A Refinement of the AFEN Response Matrix Method in the Two-dimensional Trigonal Geometry

Jae Man Noh

May 12, 2021

Korea Atomic Energy Research Institute

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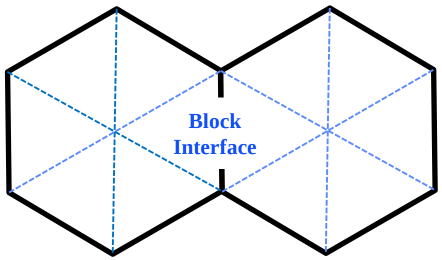
Introduction

Purpose

- Improve triangular AFEN method by refining it with Transverse Gradient Basis Functions and Interface Flux Moments

Background

- Triangular AFEN has been tried.
  - To treat the intra-block asymmetric heterogeneity
  - Tried is Original AFEN without Flux moments
    - Number of nodes increases by six times compared to the hex refined AFEN
    - Number of interface unknowns increases by 1.5 times
  - Poorer performance than Hex AFEN
    - Possibly due to a looser flux continuity constraint across each hex-block interface
      - ✓ Hex AFEN : tighter half-interface continuity
      - ✓ Tri AFEN : looser full-interface continuity



Hex AFEN : Flux and Flux Moment
Tri AFEN : Flux only

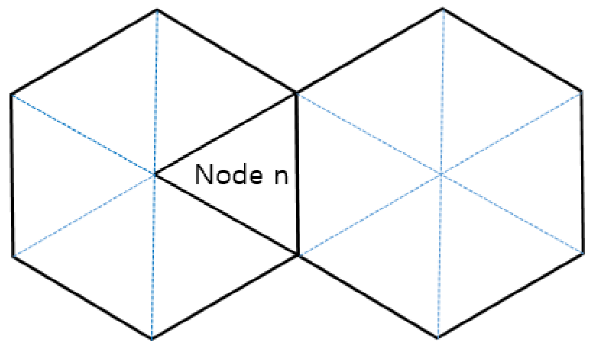
Introduction

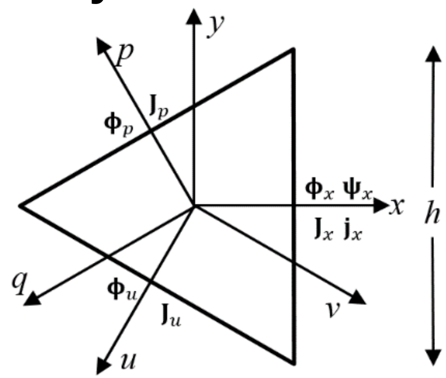
- **Refinement to improve Tri AFEN**
  - **Full scope refined AFEN employing flux moment at every node interface**
    - **Increases number of interface unknowns by 6 times**
    - **too much refinement (?)**
  - **Simple refined AFEN employing flux moment only at hex-block interface**
    - **Increases number of interface unknowns by 2.5 times**
    - **Interface constraints are always tighter than those of the hex refined AFEN**
    - **Unknown set always contains that of hex refined AFEN.**
    - **Expect better performance than hex refined AFEN**

Methodology

Refinement of Tri AFEN

- **Introduce interface flux moment only at interface between two hex blocks**





- **Flux moment and current moment : asymmetric unknown**

$$\boldsymbol{\psi}_x = \frac{1}{h}\int_{-\frac{h}{2}}^{\frac{h}{2}} w(y)\boldsymbol{\phi}\left(\frac{\sqrt{3}}{6}h, y\right) dy \qquad \mathbf{j}_x = \frac{\mathbf{D}}{h}\int_{-\frac{h}{2}}^{\frac{h}{2}} w(y)\frac{\partial}{\partial x}\boldsymbol{\phi}(x,y)\, dy \Bigg|_{x=\frac{\sqrt{3}}{6}h}$$

  - A step function with sign changing across y = 0 is used for the weighting function w[y].

- **Decoupling transformation of conventional symmetric unknowns**

$$\boldsymbol{\phi}_\theta = \frac{\boldsymbol{\phi}_x + \boldsymbol{\phi}_u + \boldsymbol{\phi}_p}{3} - \bar{\boldsymbol{\phi}}, \boldsymbol{\phi}_\varepsilon = \frac{2\boldsymbol{\phi}_x - \boldsymbol{\phi}_u - \boldsymbol{\phi}_p}{3}, \boldsymbol{\phi}_\chi = \frac{\boldsymbol{\phi}_u - \boldsymbol{\phi}_p}{3}$$

$$\mathbf{J}_\theta = \frac{\mathbf{J}_x + \mathbf{J}_u + \mathbf{J}_p}{3}, \qquad \mathbf{J}_\varepsilon = \frac{2\mathbf{J}_x - \mathbf{J}_u - \mathbf{J}_p}{3}, \qquad \mathbf{J}_\chi = \frac{\mathbf{J}_u - \mathbf{J}_p}{3}$$

