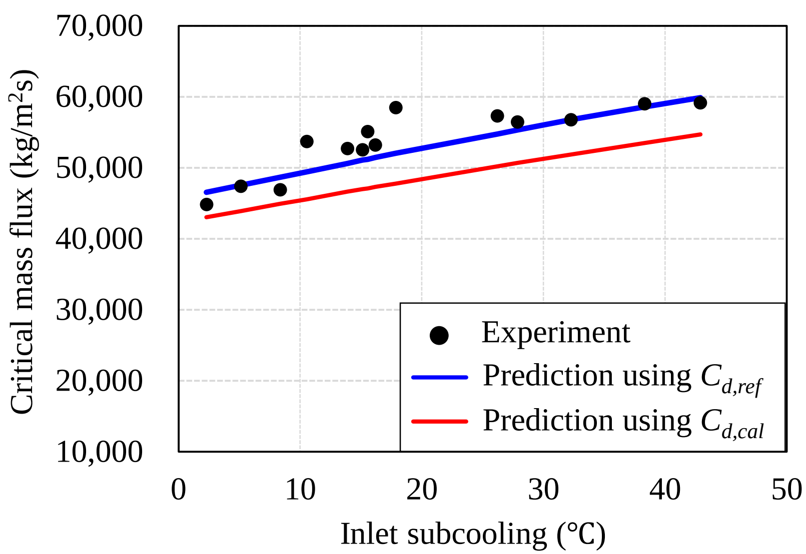
- The present method shows more accurate critical mass flux than the Henry-Fauske model
- The critical pressure ratio by the present method is the same with the Fauske's investigation.

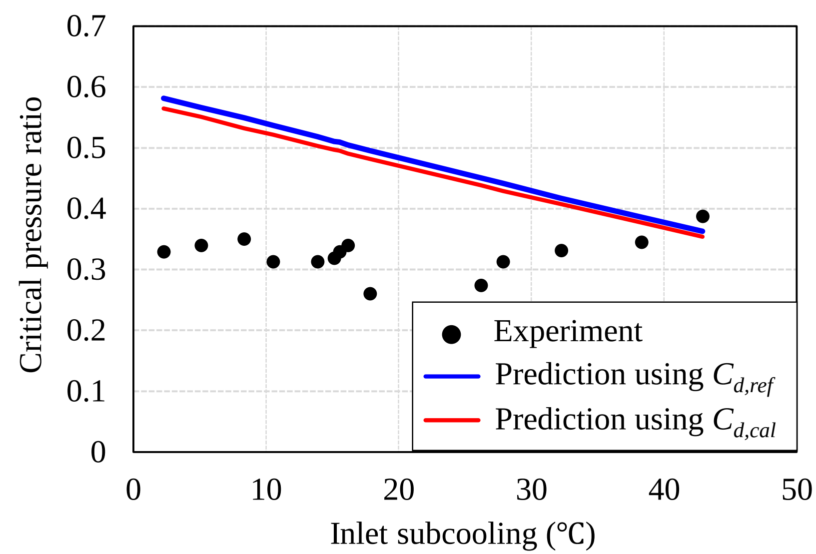
Comparison with experiment, short tube (1/2)

Sozzi and Sutherland's experiments [3]

P_0 (MPa)	ΔT_{sub} (°C)	Geometry	L (mm)	D (mm)	L/D	$C_{d,ref}$	$C_{d,cal}$
6.5	2.3 ~ 42.9	Tube	4.7	12.7	0.37	0.73	0.66

Comparison of prediction and experimental results [3]





- The predicted critical mass flux using $C_{d,ref}$ shows -4.3 ~ 2.3 % difference with the measured critical mass flux
  using $C_{d,cal}$ shows -10.3 % difference with the measured critical mass flux
- The measured critical pressure ratio shows almost constant values (~0.3).
  On the other hand, the calculated pressure ratio is 0.56 at low subcooling and reduced as the subcooling increases.
- Physically, the pressure ratio is inversely proportional to the mass flow rate.
  The critical pressure ratio would be difficult to measure accurately

Park et al.'s experiments [4]

