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A Global-Local Iteration Method using the Hexagonal and Trigonal AFEN Methods to handle Intra-Block Heterogeneity in the HTGR Core

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1. Introduction

The trigonal node based Analytic Function Expansion Nodal (AFEN) method has been developed to deal with the asymmetric heterogeneity inside fuel or reflector blocks in the hexagonal reactor core. [1,2] At first, a relatively simple form of the AFEN method was tried, considering that the number of nodes increases by six times in the trigonal AFEN method compared to the hexagonal method. While a more sophisticated AFEN method uses additionally nodal unknowns such as the corner point fluxes or interface flux moments[3-8], this method uses only the neutron flux as the nodal unknown per interface. Since the performance of this method has been disappointing, we then tried to improve it by adding the flux moments only at the hexagonal block interfaces as the nodal unknowns. Because it encompasses all the nodal unknowns and flux constraints of the hexagonal refined AFEN method, the improved method was expected to be at least as accurate as the hexagonal refined AFEN method. However, contrary to this hopeful expectation, this method showed the accuracy of the simple trigonal AFEN method tried earlier rather than that of the hexagonal refined AFEN method.

Although inefficiency of the AFEN expansion function in representing the neutron flux distribution in a trigonal node is suspected to cause the inaccuracy of the tried trigonal AFEN methods, it is extremely burdensome to elucidate and overcome the obvious reason. Note that computation speed of the trigonal methods is already significantly slow by increasing the number of nodal unknowns several times compared to the hexagonal refined AFEN method. As recommended in Reference 2, we propose a global-local iteration method to handle the heterogeneity inside hexagonal blocks alternatively. In this method, the trigonal nodal method, along with the hexagonal nodal method constitutes an iterative process in which the trigonal nodal method solves a single asymmetrically heterogeneous hexagonal block with the boundary condition obtained by the global hexagonal nodal calculation and passes the homogenization constants of the block to the global hexagonal nodal calculation. These constants include the hexagonal interface discontinuity factors designed to correct only the effect of heterogeneity within the hexagonal node, excluding the effect of inaccuracy of the trigonal AFEN method. The equivalence theory[9,10] guarantees that this global-local iteration method treats the heterogeneous hexagonal blocks in the global homogeneous calculation. Noting that there are a few hexagonal blocks with internal heterogeneity in a typical high temperature gas-cooled reactor (HTGR) core, the increase in computation time compared to the hexagonal AFEN case will also be negligible.

2. Methodology

Fig. 1 schematically shows the proposed global-local iteration scheme. This kind of scheme has proven its effectiveness theoretically and practically and is widely used in the reactor physics analysis for various purposes. In the global stage of our scheme, the whole core composed of homogeneous hexagonal blocks is solved by the hexagonal AFEN method. A typical inner and outer iteration process is used to solve an eigenvalue elliptic problem with boundary conditions. At a certain moment of the iteration process, the global stage passes the interface currents only for the internally heterogeneous blocks to the local stage. In the local stage, each single block with internal heterogeneity is solved by the trigonal AFEN method. The single block is heterogeneous, i.e. consists of trigonal nodes with different cross-sections. Given the fixed boundary currents, this problem becomes a fixed source elliptic problem. Once the problem is solved by the trigonal AFEN method, the flux weighted cross-sections and the interface discontinuity factors for the hexagonal block are evaluated and passed to the global stage.

