# Comparison between Generalized Equivalence Theory (GET) and Super-Homogenization (SPH) Method in the Framework of Pinwise Nodal Analysis

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# Introduction: Motivations & Objectives

**Philosophy of Two-Step Procedure**

– Fuel assembly calculation based on high-fidelity transport method(s)
  - Deterministic Transport Calculation .vs. Stochastic Transport Calculation
  - More realistic (reliable) result but noticeable computational burden

– Whole core calculation based on diffusion method(s)
  - FDM, NEM, SENM, AFEN, …
  - Low computational burden but less accurate result (diffusion approximation)

– Combine the benefits of aforementioned calculations
  - Fuel Assembly (Transport) → Whole core (Diffusion)
  - How to properly convey the high-fidelity local information into global calculation?

**Homogenization of Local Information**

– Preservation of current (and its associated diffusion coefficient) is an intricate problem

– Two different approaches had been devised
  - Generalized Equivalence Theory (GET)
  - Super-Homogenization (SPH) Method

# Introduction: Motivations & Objectives

**A detailed localized (pinwise) information is required**

- Often, reconstruction procedure is implemented to meet such a goal
  - Usage of form-function, Point source expansion method, …
  - Inevitably, a certain degree of assumption(s) is stipulated
- Pinwise diffusion calculation
  - With an increase in the computing resources, the need and availability of pin-by-pin diffusion approach are being recognized
  - A concept of Hybrid Coarse-Mesh Finite Difference (HCMFD) method had been proposed to dwindle the computational burden
    - Exploits hierarchy of acceleration with nodal expansion method (NEM) on a pin-level
    - Appropriate for efficient parallel computing

**Investigation concerning the effectiveness of GET and SPH method on the pinwise analysis**

- UOX/MOX fueled two-dimensional reactor configuration has been considered

# Homogenization Methods

**The success of two-step procedure hinges upon the retainment of high-fidelity fuel assembly calculation result in the whole core diffusion calculation**

- Preservation of reaction rate, eigen-value, and (net) current

**Generalized Equivalence Theory (GET)**

- Intentional discontinuity between contiguous homogeneous surface fluxes via discontinuity factor (DF), which retains the information regarding reference surface flux and current

$$DF_{gsi} \coloneqq \frac{\phi_{gsi}^{het}}{\phi_{gsi}^{hom}} \tag{1}$$

  - $\phi_{gsi}^{hom}$ and $\phi_{gsi}^{het}$ denote homogeneous and heterogeneous (reference) surface flux for group $g$ at the surface $s$ of mesh of interest $i$.

- $\phi_{gsi}^{hom}$ is determined based on the homogenized XSs with a specific diffusion-based method
  - Signifies DF is inherently methodology dependent

- Preserves both surface current and reaction rate, hence $k_{eff}$ is also retained

# Homogenization Methods

**Super-Homogenization (SPH) Method**

– Correction in the homogenized XSs is made through the usage of SPH factor $\mu_g$ in order to preserve the reaction rates

$$\Sigma_g^{ref}\phi_g^{ref} = \mu_g \Sigma_g^{ref}\phi_g^{hom} \tag{2}$$

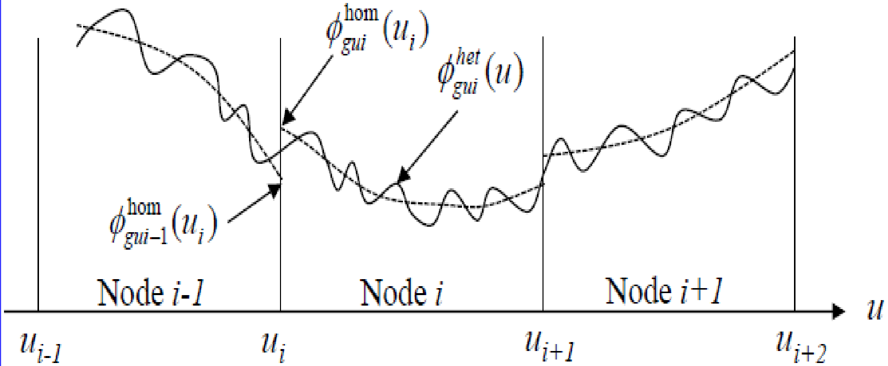
– Cell and group wise $\mu_g$ dictates $\phi_g^{hom}$, and $\phi_g^{hom}$ contributes to the determination of $\mu_g$

$$\mu_g = \frac{\phi_g^{ref}}{\bar{\phi}_g^{hom}} \tag{3}$$

$$\bar{\phi}_g^{hom} = \phi_g^{hom}\frac{\sum_i V_i \phi_g^{ref}}{\sum_i V_i \phi_g^{hom}} \tag{4}$$