Methodology

Intranodal flux expansion function

$$\boldsymbol{\phi}(x,y)=\boldsymbol{\phi}_{\boldsymbol{S}}(x,y)+\boldsymbol{\phi}_{\boldsymbol{A}}(x,y)$$

- **Symmetric original tri AFEN expansion function**

$$\boldsymbol{\phi}_{\boldsymbol{S}}(x,y)=\mathbf{A}_{\theta}\boldsymbol{\varphi}_{\theta}^{sn}(x,y)+\mathbf{A}_{\varepsilon}\boldsymbol{\varphi}_{\varepsilon}^{sn}(x,y)+\mathbf{A}_{\chi}\boldsymbol{\varphi}_{\chi}^{sn}(x,y)$$

where

$$\boldsymbol{\varphi}_{\theta}^{sn}(x,y)=\left\{-2\sinh\left(\frac{\boldsymbol{k}}{2}x\right)\cosh\left(\frac{\sqrt{3}\boldsymbol{k}}{2}y\right)+\sinh(\boldsymbol{k}x)\right\}$$ corresponding to ϕ_θ and J_θ

$$\boldsymbol{\varphi}_{\varepsilon}^{sn}(x,y)=\left\{\sinh\left(\frac{\boldsymbol{k}}{2}x\right)\cosh\left(\frac{\sqrt{3}\boldsymbol{k}}{2}y\right)+\sinh(\boldsymbol{k}x)\right\}$$ corresponding to ϕ_ε and J_ε

$$\boldsymbol{\varphi}_{\chi}^{sn}(x,y)=-3\cosh\left(\frac{\boldsymbol{k}}{2}x\right)\sinh\left(\frac{\sqrt{3}\boldsymbol{k}}{2}y\right)$$ corresponding to ϕ_χ and J_χ

- **Asymmetric additive expansion function by adopting flux moment**

$$\boldsymbol{\phi}_{A}(x,y)=\mathrm{y}\sinh(\sqrt{\boldsymbol{\Lambda}}x)\mathbf{B}_{\boldsymbol{x}}$$

  - **This additive function also complies the even-odd test to test its physical validity.**

Methodology

Relationship between interface flux and current

- **The original 3X3 system is decoupled into two scholar systems and one 2x2 matrix system.**
- **For θ and ε components, two scholar systems are obtained:**

$$\boldsymbol{\phi}_\theta = \mathbf{P}_\theta \mathbf{A}_\theta, \qquad \boldsymbol{\phi}_\varepsilon = \mathbf{P}_\varepsilon \mathbf{A}_\varepsilon$$

$$\mathbf{J}_\theta = \mathbf{D}\mathbf{Q}_\theta \mathbf{A}_\theta, \qquad \mathbf{J}_\varepsilon = \mathbf{D}\mathbf{Q}_\varepsilon \mathbf{A}_\varepsilon$$

- **For χ component, one 2x2 system is obtained:**

$$\begin{pmatrix} \boldsymbol{\phi}_\chi \\ \boldsymbol{\psi}_\chi \end{pmatrix} = \mathbf{P}_\chi \begin{pmatrix} \mathbf{A}_\chi \\ \mathbf{B}_\chi \end{pmatrix}, \qquad \begin{pmatrix} \mathbf{J}_\chi \\ \mathbf{j}_\chi \end{pmatrix} = \mathbf{D}\mathbf{Q}_\chi \begin{pmatrix} \mathbf{A}_\chi \\ \mathbf{B}_\chi \end{pmatrix}$$

- **Eliminate coefficients,**

$$\boldsymbol{\phi}_\alpha = \mathbf{T}_\alpha \mathbf{J}_\alpha, \qquad \begin{pmatrix} \boldsymbol{\phi}_\chi \\ \boldsymbol{\psi}_\chi \end{pmatrix} = \mathbf{T}_\chi \begin{pmatrix} \mathbf{J}_\chi \\ \mathbf{j}_\chi \end{pmatrix}$$

where α is θ or ε and $\mathbf{T}_\beta = \mathbf{P}_\beta \mathbf{Q}_\beta^{-1} \mathbf{D}^{-1}, \quad \beta = \theta, \varepsilon \; or \; \chi$.

