# Study on APEC-corrected Macroscopic Depletion in 2-D Nodal Analysis

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### **1. Introduction**

The conventional two-step method aided by  $B_1$ leakage correction has been considered as a standard tool for a design of the commercial pressurized water reactor (PWR) due to its acceptable accuracy [1, 2]. However, the trend of modern reactor physics is to avoid using B<sub>1</sub> leakage correction due to the limitation that it cannot take into account actual leakage through the surfaces [3]. Recently, the albedo-corrected parameterized equivalence constants (APEC) method was introduced to correct the homogenized group constants (HGCs), i.e., cross-sections (XSs) and discontinuity factors (DFs), by reflecting actual leakage information via normalized parameters [4, 5]. It was proven that the APEC method could improve the nodal equivalence not only in the case of the beginning of the cycle (BOC), but also microscopic depletion calculation [6]. It was also found that the previous APEC leakage correction may not be good enough, mainly caused by DF modeling, so the improved discontinuity factor modeling was introduced and validated at BOC condition [7, 8]. In this study, the macroscopic depletion in two-dimensional nodal analysis corrected by the burnup-independent APEC method is discussed by analyzing UOX- and partially MOX- loaded SMR cores.

### 2. APEC-corrected Macroscopic Depletion Analysis

The reference solution of the SMR and the lattice calculation of each fuel assembly (FA) were calculated by DeCART2D code [9]. The APEC-corrected macroscopic depletion based on the two-step method was performed by in-house 2-group, 2D/3D 1x1 NEM nodal code that has the algorithm as shown in Fig. 1.

### 2.1 Macroscopic Depletion

The in-house code can handle the equilibrium and transient Xe/Sm calculation. In this study, the transient Xe calculation is only applied to minimize the error that occurred by the simplified decay chain model of Sm. The simplified transient Xe number density can be calculated by Eq. (1) and (2).

$$N_{I}(t_{n}+\Delta t) = N_{I}(t_{n}) + \Delta t \left( \gamma_{I} \sum_{g=1}^{2} \Sigma_{f,g}(t_{n}) \phi_{g}(t_{n}) - \lambda_{I} N_{I}(t_{n}) \right)$$
(1),

$$N_{Xe}(t_n + \Delta t) = N_{Xe}(t_n) + \Delta t (\lambda_I N_I(t_n) + \gamma_{Xe} \sum_{g=1}^{2} \Sigma_{f,g}(t_n) \phi_g(t_n)$$
(2)

$$-\lambda_{\chi_{e}}N_{\chi_{e}}(t_{n}) - \sum_{g=1}^{2} \sigma_{\chi_{e,a,g}}(t_{n})N_{\chi_{e}}(t_{n})\phi_{g}(t_{n}))$$



Fig. 1. Algorithm scheme of macroscopic depletion 2D/3D nodal analysis.

where,

 $N_i$ :nuclei number density of isotope i,

 $\gamma_i$  : effective yield of isotope i,

 $\lambda_i$  : decay constant of isotope i.

The approximated macroscopic depletion calculation was implemented, as shown in Eq. (3).

$$\Delta B_i = \Delta B_c \frac{P_i}{G_i} / \frac{P_c}{G_c}$$
(3),

where,

 $\Delta B_{iorc}$ :  $i_{th}FA$  burnup or core average burnup increment in one step,

 $G_{iorc}$ : the heavy metal loading in  $i_{th}$  region or total heavy metal loading in the core,  $P_{iorc}$ : power in  $i_{th}$  region or total power in the core.

#### 2.2 APEC Functions

The APEC XS and 1x1 NEM based DF functions were defined as shown in Ref. [7]. The main principle of the APEC method is to predetermine the coefficients of the APEC XS and DF functions by multiple linear regression using the results of the color-set calculation. HGCs can then be corrected in the nodal analysis by taking into account actual leakage via normalized parameters such as current to flux ratio (CFR).

## 3. Numerical Results

3.1 UOX- and Partially MOX-loaded SMR Benchmark Problem



Fig. 2. Core configuration of UOX- and partially MOXloaded SMR.

As shown in Fig. 2, the UOX- and partially MOXloaded SMR benchmark problems were set up to analyze the impact of the APEC leakage correction in the macroscopic depletion calculation. In this analysis, the macroscopic XSs were functionalized in terms of burnup by FA-wise lattice calculation, and the temperature is assumed to be constant.