- Iteration process is necessary to acquire $\mu_g$ that preserves the reaction rate
- Does not guarantee the preservation of current information or multiplication factor

# Homogenization Methods

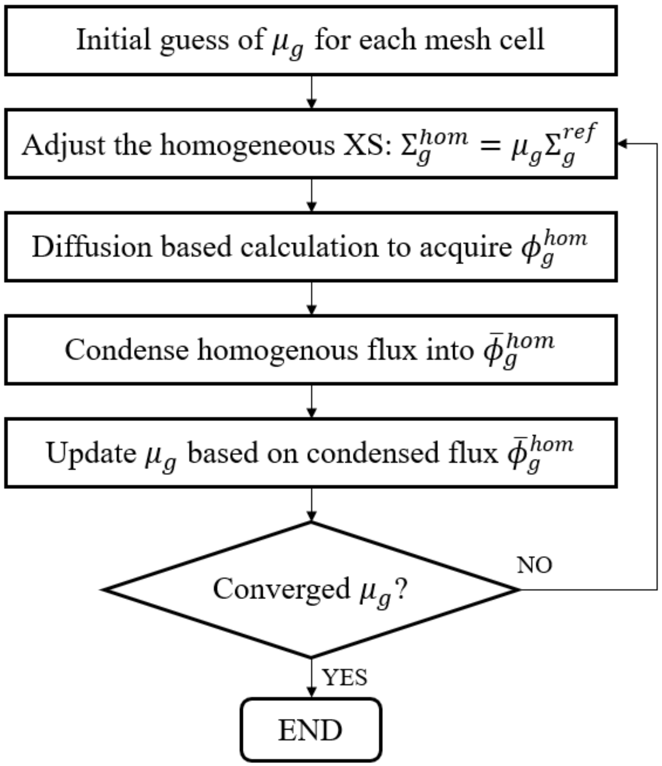



$$f_{gi}^{u-} \equiv \frac{\phi_{gui}^{het}(u_i)}{\phi_{gui}^{\text{hom}}(u_i)}, \quad f_{gi}^{u+} \equiv \frac{\phi_{gui}^{het}(u_{i+1})}{\phi_{gui}^{\text{hom}}(u_{i+1})}$$

$$\phi_{gui-1}^{\text{hom}}(u_i) f_{gi-1}^{u+} = \phi_{gui}^{\text{hom}}(u_i) f_{gi}^{u-}$$