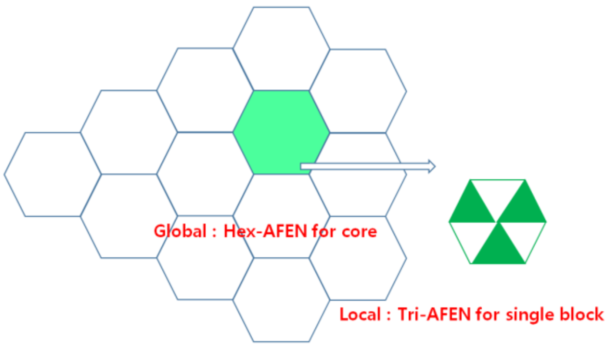

Fig. 1. Global-local iteration scheme

Using the hexagonal block cross-sections weighted by the flux shape calculated in the local stage and the discontinuity factors defined by the ratio of the interface flux calculated in the local stage to that calculated by the global stage, the equivalence theory[9,10] guarantees that the flux solution of the global stage shall be equivalent to the flux solution if we solve the global core by the local method. However, note that we do not want the accuracy of solving the entire core by the local method, the trigonal AFEN method because it is less accurate than the global method, the hexagonal AFEN method. We need to capture only the effect of heterogeneity within the hexagonal block in the local stage calculation, discarding the effect of inaccuracy of the trigonal AFEN method. This can be done by defining the discontinuity factor as the ratio of two interface fluxes calculated by the same trigonal AFEN method: one for the heterogeneous block and the other for the homogeneous block.

Considering that a HTGR core usually has a few heterogeneous blocks, our global-local iteration scheme can then maintain the accuracy and the computation time of the hexagonal AFEN method.

2.1 Hexagonal AFEN method for global core problem

The hexagonal AFEN method to solve the global whole core problem is the response matrix method based on the refined AFEN method that uses both the partial current and the partial current moment as the nodal unknown per interface. The step function is used in defining the interface partial current moment. The excellent accuracy and efficiency of this method is well documented in References 4 and 8.

This method is well described in References 4 and 8 if the discontinuity factors are not involved. We explain here how to implement the discontinuity factors to the method.

Assuming that the net current at the interface s of a hexagonal node in Fig. 2 is defined in the direction outgoing from the node, the heterogeneous partial currents are given by

$$\boldsymbol{j}_s^{\text{out}} = \frac{1}{2}\boldsymbol{J}_s + \frac{1}{4}\boldsymbol{\phi}_s\,, \quad \boldsymbol{j}_s^{\text{in}} = -\frac{1}{2}\boldsymbol{J}_s + \frac{1}{4}\boldsymbol{\phi}_s \tag{1}$$

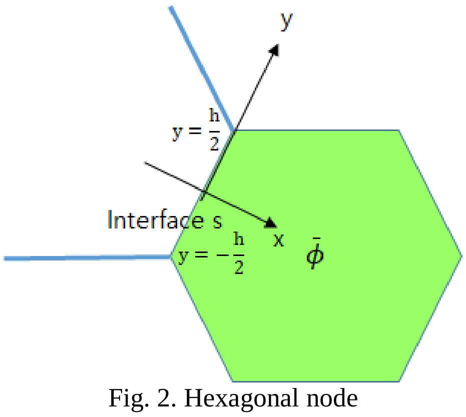

Fig. 2. Hexagonal node

Their homogeneous partners are given by

$$\hat{\boldsymbol{j}}_s^{\text{out}} = \frac{1}{2}\boldsymbol{J}_s + \frac{1}{4}\hat{\boldsymbol{\phi}}_s\,, \quad \hat{\boldsymbol{j}}_s^{\text{in}} = -\frac{1}{2}\boldsymbol{J}_s + \frac{1}{4}\hat{\boldsymbol{\phi}}_s \tag{2}$$

where the symbol hat ^ indicates a homogeneous quantity and the fact that the homogeneous current preserves the heterogeneous current due to the equivalence theory[9,10] is reflected. Note that the discontinuity factor is defined by

$$\boldsymbol{f}_s = \frac{\boldsymbol{\phi}_s}{\hat{\boldsymbol{\phi}}_s} \tag{3}$$

Then the homogeneous partial currents become

$$\hat{\boldsymbol{j}}_s^{\text{out}} = \boldsymbol{J}_s + \frac{1}{4}\frac{\boldsymbol{\phi}_s}{\boldsymbol{f}_s}\,, \quad \hat{\boldsymbol{j}}_s^{\text{in}} = -\frac{1}{2}\boldsymbol{J}_s + \frac{1}{4}\frac{\boldsymbol{\phi}_s}{\boldsymbol{f}_s} \tag{4}$$

