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Comparison between Generalized Equivalence Theory (GET) and Super-Homogenization (SPH) Method in the Framework of Pinwise Nodal Analysis

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1. Introduction

Profound analyses regarding the nuclear reactor system are imperative for both realistic design and safety analyses. Specifically, a reliable estimation of pin-power distribution and multiplication factor is demanded. However, procurement of such information must be done within an affordable computational burden, hence a concept of two-step procedure had been devised and is widely accepted.

In the conventional two-step procedure, the assembly wise problem, which is referred to as lattice calculation, is dealt with high fidelity approach, i.e., transport calculation, and the result is properly collapsed for the assembly wise whole core calculation through diffusion-based methodologies. Intuitively, such a concept signifies that the accuracy of the final outcome from the whole core calculation hinges upon the degree of preservation of high-fidelity information. For such purpose, two different homogenization approaches, namely generalized equivalence theory (GET) [1] and super-homogenization (SPH) method [2], are commonly employed. Note that reconstruction process is required for acquiring the pinwise quantities of interest for the presented method.

To avoid the usage of reconstruction method, which inevitably introduces assumption(s) in the calculation, continuous efforts concerning the pin-by-pin diffusion approach had been made [3]. With an increase in the computing resources, the need and availability for accurate pinwise analysis are further being recognized. Recently, a concept of Hybrid Coarse-Mesh Finite Difference (HCMFD) method, which exploits hierarchy of acceleration with nodal expansion method (NEM) on the pin-level, had been proposed, which dwindles the computational cost through parallelization [4].

In this study, a thorough investigation regarding the implementation of GET and SPH method in the pinwise reactor analysis through HCMFD algorithm is conducted. Both multiplication factor and pinwise information for UOX/MOX fueled 2D reactor configuration are obtained to assess the performance of each homogenization technique.

2. Homogenization Methods

The success of two-step procedure relies on the manifestation of high-fidelity lattice calculation result in the diffusion calculation through proper homogenization. Both GET and SPH method are commonly utilized to suffice such goal.

2.1 Generalized Equivalence Theory (GET)

In the GET, an intentional discontinuity between the homogeneous surface fluxes is made for each interfacing surface via discontinuity factor (DF), which is defined as

$$DF_{gsi} := \frac{\phi_{gsi}^{het}}{\phi_{gsi}^{hom}}, \tag{1}$$

where ϕ_{gsi}^{hom} and ϕ_{gsi}^{het} denote homogeneous and heterogeneous (reference) surface flux for group *g* at the surface *s* of mesh of interest *i*. Concerning the boundary surfaces, the current-to-flux ratio (CFR) was intentionally employed to preserve the reference net current. Note that ϕ_{gsi}^{hom} is determined based on the homogenized XSs with a specific diffusion-based method, hence the DF is inherently methodology dependent.

Through the usage of DF, the diffusion-based whole core solution will precisely coincide with ϕ_{gsi}^{hom} at each surface, preserving the heterogeneous quantities including both reaction rate and surface current. Consequently, the multiplication factor will also be preserved due to the uniqueness of the solution.

2.2 Super-Homogenization (SPH) Method

In the SPH method, a correction in the homogenized XSs is made through the usage of SPH factor μ_g to preserve the reaction rates of the heterogeneous calculation for each mesh of interest.

$$\Sigma_g^{ref}\phi_g^{ref} = \mu_g\Sigma_g^{ref}\phi_g^{hom}, \tag{2}$$

where Σ_g^{ref}, ϕ_g^{ref}, and ϕ_g^{hom} represent reference XS, reference heterogeneous flux, and homogeneous flux respectively.