**Generalized Equivalence Theory (GET)**

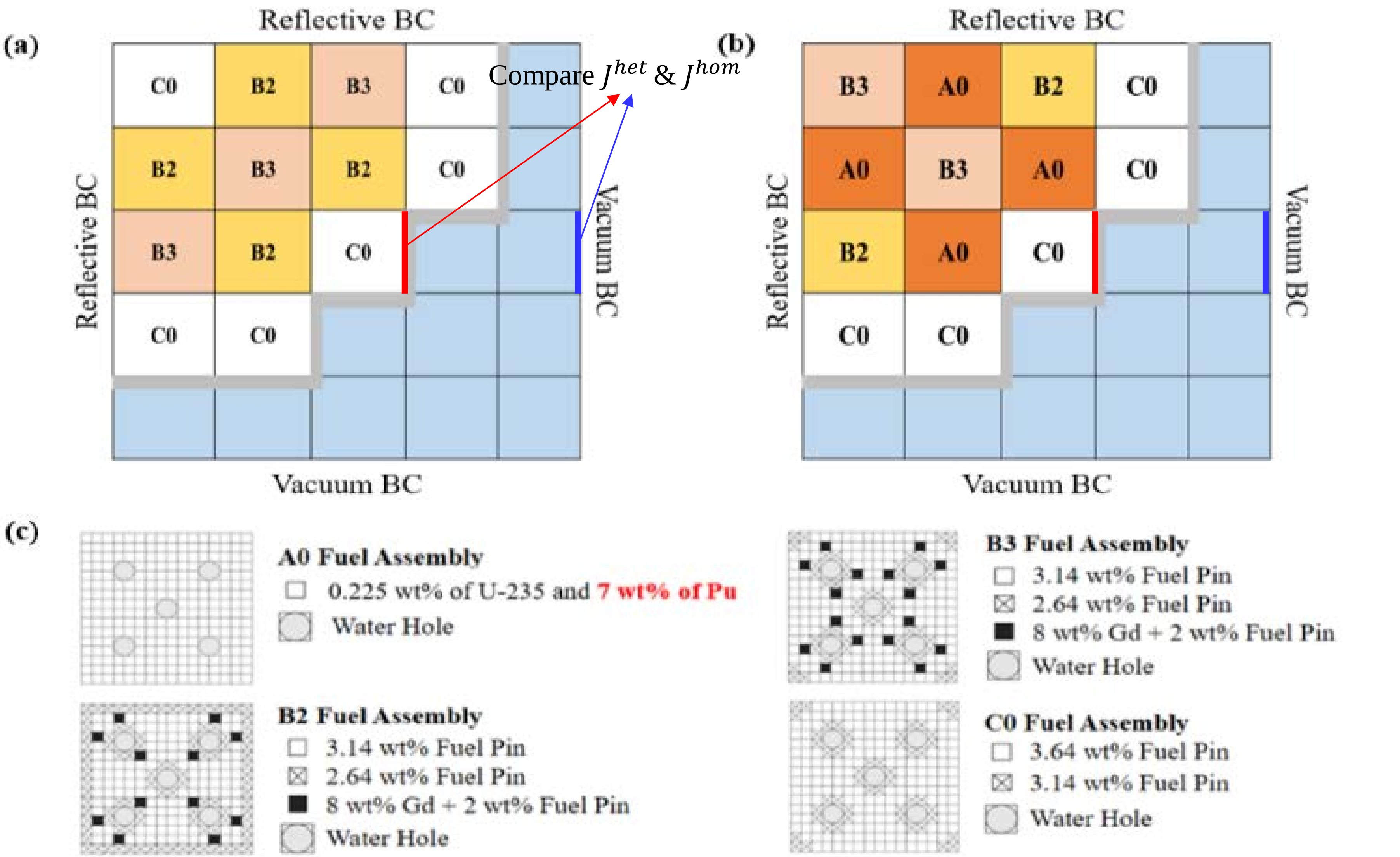
- Allowance of discontinuity between homo-generous surface fluxes
- No iteration is required
- Preserves reaction rate, current, and $k_{eff}$




**Super-Homogenization (SPH) Method**

- Adjustment in the homogeneous cross-section values to preserve reaction rate
- Iteration imperative
- Cannot preserve both current and $k_{eff}$