### 3.2 Color-set Models for APEC Functions



(c) L-shape color-set type 1 (d) L-shape color-set type 2

Fig. 3. Color-set models for constructing APEC functions.

Table I: List of Color-set models

Color-set Model	Combination of FAs			
Checkerboard	(B2,B3,C0), (B3,C0,B2), (C0,B3,B2), (A0,B2,B3), (B3,C0,A0), (C0,A0,B2)			
L-Shape Type1	(B2,B3,C0), (B2,C0,B3), (B3,C0,B2), (B3,B2,C0), (C0,B2,B3), (C0,B3,B2) (A0,B2,C0), (A0,C0,B3), (B2,A0,C0), (B2, B3, A0)			
L-Shape Type2	(B2,B3,C0), (B2,C0,B3), (B3,C0,B2), (B3,B2,C0), (C0,B2,B3), (C0,B3,B2) (B2,B3,B2), (C0,B2,C0), (B3,C0,B3), (C0,B2,A0), (A0,B3,C0), (B2,A0,B3), (B3,B2,A0),(A0,C0,A0), (C0,A0,C0), (B3,A0,B3)			

The APEC XS and DF functions were constructed by solving color-set calculation, as shown in Fig. 3 and Table I. Note that the way to use of combination is different depending on the benchmark problem. Specifically, the combinations highlighted in black color are used for APEC functions of the UOX-loaded SMR core, and additional combinations highlighted in red color are used for APEC functions of the MOX-loaded SMR core. The reference HGCs at the BOC of the reflector region were fixed during the macroscopic depletion calculation and the APEC leakage correction for the baffle-reflector region was not applied.

#### 3.3 Results of APEC-corrected Macroscopic Depletion

The evolution of the infinite multiplication factors was evaluated, as shown in Fig. 4. The main features of each FA are determined depending on the existence of burnable absorbers (BAs). In the B2 and B3 types, the evolution of the infinite multiplication factor has a peak point, when the BAs are burnt out. In this regard, it may be necessary to apply the burnup-dependent APEC leakage correction.



Fig. 4. Evolution of infinite multiplication factor of each FA.



Fig. 5. Effective multiplication factor and reactivity error (%) of UOX-loaded SMR core.



Fig. 6. Effective multiplication factor and reactivity error (%) of partially MOX-loaded SMR core.

The macroscopic depletion results showed that the APEC leakage correction improves the nodal equivalence at the BOC in terms of reactivity error and relative error in assembly power, and tries to keep the enhanced accuracy during the macroscopic depletion calculation. In the case of the effective multiplication factor, as shown in Fig 5 and 6, it is shown that the accuracy can be improved in all of the burnup points where the APEC leakage correction is applied. Note that the tendency of the reactivity error of the APECcorrected NEM is similar to that of the simplified equivalence theory (SET) based NEM, but the actual reactivity error of the former is quite close to zero compared to that of the latter. It means that the burnupindependent APEC functions generated by color-set at the BOC condition could be appropriate for conducting macroscopic depletion calculation.

In both cases, it is observed that the reactivity errors are suddenly increased around 9 MWD/kgHM due to the effect generated by burning out the BAs in the B2 and B3 FAs. It is expected that the accuracy of the reactivity error could be improved by the burnupdependent APEC functions.

HGCs	k <sub>eff</sub>	$\Delta \rho$	$RRMS^a$	$Min.^{b}$	$Max.^{c}$	
		(pcm)	(%)	(%)	(70)	
Ref.	1.042409	0.00 MWD/kgHM				
$\operatorname{GET}^d$	1.042409	-0.02	0.003	-0.006	0.002	
SET <sup>e</sup>	1.044532	195.01	1.353	-2.419	1.625	
APEC <sup>f</sup>	1.042393	-1.52	0.396	-0.759	0.676	
Ref.	1.003631	2.00 MWD/kgHM				
GET	1.003663	3.14	0.120	-0.123	0.224	
SET	1.005950	229.68	1.308	-2.249	1.645	
APEC	1.004274	63.78	0.380	-0.630	0.617	
Ref.	1.000213	4.50 MWD/kgHM				
GET	1.000224	1.10	0.192	-0.198	0.353	
SET	1.002396	217.75	1.537	-2.405	2.115	
APEC	1.001100	88.60	0.608	-0.855	0.845	
Ref.	1.001227	7.50 MWD/kgHM				
GET	1.001064	-16.23	0.305	-0.397	0.546	
SET	1.002745	151.22	1.944	-2.853	2.843	
APEC	1.001955	72.56	1.096	-1.764	1.654	
Ref.	0.962057	15.00 MWD/kgHM				
GET	0.962393	36.28	0.226	-0.286	0.456	
SET	0.967287	561.98	0.582	-0.712	0.812	
APEC	0.965341	353.58	0.564	-0.995	0.673	