Unlike GET, SPH method necessitates an iterative procedure that determines both μ_g and corresponding ϕ_g^{hom} to preserve the reaction rate. For each iteration, the SPH factor is updated as

$$\mu_g = \frac{\phi_g^{ref}}{\bar{\phi}_g^{hom}}. \tag{3}$$

Normalized homogeneous flux $\bar{\phi}_g^{hom}$ is defined as

$$\bar{\phi}_g^{hom} = \phi_g^{hom} \frac{\sum_i V_i \phi_g^{ref}}{\sum_i V_i \phi_g^{hom}}, \quad (4)$$

where i indicates a certain pin with a volume of V_i that comprises the configuration of interest [2].

Utilization of SPH factor in the whole-core calculation preserves the reaction rate, i.e., pinwise power, but mathematically, it does not guarantee the preservation of surface current information.

3. Numerical Results

Figure 1 illustrates the UOX-loaded and MOX-loaded 2D SMR cores that had been considered throughout the presented study, where each fuel assembly consists of 16x16 fuel rods with a pitch of 1.2658 [cm]. Reference (high-fidelity) solution and pinwise homogenized XSs, DFs, and SPH factors were obtained through deterministic transport calculation via DeCART2D [5].

Two different evaluations were made: (1) reference solution was directly processed to determine XSs, DFs, and SPH factors for every pin residing in the core, and (2) conventional lattice calculation was performed to estimate XSs, DFs, and SPH factors for active core regions. The reference flux-based quantities were employed regarding the baffle reflector regions universally. Note that four 3.1 GHz Intel Core i5 processors were parallelly used for the calculation.

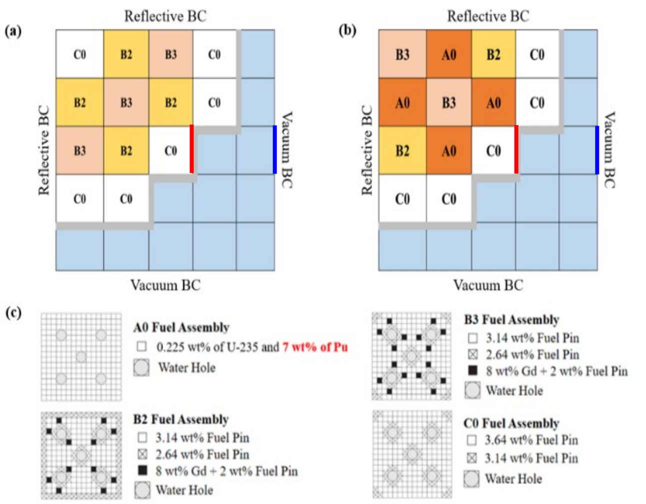

Fig. 1. Illustration of the problem (a) UOX-loaded, (b) MOX-loaded, and (c) description of each fuel assembly

3.1 Reference Solution based two-step procedure

Three different energy group structures were involved in the analysis, e.g., 2-group, 4-group, and 7-group [6, 7], and both GET and SPH approaches successfully retained the reaction rate. Figure 2 illustrates the average power density normalized reference power distribution for UOX and MOX loaded SMR cores.

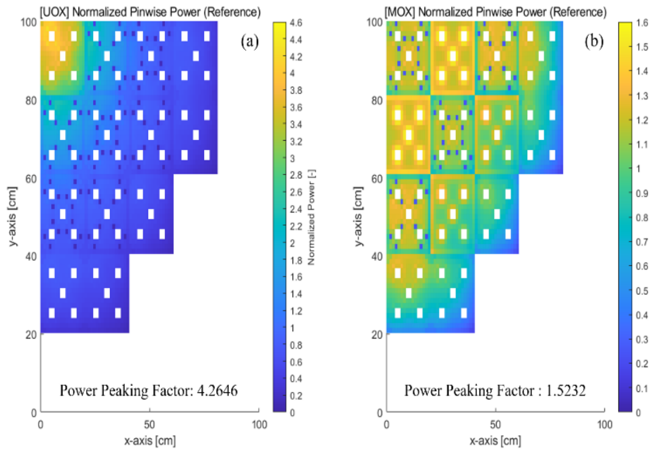

Fig. 2. Normalized pinwise power distribution for (a) UOX and (b) MOX loaded cores