# Description of 2D SMR Cores

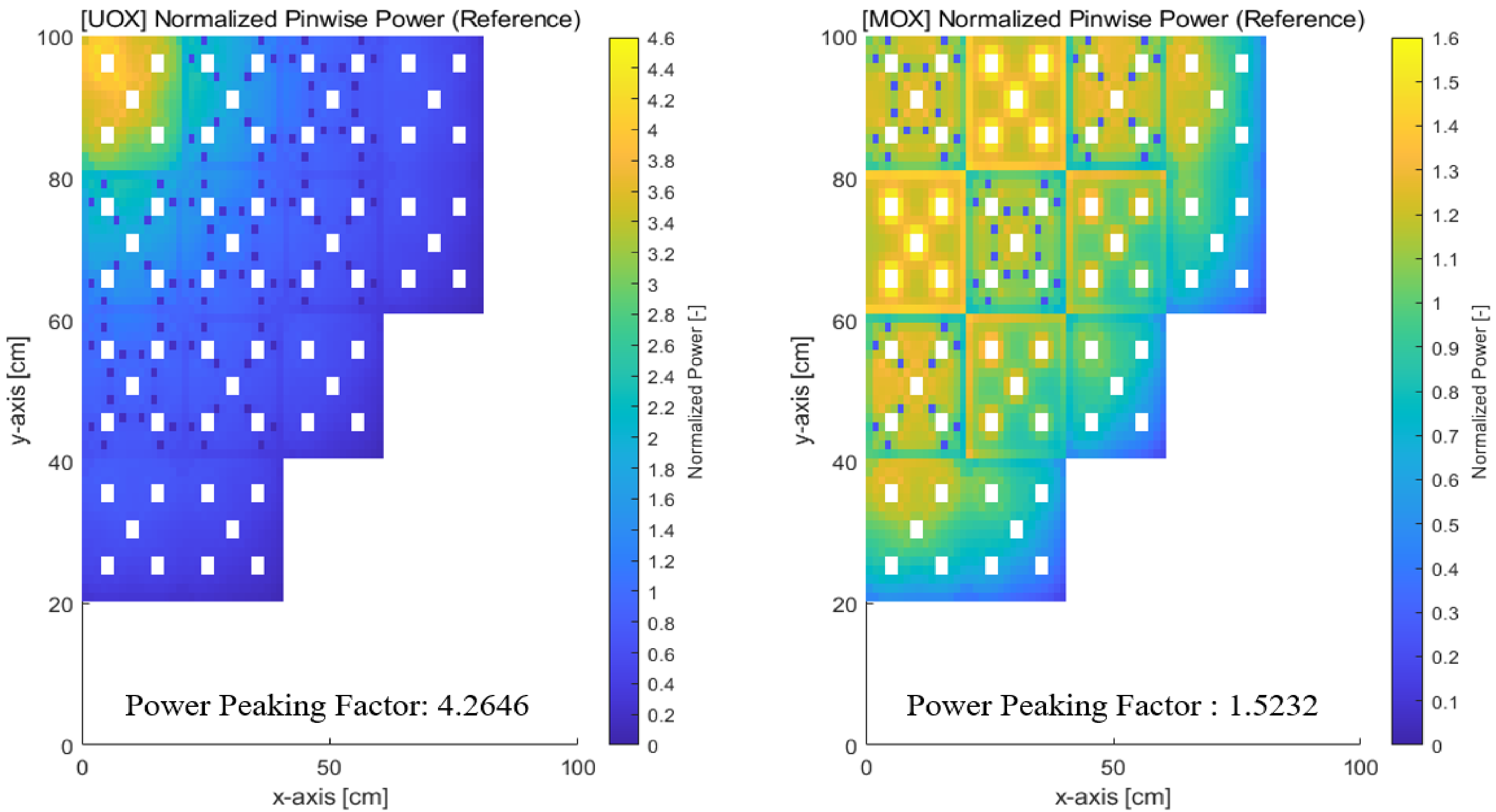



(a) UOX-loaded, (b) MOX-loaded, and (c) description of each fuel assembly

# Reference flux based two-step procedure

**Reference transport calculation result was obtained from DeCART2D**

– When the reference solution is known, both GET and SPH method are capable of preserving the reaction rate for each node, i.e., pinwise power.



# Reference flux based two-step procedure

**Only the GET based approach was able to reconstruct the reference multiplication factor**

– Discrepancy in the $k_{eff}$ value from SPH method became more conspicuous for MOX-core
  - Enhanced leakage due to the inclusion of MOX-fuel and relatively flat power profile

| UOX Loaded Core | | $k_{eff}$ | $\Delta\rho$ [pcm] | Time [s] |
|---|---|---|---|---|
| Ref | | 1.078179 | - | - |
| 2-group | GET | 1.078179 | 0 | 4.25 |
| | SPH | 1.078536 | 31 | 4.27 |
| 4-group | GET | 1.078179 | 0 | 13.92 |
| | SPH | 1.078622 | 38 | 14.58 |
| 7-group | GET | 1.078179 | 0 | 46.77 |
| | SPH | 1.078637 | 39 | 59.30 |

| MOX Loaded Core | | $k_{eff}$ | $\Delta\rho$ [pcm] | Time [s] |
|---|---|---|---|---|
| Ref | | 1.056064 | | - |
| 2-group | GET | 1.056064 | 0 | 3.24 |
| | SPH | 1.056611 | 49 | 3.23 |
| 4-group | GET | 1.056064 | 0 | 9.81 |
| | SPH | 1.056754 | 62 | 10.21 |
| 7-group | GET | 1.056064 | 0 | 42.17 |
| | SPH | 1.056779 | 64 | 32.90 |