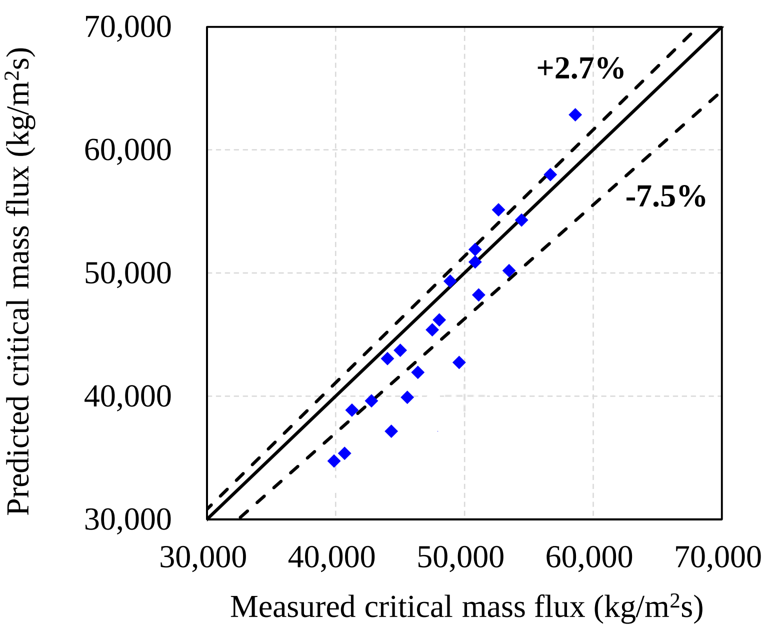
P_0 (MPa)	ΔT_{sub} (°C)	Geometry	L (mm)	D (mm)	L/D	$C_{d,ref}$	$C_{d,cal}$
4.0	2.0 ~ 105.6	Tube	2	4	0.5	0.67	0.66
			4	4	1	0.72	0.76
			8	4	2	0.77	0.82
			4	8	0.5	0.63	0.73
			8	8	1	0.61	0.78

D = 4 mm : with an increase of L/D, the difference between $C_{d,ref}$ and $C_{d,cal}$ increase

D = 8 mm : difference between $C_{d,ref}$ and $C_{d,cal}$ becomes higher than 0.1 this causes inaccuracy prediction of G_{cri}

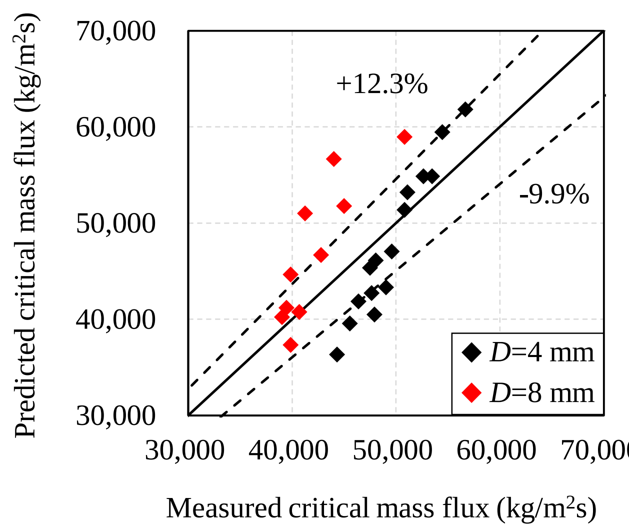
Comparison of prediction and experimental results [4]

- **Prediction using $C_{d,ref}$**



Mean relative error : $\bar{X}_m = \frac{1}{n}\sum_{i=1}^{n} X_{im}$, $X_{im} = \frac{G_{c,pre} - G_{c,exp}}{G_{c,exp}}$

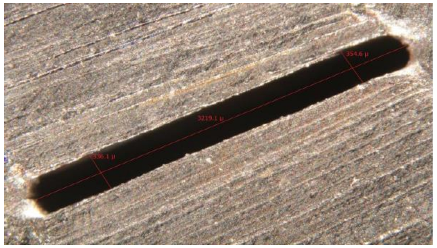
- **Prediction using $C_{d,cal}$**

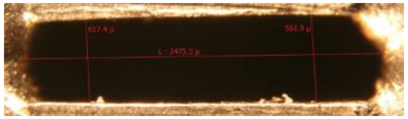


Comparison with experiment, slit

Revankar et al.’s experiments [1]