Methodology

Response Matrix

- **Interface partial current and partial current moment at the interface *s***

$$P_s^f = \frac{J_s^f}{2} + \frac{\phi_s}{4} \qquad p_x^f = \frac{j_x^f}{2} + \frac{\psi_x}{4}$$

where f = *in* or *out*

- **The interface flux and current are equivalently given by**

$$J_s^{in} = P_s^{in} - P_s^{out}, \qquad \phi_s = 2\left(P_s^{in} + P_s^{out}\right)$$

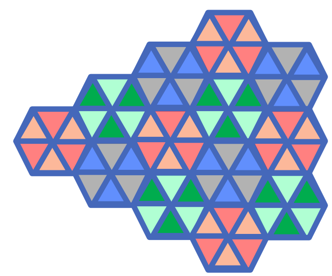
- **The interface flux moment and current moment are also given by**

$$j_x^{in} = p_x^{in} - p_x^{out}, \qquad \psi_x = 2\left(p_x^{in} + p_x^{out}\right)$$

- **The decoupling transformation is also applicable for this relationship.**

$$J_\alpha^{in} = P_\alpha^{in} - P_\alpha^{out}, \qquad \phi_\alpha = 2\left(P_\alpha^{in} + P_\alpha^{out}\right), \qquad \alpha = \theta, \varepsilon, or\ \chi$$

- **Finally, the refined AFEN response matrix becomes**



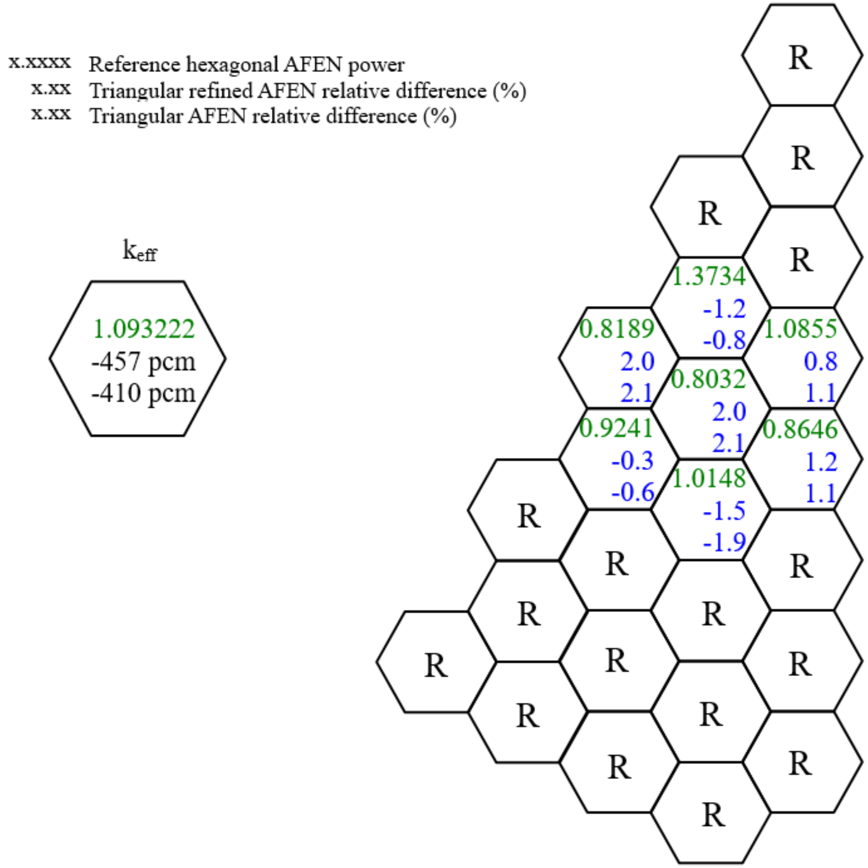
$$P_\alpha^{out} = R_\alpha P_\alpha^{in}, \qquad \alpha = \theta\ or\ \varepsilon \qquad \begin{pmatrix} P_\chi^{out} \\ p_x^{out} \end{pmatrix} = R_\chi \begin{pmatrix} P_\chi^{in} \\ p_x^{in} \end{pmatrix}$$

where $R_\alpha = -(2I + T_\alpha)^{-1}(2I - T_\alpha)$ for $\alpha = \theta, \varepsilon$, or χ.