Four 3.1 GHz Intel Core i5 processors were parallelly used for the calculation

# Reference flux based two-step procedure

**Comparison between heterogeneous and homogeneous currents**

– Root-mean-square (RMS) errors in the calculated 2-group net currents at red/blue interfaces

| UOX Loaded Core | | GET | SPH |
|---|---|---|---|
| Red | Group1 | 0.00 | 3.69 |
| | Group2 | 0.00 | 0.92 |
| Blue | Group1 | 0.00 | 24.19 |
| | Group2 | 0.00 | 0.18 |

| MOX Loaded Core | | GET | SPH |
|---|---|---|---|
| Red | Group1 | 0.00 | 3.69 |
| | Group2 | 0.00 | 0.87 |
| Blue | Group1 | 0.00 | 24.26 |
| | Group2 | 0.00 | 0.50 |

– GET based result perfectly retains the reference current information

– SPH based result does not exhibit preservation of reference current information

- Higher difference in the periphery of the core configuration
- Corresponds to the mismatch in multiplication factor
  – Larger the difference in $k_{eff}$, i.e., $\Delta\rho$, accordingly higher RMS error in the current
  – Mismatch in $k_{eff}$ concerning the SPH solution could be quite bigger in leakier core configurations

# Lattice Calculation based two-step procedure

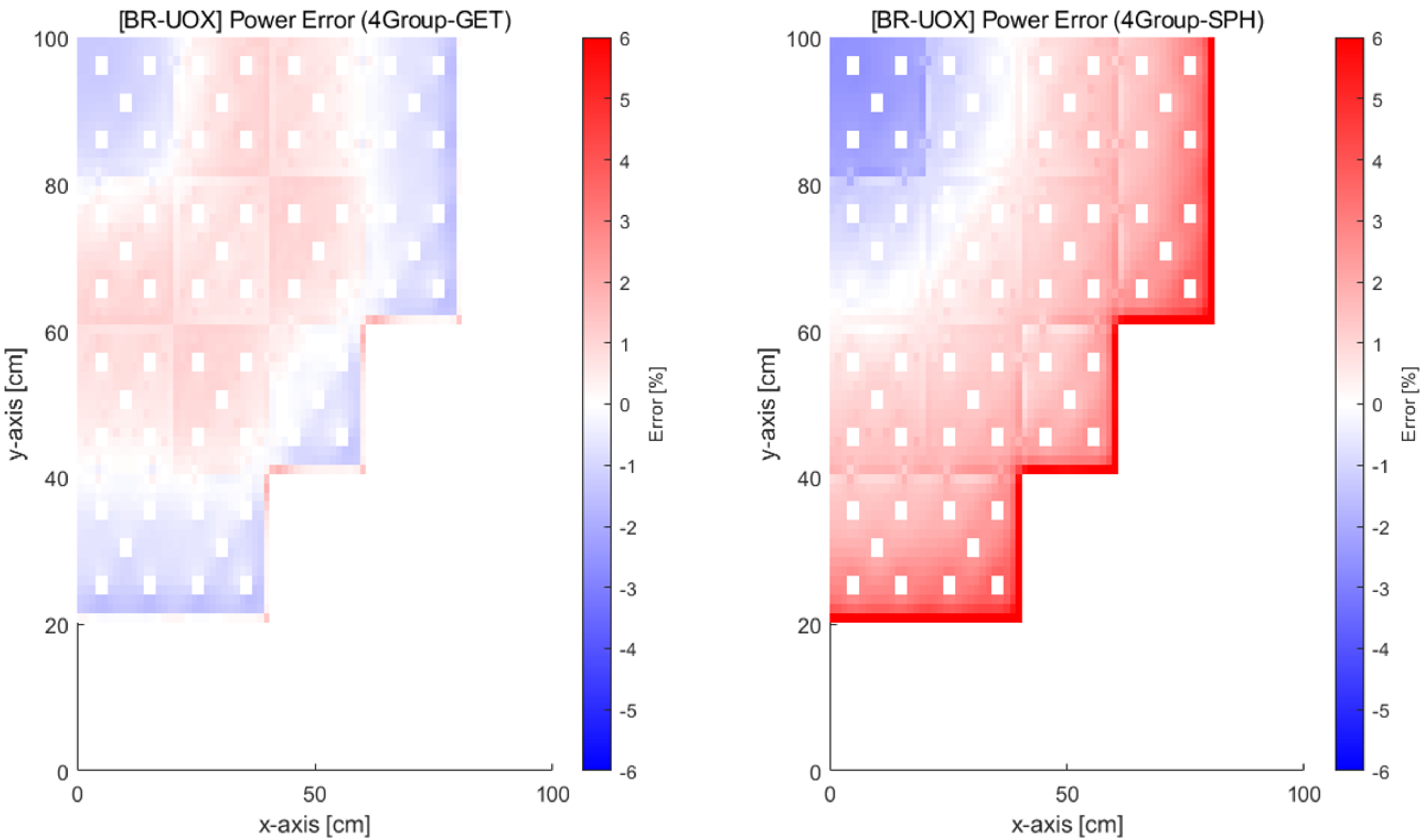
**Transport calculation performed with reflective boundary condition for fuel assemblies**