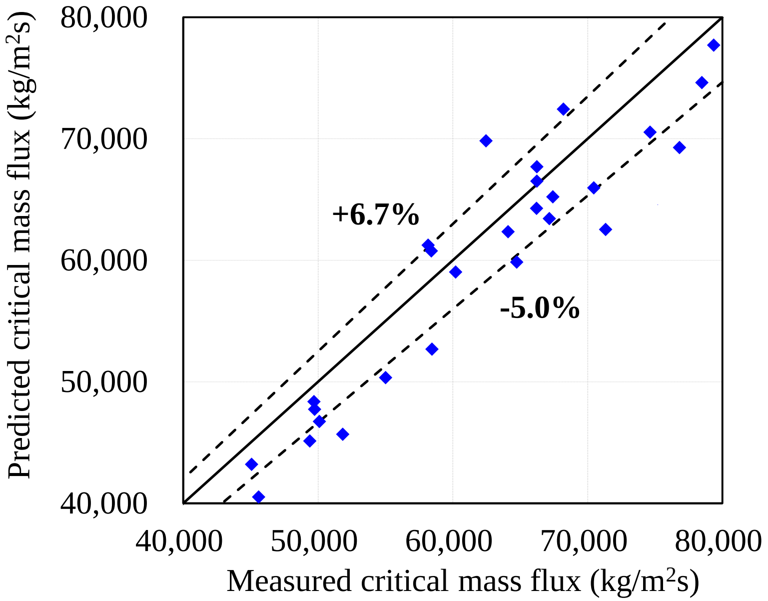
P_0 (MPa)	ΔT_{sub} (°C)	Geometry	L (mm)	D (mm)	L/D	$C_{d,ref}$
6.8	14~ 51	Slit	1.3	0.62 ~ 0.98	1.33 ~ 2.11	0.54~ 0.9



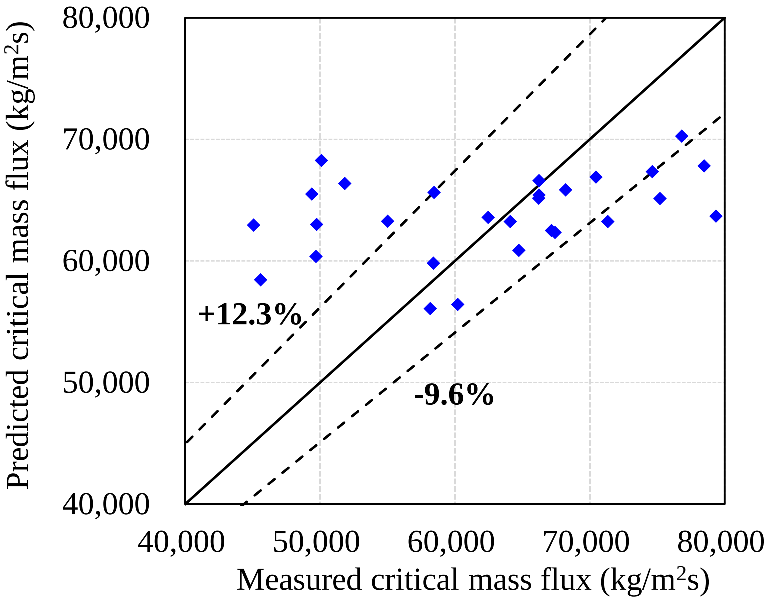


Comparison of prediction and experimental results [slit]

- **Prediction using $C_{d,ref}$**



- **Prediction using $C_{d,cal}$**



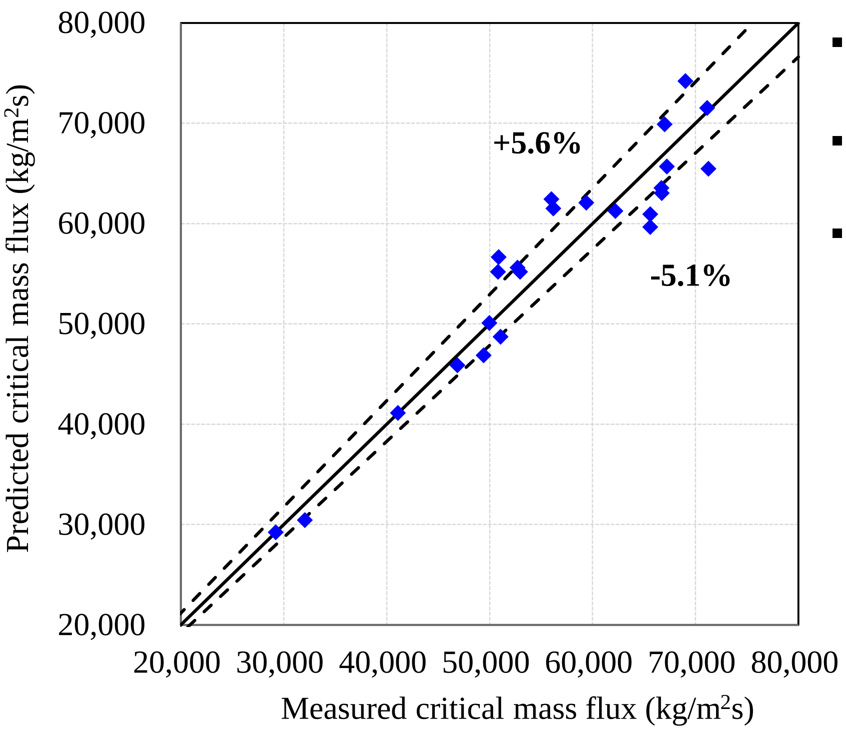
- $C_{d,cal}$ is in the range of 0.79~0.81. However, for some cases $C_{d,ref}$ is 0.54 and 0.61.
- For slit, the geometry effects such as aspect ratio should be considered in the calculation of $C_{d,cal}$