- **The RGB-BW sweeping scheme is applicable during each inner iteration.**

Results and Discussion

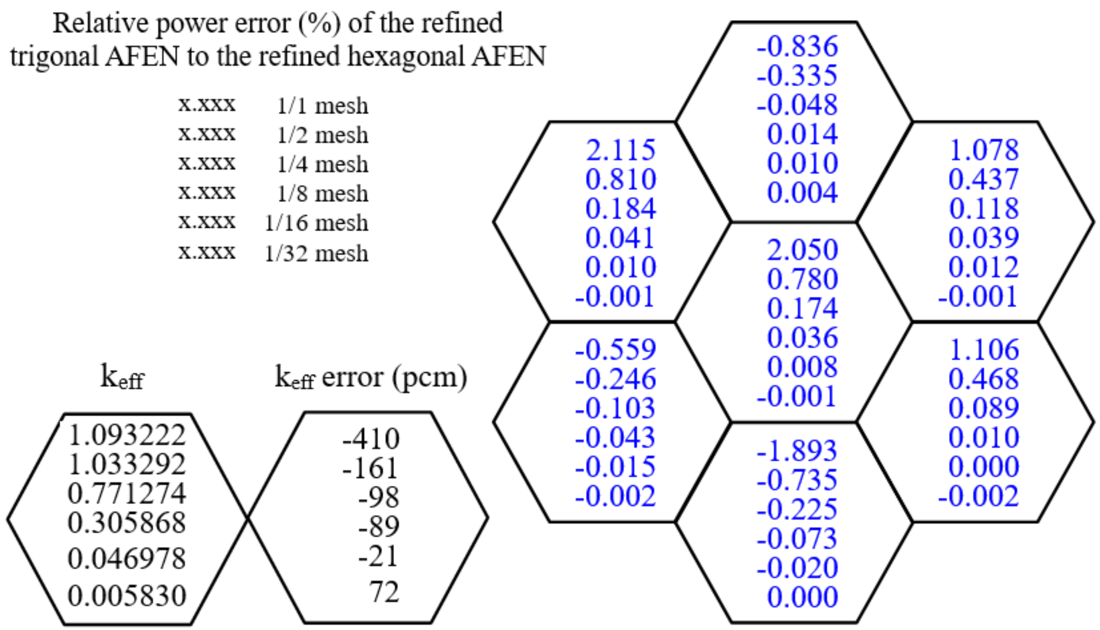
- **Results of MHTGR-350 Problem**



- **The refined tri AFEN clearly but insignificantly improves the original tri AFEN.**
  - **It should have shown better performance than even the refined hex AFEN, noting that its set of unknowns and constraints contains that of the hex AFEN.**

Results and Discussion

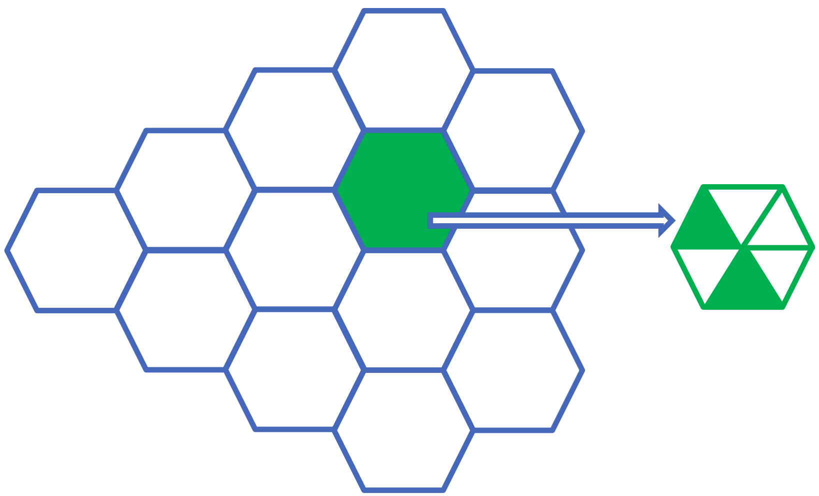
- **Check if bugs are involved by reducing the block size while maintaining the core configuration as it is.**
  - **Note that the core made of hex blocks cannot be divided into smaller hexagons.**



  - **As the node size decreases, the solutions of two methods converge to each other. => This result reduces the likelihood that the trigonal AFEN's low performance is due to hidden bugs.**
  - **The remaining cause to be suspected is that the tri AFEN expansion function is inefficient in representing the flux distribution within a tri node.**

Conclusion

- To improve tri AFEN, we proposed the simple refined AFEN that adopts flux moments only at interfaces between adjacent hex blocks.
  - It should have shown better results than the hex refined AFEN, because it compasses all the unknowns and constraints of the hex AFEN.
- However, It is not only inferior to the hex method, but also shows similar accuracy to the original tri AFEN which we intended to improve.
- Alternatively, the global-local iteration method can be suggested to treat the intra-hex-block asymmetric heterogeneity.



Global : Hex AFEN for core

Pass block-interface currents to Local

Local : Tri AFEN for single block

Solve a block with BC of block-interface currents and pass CXs and DFs to Global