- Often utilized to acquire assembly discontinuity factor (ADF)
  - However, for pinwise consideration, each pin will have different DF / GET values
- The reference solution based DF / SPH-factor were utilized regarding the reflector regions
- Mimics the pragmatic two-step procedure in the designing procedure of a reactor system

| UOX Loaded Core | | $k_{eff}$ | $\Delta\rho$ [pcm] | Time [s] |
|---|---|---|---|---|
| Ref | | 1.078179 | | - |
| 2-group | GET | 1.079946 | 152 | 4.27 |
| | SPH | 1.080101 | 165 | 4.42 |
| 4-group | GET | 1.077786 | -34 | 13.77 |
| | SPH | 1.078107 | -6 | 14.49 |
| 7-group | GET | 1.077157 | -88 | 46.60 |
| | SPH | 1.077456 | -62 | 81.57 |

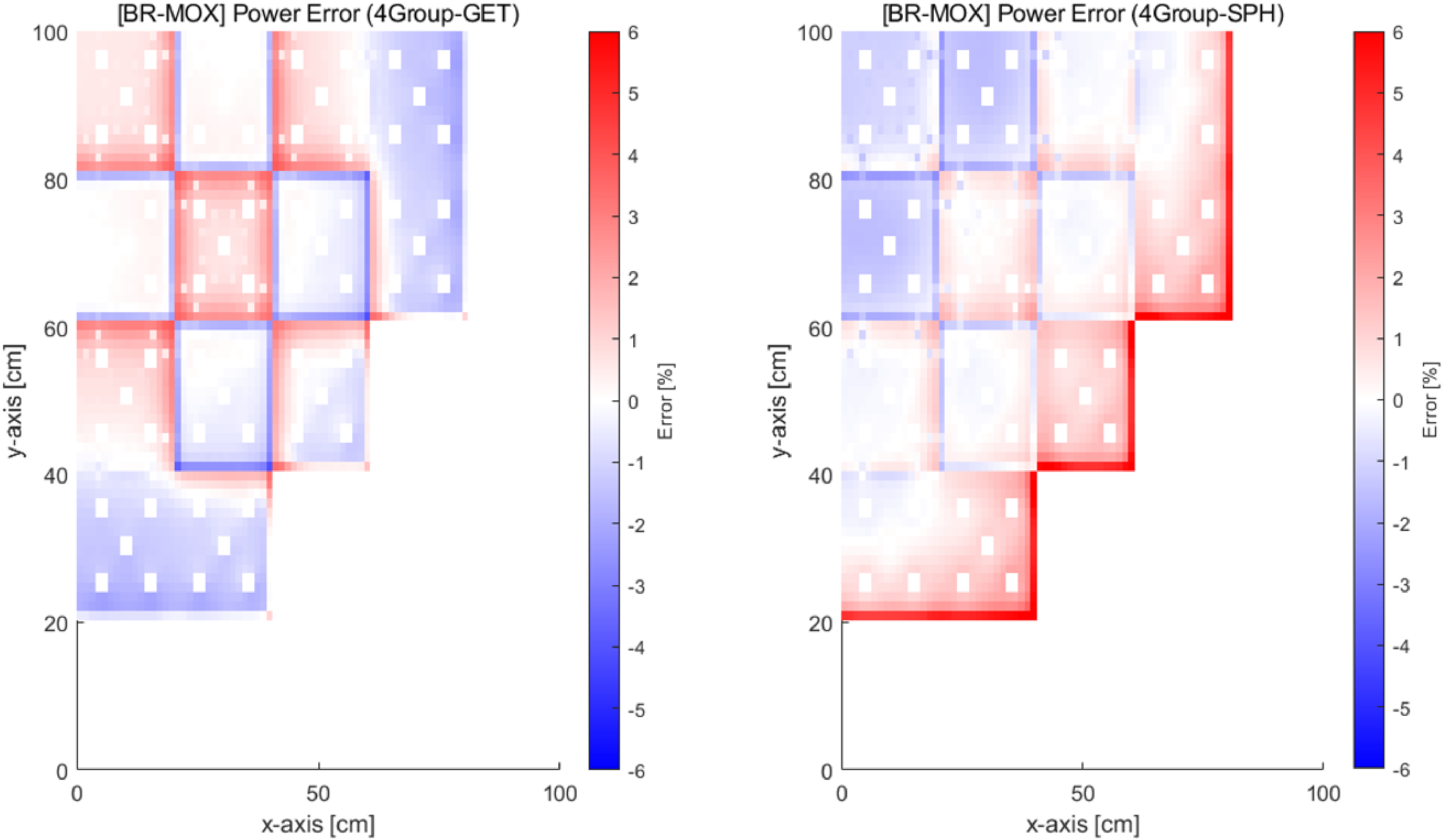
| MOX Loaded Core | | $k_{eff}$ | $\Delta\rho$ [pcm] | Time [s] |
|---|---|---|---|---|
| Ref | | 1.056064 | | - |
| 2-group | GET | 1.058364 | 206 | 3.35 |
| | SPH | 1.059479 | 305 | 3.36 |
| 4-group | GET | 1.057121 | 95 | 10.32 |
| | SPH | 1.058743 | 240 | 10.19 |
| 7-group | GET | 1.055968 | -9 | 31.41 |
| | SPH | 1.054940 | 128 | 41.12 |