Comparison with experiment, crack

Revankar et al.'s experiments [5]

P_0 (MPa)	ΔT_{sub} (°C)	Geometry	L (mm)	D (mm)	L/D	$C_{d,ref}$
1.3~6.89	12 ~ 55	Crack	1.2	0.45, 0.65	1.85, 2.67	-

Comparison of prediction and experimental results [crack]



- Since $C_{d,ref}$ was not informed in the reference [5], the critical mass flux is predicted based on $C_{d,cal}$ by Idelchik correlation.
- As mentioned before, the correlation of Idelchik has limitations to use for the geometries which are not circular shape.
- However, the predicted mass flux shows good agreement with experimental results

Summary

- Based on the characteristics of two-phase critical flow through short channel, **HFM** and **Mass flux calculation with $C_{d,ref}$** are used to predict the critical mass flux.

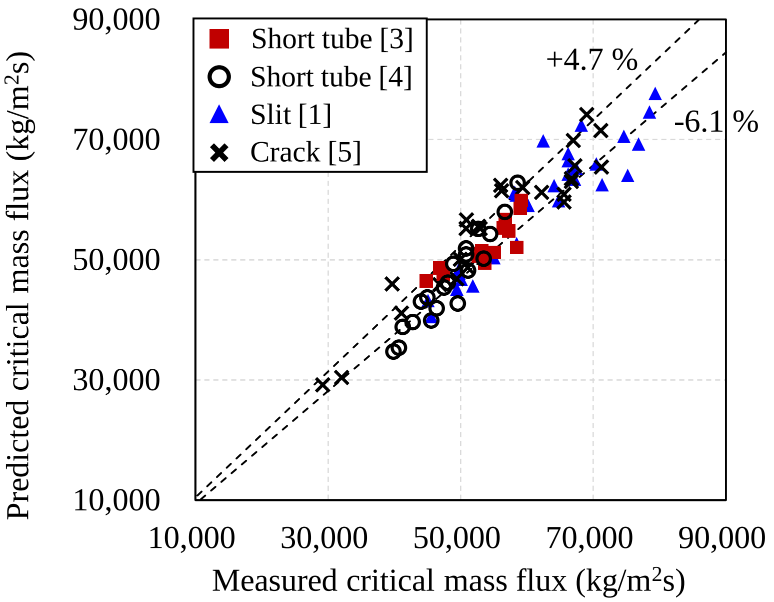
- The present method shows good agreement in various geometries and the existing correlations for short channel are not applicable for various geometries

[Mean relative error of present method and existing correlations compared to experimental data]

Experimental data \ Prediction	Present method	Park et al. [4] (Correlation)	Revankar et al. [1] (Correlation)
Sozzi & Sutherland [3](short tube)	-4.3 ~ 2.3 %	17.3 %	-20.4 ~ 8.4 %
Park et al. [4] (short tube)	-7.5 ~ 2.7 %	-1.3 ~ 2.5 %	-3.2 ~ 29.7 %
Revankar et al. [1] (slit)	-6.7 ~ 5.0 %	-2.3 ~ 11.5 %	-8.5 ~ 25.4 %
Revankar et al. [5] (crack)	-5.1 ~ 5.6 %	15.6 %	-16.3 ~ 13.0 %

- The present method can be applied for the short length channel with various cross-sections in the range of ***L/D* < 3 , and $\Delta T_{sub} \leq 100$℃.**

- For some cases, the results of **critical mass flux using calculated C_d are not comparable** with experiments. It is **necessary to consider the effects of diameter and cross-section shape.**



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