The homogeneous response matrix equation is given by

$$\hat{\boldsymbol{j}}_s^{\text{out}} = \hat{\mathbf{R}}\,\hat{\boldsymbol{j}}_s^{\text{in}} \tag{5}$$

Finally, the heterogeneous response matrix equation is obtained by solving Eqs. (1), (4) and (5) for $\boldsymbol{j}_s^{\text{out}}$ by eliminating $\boldsymbol{J}_s$, $\boldsymbol{\phi}_s$, $\hat{\boldsymbol{j}}_s^{\text{in}}$, and $\hat{\boldsymbol{j}}_s^{\text{out}}$.

$$\boldsymbol{j}_s^{\text{out}} = \mathbf{R}\,\boldsymbol{j}_s^{\text{in}} \tag{6}$$

where the heterogeneous response matrix **R** is given by

$$\mathbf{R} = \{\boldsymbol{f_s} + \mathbf{1} + (\boldsymbol{f_s} - \mathbf{1})\hat{\boldsymbol{R}}\}^{-\mathbf{1}}\{\boldsymbol{f_s} - \mathbf{1} + (\boldsymbol{f_s} + \mathbf{1})\}\hat{\boldsymbol{R}} \tag{7}$$

We will use the simple trigonal AFEN method as the solver for the local single block problem which does not have the discontinuity factor for the flux moment because the flux moment is not defined in the method. In this case, we will take a look at which discontinuity factor to use for the flux moment in the global calculation. We define two half interface fluxes at the interface s in Fig. 2 as follows:

$$\hat{\boldsymbol{\Phi}}_B = \frac{2}{h}\int_{-\frac{h}{2}}^{0}\boldsymbol{\phi}(x_s,y)\,dy \qquad \hat{\boldsymbol{\Phi}}_T = \frac{2}{h}\int_{0}^{\frac{h}{2}}\boldsymbol{\phi}(x_s,y)\,dy \tag{8}$$

Then, the interface flux moment defined with the step weighting function that alternating sign across y=0 becomes

$$\hat{\boldsymbol{\Psi}}_s = \frac{1}{h}\left(\int_{\mathbf{0}}^{\frac{1}{2}}\boldsymbol{\phi}(x_s,y)\,dy - \int_{-\frac{h}{2}}^{\mathbf{0}}\boldsymbol{\phi}(x_s,y)\,dy\right) = \frac{\hat{\boldsymbol{\Phi}}_T - \hat{\boldsymbol{\Phi}}_B}{\mathbf{2}} \tag{9}$$

Similarly, the interface flux becomes the arithmetic mean of $\boldsymbol{\phi}_A$ and $\boldsymbol{\phi}_B$. We assume that

$$\mathbf{f}_s = \frac{\boldsymbol{\Phi}_s}{\hat{\boldsymbol{\Phi}}_s} = \frac{\boldsymbol{\Phi}_T}{\hat{\boldsymbol{\Phi}}_T} = \frac{\boldsymbol{\Phi}_B}{\hat{\boldsymbol{\Phi}}_B} \tag{10}$$

Then,

$$\boldsymbol{\Psi}_s = \frac{\boldsymbol{\Phi}_T - \boldsymbol{\Phi}_B}{\mathbf{2}} = \frac{\mathbf{f}_s(\hat{\boldsymbol{\Phi}}_T - \hat{\boldsymbol{\Phi}}_B)}{\mathbf{2}} = \mathbf{f}_s\hat{\boldsymbol{\Psi}}_s \tag{11}$$

Therefore, we can approximately apply the flux discontinuity factor to the flux moment as well.