# Lattice Calculation based two-step procedure




Pinwise power % error for UOX Loaded Core (4-group)

# Lattice Calculation based two-step procedure




Pinwise power % error for MOX Loaded Core (4-group)

# Lattice Calculation based two-step procedure

**The SPH-based approach returned an accurate $k_{eff}$ value for UOX-loaded core (4-group)**

- Which is even closer than that of reference solution-based two-step procedure
- Should be articulated that such an observation cannot be apprehended as a better performance of SPH-based approach
  - SPH based method intrinsically does not retain the $k_{eff}$ information

**The GET-based approach results in an improved estimation for MOX-loaded case**

- Which also tends to improve with enlarged group numbers, i.e., dwindling $\Delta\rho$

**SPH-based calculation of pinwise power is susceptible to salient error at the periphery of the fuel assemblies that face the reflector regions**

- Further intensification of pinwise power error between adjacent fuel assemblies is observed when the MOX-fuel is embedded in the core
- Can be understood along with the fact that SPH based approach does not take preservation of current into account, and insertion of MOX enhances the leakage of neutrons

**Related to the periphery fuel assembly regions, error from GET-based approach diminishes as number of groups increases (shown in the next slide)**

# Lattice Calculation based two-step procedure

**GET (2-group)**

| | | | | |
|---|---|---|---|---|
| **Max** | 1.89 | -1.08 | -1.78 | 1.86 |
| **RMS** | 1.48 | 0.73 | 1.35 | 1.25 |
| | | -1.84 | -1.15 | -3.41 |
| | | 1.49 | 0.78 | 1.26 |
| | | | -3.67 | |
| | | | 1.18 | |

**SPH (2-group)**

| | | | | |
|---|---|---|---|---|
| **Max** | -1.42 | -1.81 | -1.80 | 4.08 |
| **RMS** | 0.72 | 1.34 | 1.15 | 2.88 |
| | | -2.01 | 1.94 | 4.01 |
| | | 1.57 | 0.75 | 2.52 |
| | | | 3.55 | |
| | | | 1.42 | |

**GET (4-group)**

| | | | | |
|---|---|---|---|---|
| **Max** | -1.26 | 1.15 | 0.99 | -1.56 |
| **RMS** | 0.91 | 0.73 | 0.56 | 0.74 |
| | | 0.94 | 1.16 | 1.84 |
| | | 0.61 | 0.74 | 0.75 |
| | | | 1.81 | |
| | | | 0.60 | |

**SPH (4-group)**

| | | | | |
|---|---|---|---|---|
| **Max** | -2.54 | -1.92 | 2.06 | 7.21 |
| **RMS** | 2.22 | 0.66 | 1.11 | 2.76 |
| | | 1.32 | 4.35 | 10.11 |
| | | 0.56 | 1.55 | 3.68 |
| | | | 8.81 | |
| | | | 2.90 | |