Given the reference solution, both methods reproduce the power profile, as expected. However, a conspicuous difference in the multiplication factor was observed between two approaches as shown in Tables 1 and 2, along with an increase in the computing time with the enlarged group numbers. Only the GET-based calculation achieved the conservation of k_{eff}, and the error in the SPH-based calculation was noticeably larger for the MOX-loaded SMR core. Such result can be ascribed to the enhanced leakage due to the inclusion of MOX-fuel and relatively flat power profile, as depicted in Fig. 2, and an inherent trait of SPH method, which does not preserve the surface current information. The root-mean-square (RMS) errors in the calculated 2-group net currents at red and blue surfaces, which is illustrated in Fig. 1, are tabulated in Tables 3 and 4 for each approach. Only the GET-based calculation managed to preserve the current information. It should be mentioned that the k_{eff} mismatch in the SPH solution could be quite bigger in leakier core configurations.

Table 1. UOX-loaded Core (Reference solution based)

Method		k_{eff}	$\Delta\rho$ [pcm]	Time [s]
Ref		1.078179	-	-
2-group	GET	1.078179	0	4.25
	SPH	1.078536	31	4.27
4-group	GET	1.078179	0	13.92
	SPH	1.078622	38	14.58
7-group	GET	1.078179	0	46.77
	SPH	1.078637	39	59.30

Table 2. MOX-loaded Core (Reference solution based)

Method		k_{eff}	$\Delta\rho$ [pcm]	Time [s]
Ref		1.056064		-
2-group	GET	1.056064	0	3.24
	SPH	1.056611	49	3.23
4-group	GET	1.056064	0	9.81
	SPH	1.056754	62	10.21
7-group	GET	1.056064	0	42.17
	SPH	1.056779	64	32.90

Table 3. UOX-loaded Core (RMS error % for current)

Description		GET	SPH
Red Surface	Group1	0.00	3.69
	Group2	0.00	0.92
Blue Surface	Group1	0.00	24.19
	Group2	0.00	0.18

Table 4. MOX-loaded Core (RMS error % for current)

Description		GET	SPH
Red Surface	Group1	0.00	3.69
	Group2	0.00	0.87
Blue Surface	Group1	0.00	24.26
	Group2	0.00	0.50

3.2 Lattice Calculation based two-step procedure

The pinwise information regarding the fuel assemblies was estimated by imposing the reflective boundary conditions for each surface. The baffle reflector regions on the other hand directly utilized the reference flux-based quantities, i.e., identical to that of Section 3.1.

Error in the estimated multiplication factor and the associated computing time from each approach are juxtaposed in Tables 5 and 6. It could be noticed that GET-based approach results in an improved estimation for MOX-loaded case, which also tends to improve with enlarged group numbers. For UOX-loaded core, the 4-group SPH-based approach returned an accurate k_{eff} value, which is even closer than that of reference solution-based two-step procedure. However, it should be articulated that such observation cannot be apprehended as a better performance of SPH-based approach, which intrinsically does not retain the k_{eff} information. Furthermore, a noticeable error in the pinwise power at the fringe of the fuel assembly exists for SPH-based result as depicted in Fig. 3.

As demonstrated in Figs. 3 and 4, the SPH-based calculation of pinwise power is susceptible to salient error at the periphery of the fuel assemblies facing the reflector regions. Further intensification of pinwise power error between adjacent fuel assemblies is observed when the MOX-fuel is embedded in the core, which is illustrated in Fig. 4.

Such phenomenon can be understood in line with the fact that SPH method does not take preservation of current into account, and the aforementioned treatments renders the leakage of neutrons to be prominent.