2.2 Trigonal AFEN method for local block problem

2.2.1 Response matrix of trigonal AFEN method

We introduced two trigonal AFEN methods in our previous works: a simple one[1] and a refined one[2]. The method we chosen to solve the local single heterogeneous hexagonal block problem is the simple trigonal AFEN method, because even the simple method is not much less accurate than the refined one.

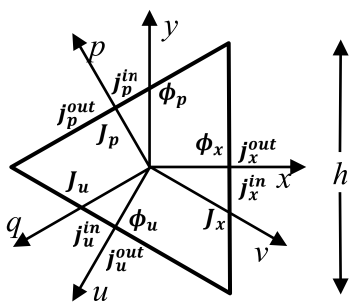

Fig. 3. Trigonal node

The single hexagonal block problem with the net current boundary conditions on the outer interfaces is a fixed source elliptic problem. This problem is divided into six trigonal nodes with different cross-sections and solved by the response matrix method based on the trigonal AFEN method.

The procedure to derive the response matrix for a trigonal node in Fig. 3 was well described in Reference 1. Here, we summarize the procedure shortly.

The intranodal flux expansion function which is symmetric to three coordinates in Fig. 3 and harmonious to the even and odd basis functions is given by

$$\boldsymbol{\phi}_s(x,y) = \begin{array}{l} \left\{2\cosh\left(\frac{\sqrt{\Lambda}}{2}x\right)\cosh\left(\frac{\sqrt{3\Lambda}}{2}y\right)+\cosh(\sqrt{\boldsymbol{\Lambda}}x)\right\}\mathbf{A}_\theta \\ +\left\{\sinh\left(\frac{\sqrt{\Lambda}}{2}x\right)\cosh\left(\frac{\sqrt{3\Lambda}}{2}y\right)+\sinh(\sqrt{\boldsymbol{\Lambda}}x)\right\}\mathbf{A}_\varepsilon \\ -3\cosh\left(\frac{\sqrt{\Lambda}}{2}x\right)\sinh\left(\frac{\sqrt{3\Lambda}}{2}y\right)\mathbf{A}_\chi \end{array} \quad (12)$$

where A_θ, A_ε and A_χ are the three transformed coefficients resulted from the direction decoupling transformation described in Reference 1. The transformed nodal unknowns are defined as follow in this transformation:

$$\boldsymbol{\phi}_\theta = \frac{\phi_x+\phi_u+\phi_p}{3} - \bar{\boldsymbol{\phi}}, \boldsymbol{\phi}_\varepsilon = \frac{2\phi_x-\phi_u-\phi_p}{3}, \boldsymbol{\phi}_\chi = \frac{\phi_u-\phi_p}{3} \quad (13)$$

$$\mathbf{J}_\theta = \frac{J_x+J_u+J_p}{3},\ \ \mathbf{J}_\varepsilon = \frac{2J_x-J_u-J_p}{3},\ \ \mathbf{J}_\chi = \frac{J_u-J_p}{3} \quad (14)$$

We can get the following form of relationship between the transformed flux and the transformed current by expressing both physical quantities into the expansion coefficients and eliminating the coefficients.

$$\boldsymbol{\phi}_\alpha = \mathbf{T}_\alpha \mathbf{J}_\alpha, \quad \boldsymbol{\alpha} = \boldsymbol{\theta}, \boldsymbol{\varepsilon} \text{ or } \boldsymbol{\chi} \quad (15)$$

where $\mathbf{T}_\varepsilon = \mathbf{T}_\chi$, fortunately. Similarly to Eq. (13) or (14), we define the transformed partial currents:

$$\mathbf{j}_\theta^f = \frac{j_x^f+j_u^f+j_p^f}{3} - \frac{\bar{\phi}}{4}, \mathbf{j}_\varepsilon^f = \frac{2j_x^f+j_u^f+j_p^f}{3}, \mathbf{j}_\chi^d = \frac{j_u^f-j_p^f}{3},\ f = in \text{ or } out \quad (16)$$