Table II: Depletion Results of UOX-loaded SMR Core

a: Relative Root Mean Square Error of Assembly Power (%),

b: Minimum Relative Error in Assembly power (%),

c: Maximum Relative Error in Assembly power (%),

d: Generalized Equivalence Theory (GET: Ref. XS / Ref. DF),

e: Simplified Equivalence Theory (SET: FWC<sup>g</sup> / ADF<sup>h</sup>),

f: APEC Leakage Correction (APEC: APEC XS / APEC DF),

g: Flux-weighted Constants, h: Assembly-wise DF.

Table III: Depletion Results of MOX-loaded SMR Core

HGCs	k <sub>eff</sub>	$\Delta  ho$	RRMS <sup>a</sup>	Min. <sup>b</sup>	Max. <sup>c</sup>	
		(pcm)	(%)	(%)	(%)	
Ref.	1.053803	0.00 MWD/kgHM				
$\operatorname{GET}^d$	1.053804	0.09	0.004	-0.006	0.005	
SET <sup>e</sup>	1.057639	344.19	0.991	-1.527	1.841	
APEC <sup>f</sup>	1.053702	-9.06	0.333	-0.697	0.435	
Ref.	1.009993	2.00 MWD/kgHM				
GET	1.009940	-5.17	0.148	-0.160	0.236	
SET	1.013913	382.82	1.008	-1.485	1.657	
APEC	1.010337	33.68	0.425	-0.893	0.660	
Ref.	1.000835	4.50 MWD/kgHM				
GET	1.000805	-3.02	0.178	-0.176	0.310	
SET	1.004749	389.23	1.298	-2.200	1.784	
APEC	1.001351	51.50	0.901	-1.978	1.184	
Ref.	0.992845	7.50 MWD/kgHM				
GET	0.992821	-2.43	0.232	-0.310	0.397	
SET	0.996476	367.04	1.747	-3.364	2.136	
APEC	0.993089	24.77	1.663	-3.601	1.958	
Ref.	0.951762	15.00 MWD/kgHM				
GET	0.952230	51.62	0.180	-0.113	0.358	
SET	0.957359	614.27	0.884	-1.375	1.320	
APEC	0.954193	267.63	1.680	-3.476	2.033	



Fig. 7. Relative power error (%) of each FA in UOX-loaded SMR core.



Fig. 8. Relative power error (%) of each FA in partially MOX-loaded SMR core.

The results showed that the APEC leakage correction could improve the accuracy of the assembly-wise power, as shown in Table II and III. In the case of the partially MOX-loaded SMR core, the improvement of the accuracy by APEC leakage correction is substantial. It implies that the burnup-independent APEC leakage correction might be appropriate even for the most complicated problem, partially MOX-loaded SMR core.

The specific relative power errors depending on the position of FAs, as shown in Fig. 2, are categorized by the position index as shown in Fig. 7 and 8. The results showed that it is not guaranteed to correct HGCs at the high burnup appropriately through the burnup-independent APEC correction. It also implies that the burnup-dependent APEC leakage correction might be needed to correct the HGCs properly, especially in the reactor core including BAs.

### 4. Conclusions

The burnup-independent APEC leakage correction has been performed in the macroscopic depletion based on 1x1 NEM analysis. It is demonstrated that the burnup-independent APEC leakage correction can conditionally improve the nodal equivalence in terms of reactivity error and relative error in assembly-wise power. It is expected that the burnup-dependent APEC leakage correction might be needed to take into account the spectrum change due to the combustion of the BAs. The burnup-independent APEC correction based on the nxn NEM or burnup correction model will be applied to reflect the effect that occurred by the burnup gradient in the macroscopic depletion analysis. The burnupdependent APEC leakage correction will also be demonstrated in the near future.

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