Derivation of Limit Stress Curve Equation

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1. Introduction

For stress analysis, limit analysis is a special case of plastic analysis in which the material is assumed to be ideally plastic (nonstrain-hardening) [1]. Using this assumption, the stress limit was developed by providing a margin on the actual limit load stress curve for combined tension and bending on a rectangular section [2]. In this study, the limit stress curve equation is derived and the results are compared to reference.

2. Methods and Results

Figure 1 shows the limit stress curve for combined tension and bending stresses.

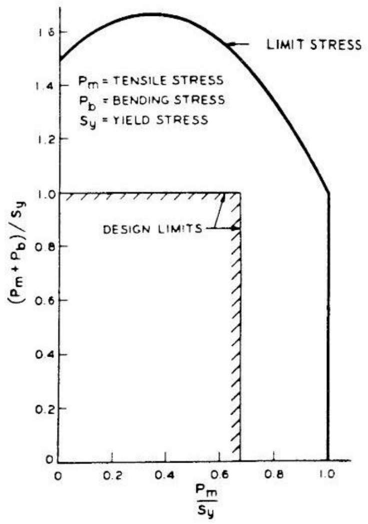

Fig. 1. Limit stress for combined tension and bending (rectangular section). (ASME, “Criteria,” Courtesy of ASME)

On a rectangular section, the limit stress curve is derived from combined tension and bending stresses as follows. Figure 2 shows the decomposition of stress components.

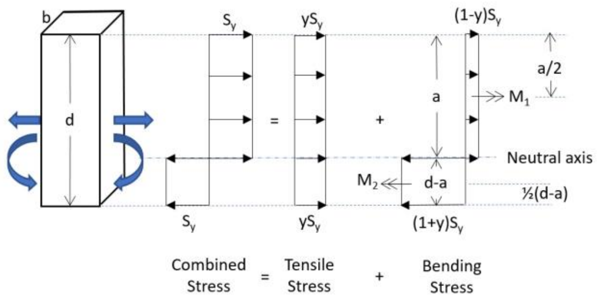

Fig. 2. Decomposition of stress components

A plastic hinge is an idealized concept used in Limit Analysis. In a beam or a frame, a plastic hinge is formed at the point where the moment, shear, and axial force lie on the yield interaction surface [1]. In the perfectly plastic state, the cross section of the neutral axis is divided into two equal areas.

$$(1-y)S_y \cdot a = (1+y)S_y \cdot (d-a) \tag{1}$$

$$\therefore a = \frac{d}{2}(1+y) \tag{2}$$

$$M_1 = (1-y)S_y \cdot ab\frac{a}{2} \tag{3}$$

$$M_2 = (1+y)S_y(d-a)\cdot b\frac{1}{2}(d-a) \tag{4}$$

Moment $M = M_1 + M_2$

$$= \frac{1}{2}S_y b\left\{(1-y)a^2 + (1+y)(d-a)^2\right\} \tag{5}$$

Substituting Eq. (2) in (5) gives

$$M = \frac{1}{4}Sybd^2(1-y^2) \tag{6}$$

Tensile stress $P_m = yS_y, y = \dfrac{P_m}{S_y}$ (7)

Bending stress $P_b = \dfrac{M}{(bd^2)/6}$ (8)

Substituting Eq. (8) in (6) gives

$$P_b = \frac{3}{2}S_y(1-y^2) \tag{9}$$

For the axes form of the limit stress curve, Eq. (7) and (9) gives

$$\frac{(P_m+P_b)}{S_y} = \frac{P_m}{S_y} + \frac{3}{2}\left\{1-(\frac{P_m}{S_y})^2\right\} \tag{10}$$

$$\therefore \frac{(P_m+P_b)}{S_y} = -\frac{3}{2}(\frac{P_m}{S_y}-\frac{1}{3})^2 + \frac{5}{3} \tag{11}$$

Figure 3 shows indicated points from Eq. (11).

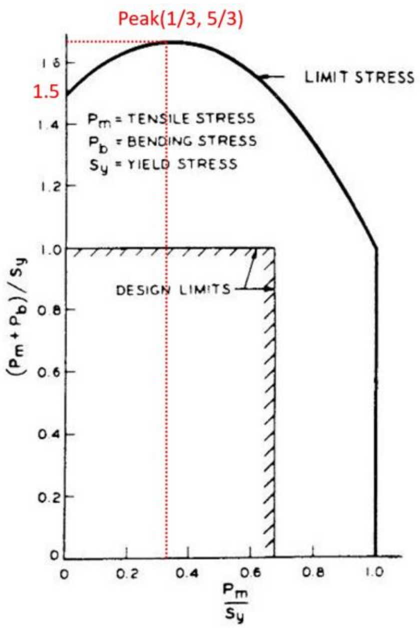

Fig. 3. Indicated points from derived equation

From this equation, we can understand the limit stress curve and analogize the whole out of a part. Figure 3 indicates that the theoretical limit stress varies from 1.5 S_y with no membrane stress present to 1.0 S_y with only membrane stress present. It should be noted that the theoretical limit stress peaks at approximately 1.67 (=5/3) with a combination of bending and membrane stress when the membrane stress is approximately (1/3)S_y [2].

3. Conclusions

In this study, the limit stress curve equation is derived from moment equilibrium and the results are compared to reference. From the results, we can figure out the meaning and purpose of the limit stress. This theoretical limit could be expanded to Class 2 and 3 Components.

ACKNOWLEDGEMENTS

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REFERENCES

[1] ASME Boiler and Pressure Vessel Code, Section III, Subsection NB, Class 1 Components, American Society of Mechanical Engineers, 2004.
[2] K. R. Rao, Companion Guide to the ASME Boiler & Pressure Vessel Code, ASME Press, 2009.