The typical following relationship between the partial currents, flux, and net current still holds for the transformed system:

$$\boldsymbol{J}_\alpha = \mathbf{j}_\alpha^{out} - \mathbf{j}_\alpha^{in},\ \ \boldsymbol{\phi}_\alpha = 2\left(\mathbf{j}_\alpha^{in} + \mathbf{j}_\alpha^{out}\right),\ \ \boldsymbol{\alpha} = \boldsymbol{\theta}, \boldsymbol{\varepsilon} \text{ or } \boldsymbol{\chi} \quad (17)$$

Substituting these relationship into Eq. (15) and solving for the transformed outgoing partial current, we finally obtain the response matrix in the transformed system as follows,

$$\mathbf{j}_\alpha^{out} = \mathbf{R}_\alpha \mathbf{j}_\alpha^{in},\ \ \boldsymbol{\alpha} = \boldsymbol{\theta}, \boldsymbol{\varepsilon} \text{ or } \boldsymbol{\chi} \quad (18)$$

where $\mathbf{R}_\alpha = -(2\mathbf{I} - \mathbf{T}_\alpha)^{-1}(2\mathbf{I} + \mathbf{T}_\alpha)$. Again, $\mathbf{R}_\varepsilon = \mathbf{R}_\chi$ because $\mathbf{T}_\varepsilon = \mathbf{T}_\chi$. The interface outgoing partial currents are inversely transformed as follows:

$$\mathbf{j}_x^f = \mathbf{j}_\theta^f + \mathbf{j}_\varepsilon^f + \frac{\bar{\phi}}{4}, \mathbf{j}_u^f = \mathbf{j}_\theta^f - \frac{j_\varepsilon^f+3j_\chi^f}{2} + \frac{\bar{\phi}}{4}, \mathbf{j}_p^f = \mathbf{j}_\theta^f - \frac{j_\varepsilon^f-3j_\chi^f}{2} + \frac{\bar{\phi}}{4} \quad (19)$$

Given the interface incoming partial currents of a trigonal node in the hexagonal block, the interface partial currents going out of the node can be calculated by the response matrix Eq. (18). Two of them (let say $\mathbf{j}_u^{out}$ and $\mathbf{j}_p^{out}$) on the interfaces inside the hexagonal block become the partial currents coming into its neighboring nodes. The other one ($\mathbf{j}_x^{out}$) on the outer interface of the hexagonal node is subtracted by the net current given as a boundary condition to yield the partial current coming from that outer interface:

$$\mathbf{j}_x^{in} = \mathbf{j}_x^{out} - \boldsymbol{J}_x \quad (20)$$

This constitutes an iterative process to be solved for the partial currents on the interfaces inside of the hexagonal block. There is only an inner iteration involved because the problem is a fixed source elliptic one.

2.2.2 Equivalence theory parameters for global core problem

Once the iteration to solve the local single block problem is converged, the equivalence theory parameters to be passed to the global core problem are evaluated. The flux weighted cross-sections for the hexagonal block are calculated by

$$\bar{\boldsymbol{\Sigma}} = \frac{\sum_{i=1}^{6} \Sigma_i \bar{\phi}_i}{\sum_{i=1}^{6} \bar{\phi}_i} \quad (21)$$

where the summation operator Σ runs over all the six trigonal nodes numbering from 1 to 6 in the hexagonal block.

The flux discontinuity factor in an outer interface *s* of the hexagonal block is given by the definition Eq. (3). The heterogeneous flux in the numerate of this definition is indisputably the flux calculated from the local single block problem. The equivalence theory[9,10] guarantees that the use of the interface flux of the global calculation as the denominator makes the solution of this global local iteration scheme equivalent to the solution calculated for the global whole core by the local method. This is why a lot of global-local iteration schemes are used to accelerate a high-order accurate but slow method by a low-order rough but fast one.