Table 5. UOX-loaded Core (Lattice calculation based)

Method		k_{eff}	$\Delta\rho$ [pcm]	Time [s]
Ref		1.078179		-
2-group	GET	1.079946	152	4.27
	SPH	1.080101	165	4.42
4-group	GET	1.077786	-34	13.77
	SPH	1.078107	-6	14.49
7-group	GET	1.077157	-88	46.60
	SPH	1.077456	-62	81.57

Table 6. MOX-loaded Core (Lattice calculation based)

Method		k_{eff}	$\Delta\rho$ [pcm]	Time [s]
Ref		1.056064		-
2-group	GET	1.058364	206	3.35
	SPH	1.059479	305	3.36
4-group	GET	1.057121	95	10.32
	SPH	1.058743	240	10.19
7-group	GET	1.055968	-9	31.41
	SPH	1.05494	128	41.12

Figures 5 and 6 enumerates the maximum and root-mean-square (RMS) errors concerning the pinwise power distribution within each fuel assembly.

$$\text{RMS } error(\%) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{p_i^* - p_i^{ref}}{p_i^{ref}}\right)^2} \times 100, \qquad (5)$$

where p_i^{ref} and p_i^* indicate reference and estimated pin wise power respectively.

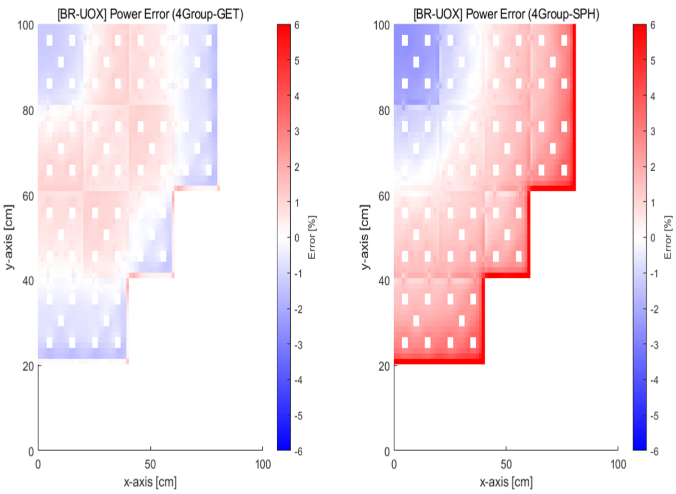

Fig. 3. Pinwise power % error of 4-group UOX-loaded core

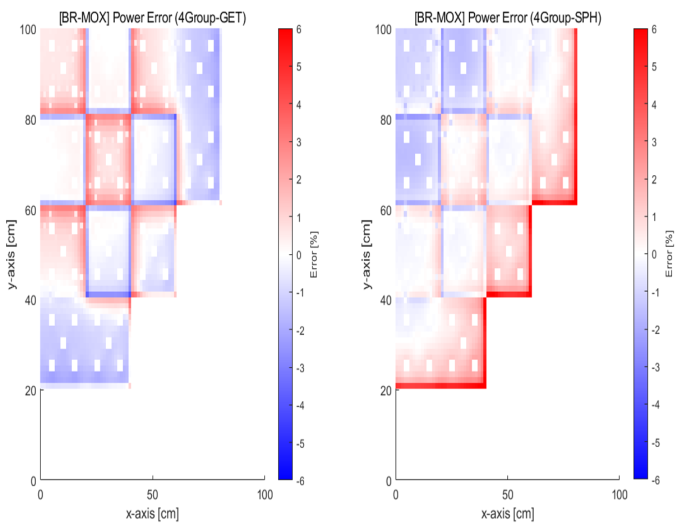

Fig. 4. Pinwise power % error of 4-group MOX-loaded core

GET (2-group)

Max	1.89	-1.08	-1.78	1.86
RMS	1.48	0.73	1.35	1.25
		-1.84	-1.15	-3.41
		1.49	0.78	1.26
			-3.67	
			1.18	

SPH (2-group)