**GET (7-group)**

| | | | | |
|---|---|---|---|---|
| **Max** | -1.72 | 1.12 | 1.09 | -0.98 |
| **RMS** | 1.27 | 0.71 | 0.78 | 0.46 |
| | | 0.99 | 1.28 | -0.93 |
| | | 0.78 | 0.98 | 0.38 |
| | | | 0.82 | |
| | | | 0.30 | |

**SPH (7-group)**

| | | | | |
|---|---|---|---|---|
| **Max** | -2.92 | -1.91 | 2.02 | 5.18 |
| **RMS** | 2.45 | 0.76 | 1.33 | 2.56 |
| | | 1.43 | 3.34 | 7.29 |
| | | 0.74 | 1.71 | 3.30 |
| | | | 6.43 | |
| | | | 2.57 | |

UOX-loaded SMR pin power error (%)

# Lattice Calculation based two-step procedure

**GET (2-group)**

| | | | | |
|---|---|---|---|---|
| **Max** | -2.83 | -3.45 | 3.27 | 1.97 |
| **RMS** | 2.05 | 1.34 | 1.16 | 1.31 |
| | | 3.64 | -3.67 | 4.01 |
| | | 1.31 | 1.02 | 1.93 |
| | | | 4.37 | |
| | | | 2.32 | |

**SPH (2-group)**

| | | | | |
|---|---|---|---|---|
| **Max** | -3.84 | -3.52 | 2.37 | 3.46 |
| **RMS** | 2.92 | 1.66 | 1.16 | 2.10 |
| | | 2.70 | -2.63 | 5.99 |
| | | 1.30 | 1.02 | 2.64 |
| | | | 5.82 | |
| | | | 2.48 | |

**GET (4-group)**

| | | | | |
|---|---|---|---|---|
| **Max** | 3.66 | -2.14 | 3.86 | -2.21 |
| **RMS** | 1.33 | 0.70 | 1.37 | 1.26 |
| | | 3.69 | -4.09 | 3.40 |
| | | 1.81 | 1.19 | 1.12 |
| | | | 3.41 | |
| | | | 1.12 | |

**SPH (4-group)**

| | | | | |
|---|---|---|---|---|
| **Max** | -1.41 | -2.67 | 1.64 | 5.24 |
| **RMS** | 0.85 | 1.38 | 0.47 | 1.40 |
| | | 2.34 | -2.08 | 8.39 |
| | | 0.72 | 0.52 | 2.50 |
| | | | 7.68 | |
| | | | 2.37 | |

**GET (7-group)**

| | | | | |
|---|---|---|---|---|
| **Max** | 2.71 | -2.08 | 2.58 | -1.82 |
| **RMS** | 1.41 | 0.69 | 1.06 | 1.18 |
| | | 2.70 | -4.30 | -1.72 |
| | | 1.34 | 1.20 | 1.00 |
| | | | 1.75 | |
| | | | 0.62 | |

**SPH (7-group)**

| | | | | |
|---|---|---|---|---|
| **Max** | -1.11 | -2.14 | -0.93 | 3.37 |
| **RMS** | 0.43 | 0.89 | 0.18 | 1.04 |
| | | -0.72 | -1.85 | 5.91 |
| | | 0.23 | 0.55 | 1.77 |
| | | | 5.29 | |
| | | | 1.37 | |

MOX-loaded SMR pin power error (%)

# Conclusions & Future Works

1. **For the preservation of reaction rate, GET and SPH method both successfully accomplished such a goal when the reference solution was employed**
   - However, only the GET successfully reproduces the multiplication factor
   - Theoretically, one can perfectly convey the high-fidelity information to the diffusion calculation via the GET approach, whereas SPH method fails to do so

2. **For practical two-step approach, pinwise power error became more conspicuous for following conditions when the SPH method was exploited**
   - The very fringe of the fuel, which faces the reflector
   - Between the MOX/UOX assemblies

3. **An increased number of energy groups effectively curtailed the error at the fringe for practical two-step procedure when GET-based approach was taken**

4. **Similar analysis will be performed for more realistic core configuration, including thermal hydraulics feedback consideration**

# Thank you for your attendance

# Q&A