In our case, the accuracy of the local method i.e. the trigonal AFEN method is inferior to that of the global method i.e. the hexagonal AFEN method. A global-local iteration scheme applied in the above way will give the poor accuracy of the trigonal AFEN method as shown in References 1 and 2. Recalling that our purpose is not to improve the accuracy of the hexagonal AFEN method but to extend its capability to handle heterogeneous hexagonal nodes each of which are composed of six different trigonal nodes, we take the denominator of the definition Eq. (3) to be the interface flux calculated for the single homogeneous hexagonal block with the cross-sections of Eq. (21) by the same method to the local method i.e. the trigonal AFEN method. Of course, the boundary conditions to be applied to calculate the denominator are also same to those to calculate the numerator. The only difference between the two single bock problems is that one is heterogeneous and the other is homogeneous. Therefore, the flux discontinuity factor allows the hexagonal AFEN method to correct only the effect of intra-nodal heterogeneity without losing accuracy. In addition, there is no need to perform the local stage calculations for the blocks without heterogeneity.

3. Numerical Results and Discussion

The intra-block asymmetric heterogeneity occurs in the HTGR core when control rods are inserted asymmetrically into a hexagonal block. Control rods are usually not designed to be inserted into the fuel blocks during normal operation due to very high temperature in the active core. Instead, they are inserted into the reflector blocks to control the reactivity during normal operation. Of course, even in the fuel block, heterogeneity may occur due to the large guide hole into which the control rod is inserted for shutdown. However, such heterogeneity can be handled by the conventional block homogenization method, because heterogeneity due to a water-free guide hole is not serious.

The ability of the global-local iteration scheme proposed in this paper to deal with heterogeneity caused by the control rods insertion in the HTGR core was verified by solving the modified MHTGR-350 problem which was also used in References 1 and 2 as a benchmark problem. The modification was done to simulate the situation where the control rods are inserted into a reflector block. The control rods are assumed to be inserted into the reflector block shaded in Fig. 4. Asymmetrical insertion of control rods are simulated by increasing the absorption cross-sections of two trigonal nodes of the reflector block facing the active core by 50 percent. Although not necessary, the absorption cross-sections of two trigonal nodes facing outside the core are decreased by 50 percent to keep balance with the original MHTGR-350 problem.

In Fig. 4, the assembly-wise relative powers of the global-local iteration scheme and the trigonal AFEN method were compared with those of the fine mesh finite difference method (FDM). The side-length of a trigonal mesh in the reference FDM calculation is only 0.325 cm. The local calculations were performed just once after the global iteration was converged with 10^{-7} of the multiplication error criterion. As shown in the figure, the global-local iteration scheme showed almost the same level of accuracy as the hexagonal refined AFEN method. This is not surprising in light of the limited use of the local method, the trigonal AFEN method, to capture only the effect of heterogeneity of heterogeneous blocks.

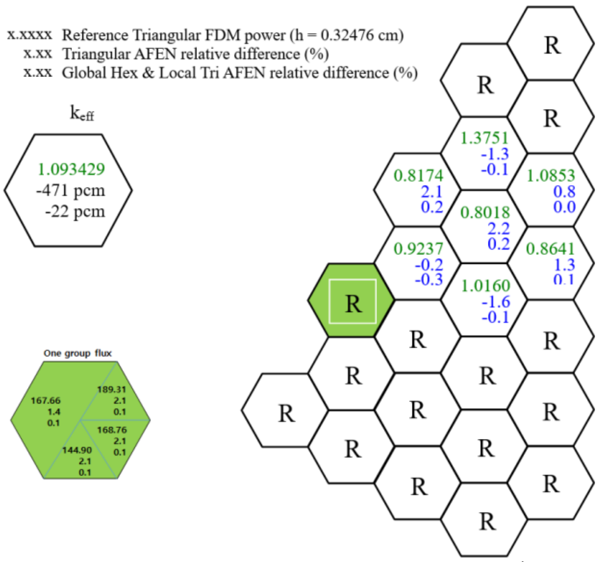

Fig. 4. Results of MHTGR-350 benchmark problem ($\frac{1}{12}$ core).