Max	-1.42	-1.81	-1.80	4.08
RMS	0.72	1.34	1.15	2.88
		-2.01	1.94	4.01
		1.57	0.75	2.52
			3.55	
			1.42	

GET (4-group)

Max	-1.26	1.15	0.99	-1.56
RMS	0.91	0.73	0.56	0.74
		0.94	1.16	1.84
		0.61	0.74	0.75
			1.81	
			0.60	

SPH (4-group)

Max	-2.54	-1.92	2.06	7.21
RMS	2.22	0.66	1.11	2.76
		1.32	4.35	10.11
		0.56	1.55	3.68
			8.81	
			2.90	

GET (7-group)

Max	-1.72	1.12	1.09	-0.98
RMS	1.27	0.71	0.78	0.46
		0.99	1.28	-0.93
		0.78	0.98	0.38
			0.82	
			0.30	

SPH (7-group)

Max	-2.92	-1.91	2.02	5.18
RMS	2.45	0.76	1.33	2.56
		1.43	3.34	7.29
		0.74	1.71	3.30
			6.43	
			2.57	

Fig. 5. UOX-loaded SMR pin power error (%)

GET (2-group)

Max	-2.83	-3.45	3.27	1.97
RMS	2.05	1.34	1.16	1.31
		3.64	-3.67	4.01
		1.31	1.02	1.93
			4.37	
			2.32	

SPH (2-group)

Max	-3.84	-3.52	2.37	3.46
RMS	2.92	1.66	1.16	2.10
		2.70	-2.63	5.99
		1.30	1.02	2.64
			5.82	
			2.48	

GET (4-group)

Max	3.66	-2.14	3.86	-2.21
RMS	1.33	0.70	1.37	1.26
		3.69	-4.09	3.40
		1.81	1.19	1.12
			3.41	
			1.12	

SPH (4-group)

Max	-1.41	-2.67	1.64	5.24
RMS	0.85	1.38	0.47	1.40
		2.34	-2.08	8.39
		0.72	0.52	2.50
			7.68	
			2.37	

GET (7-group)

Max	2.71	-2.08	2.58	-1.82
RMS	1.41	0.69	1.06	1.18
		2.70	-4.30	-1.72
		1.34	1.20	1.00
			1.75	
			0.62	

SPH (7-group)

Max	-1.11	-2.14	-0.93	3.37
RMS	0.43	0.89	0.18	1.04
		-0.72	-1.85	5.91
		0.23	0.55	1.77
			5.29	
			1.37	

Fig. 6. MOX-loaded SMR pin power error (%)

Related to the periphery fuel assembly regions, error from GET-based approach diminishes as number of groups increases, which was not observed for SPH-based estimation especially for the UOX-loaded core. It could be understood that enhanced leakage intensified the error near the baffle since SPH inherently does not conserve the surface current information.

4. Conclusions

The presented work illustrates an inherent difference between the two popular homogenization techniques, e.g., GET and SPH. A numerical assessment has been made for both UOX and MOX loaded cores based on either reference solution or lattice calculation.

In terms of preserving the reaction rate, both methods successfully accomplished such goal when the reference solution was employed. However, only the GET successfully reproduces the multiplication factor. Such observation shows that theoretically, one can perfectly convey the high-fidelity information to the diffusion calculation via the GET approach, whereas SPH method fails to do so.

In the practical two-step approach, pinwise power error at the fringe of the fuel, which faces the reflector, and between the MOX/UOX assemblies became further conspicuous when the SPH method was exploited. An increased number of energy groups effectively curtailed the error at the fringe only for the GET-based approach. Comprehensively, this work successfully demonstrates the intrinsic difference between GET and SPH methods, and found out that implementation of the latter method may introduce a deterioration in the pinwise power estimation when leakage becomes considerable. Hence, the authors cautiously conjecture that GET-based approach will be more favourable for pinwise diffusion analyses of the reactor core. A more realistic 3D configuration problem coupled with a feedback effect will be scrutinized in the near future.

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