We compared the trigonal node average fluxes collapsed into one group from ten groups in the block where control rods are assumed to be inserted in Fig. 4. The global-local iteration scheme almost eliminates the errors of trigonal AFEN method in the heterogeneous block where the local trigonal AFEN calculation is performed. This shows that the discontinuity factor we defined corrects only the inhomogeneity inside the block, excluding the effect caused by the inaccuracy of the trigonal AFEN method. The equivalence theory[8,10] which proves theoretically that the global calculation can reproduce the physical quantities of the local calculations with, for example, the form functions defined similarly to Eq. (3) is numerically confirmed by showing that the trigonal node average fluxes of the global-local iteration scheme agree well with those of the fine mesh FDM reference calculation.

4. Conclusions

In order to treat the asymmetric heterogeneity within hexagonal blocks in the HTGR core, two trigonal AFEN methods were previously tried. [1,2] The first tried method is a relatively simple one which uses only the neutron flux as the interface unknown while the second tried method is a more sophisticated one which uses the flux moment additionally to the flux. Although the number of nodal unknowns increases several times in the two AFEN methods, they did not show better accuracy compared to the hexagonal refined AFEN method.

In this paper, the hexagonal refined AFEN method, along with the trigonal AFEN method, constitutes a global-local iteration method that guarantees success without failure due to the equivalence theory[9,10]. In this method, the hexagonal refined AFEN method performs the global whole core calculation with maintaining its original accuracy and computing speed. The trigonal AFEN method performs a pair of heterogeneous and homogeneous calculations for a local single block and computes the equivalence theory parameters that correct only the effect of heterogeneity of the block. A benchmark test against the MHTGR-350 problem yielded the theoretically expected results for this global-local iteration scheme.

Acknowledgements

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References

1. J. M. NOH, " An AFEN Response Matrix Method in the Two-dimensional Trigonal Geometry," Trans. KNS Autumn Mtg, Online, October 22-23, (2020).
2. J. M. NOH, " A Refinement of the AFEN Response Matrix Method in the Two-dimensional Trigonal Geometry," Trans. KNS Spring Mtg, Online, May 13-14, (2021).
3. 1. J. M. NOH and N. Z. CHO, "A New Approach of Analytic Basis Function Expansion to Neutron Diffusion Nodal Calculation," Nucl. Sci. Eng., 116, 165 (1994).
4. S. W. WOO, N. Z. CHO, and J. M. NOH, "The Analytic Function Expansion Nodal Method Refined with Transverse Gradient Basis Functions and Interface Flux Moments," Nucl. Sci. Eng., 139, 156 (2001).
5. N. Z. CHO and J. M. NOH, "Analytic Function Expansion Nodal Method for Hexagonal Geometry," Nucl. Sci. Eng., 121, 245 (1995).
6. N. Z. CHO, Y. H. KIM, and K. W. PARK, "Extension of Analytic Function Expansion Nodal Method to Multigroup Problems in Hexagonal-Z Geometry," Nucl. Sci. Eng., 126, 35 (1997).
7. N. Z. CHO and J. J. LEE, "Analytic Function Expansion Nodal (AFEN) Method in Hexagonal-z Three

Dimensional Geometry for Neutron Diffusion Calculation," J. Nucl. Sci. and Tech., 43, 1320 (2006).
8. J. M. NOH, "A Response Matrix Method for the Refined Analytic Function Expansion Nodal (AFEN) Method in the Two-Dimensional Hexagonal Geometry and Its Numerical Performance," Nucl. Eng. Tech., 52, 2422 (2020).
9. K. KOEBKE, "A New Approach to Homogenization and Group Condensation," Proc. IAEA Technical Committee Mtg., Lugano, Switzerland, November 1978, IAEA-TECDOC 231, p.303, International Atomic Energy Agency (1978).
10. K. S. SMITH, "Spatial Homogenization Methods for Light Water Reactor Analysis," PhD Thesis, Massachusetts Institute of Technology (1980).